Risky Sex in a Risky World:
Sexual behavior in a HIV/AIDS environment

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Abstract
In spite of increased awareness of HIV/AIDS and the lack of retroviral drugs, unprotected casual sex is still widespread in many HIV infected countries. In this paper, a two-period model for sexual decisions under uncertainty is developed. The results suggest that uncertainty of future health may be an important factor driving unsafe sex practices in countries in which access to HIV drugs is limited. Furthermore, the more efficient HIV treatment becomes, the more important will health related interventions become. The results support the empirical finding of a weak link between sexual behavior, HIV frequency and HIV knowledge in poor countries, and suggest that AIDS policy needs to be calibrated to fit within different social contexts.

Key words: HIV/AIDS, sexual behavior, uncertainty, risk aversion, health risk

JEL - Classification: D81, D91, I10

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1. Introduction

The questions asked in this paper relate to how observations of seemingly irrational sexual behavior, such as unprotected casual sex in areas and populations with high HIV frequencies, fits with existing knowledge on human incentives under uncertainty of future health prospects. The results indicate that the degree of uncertainty of future health, as well as the availability and quality of HIV treatment, needs to be incorporated in the analysis in order to understand risky sexual behavior in the presence of HIV.

In 2007, 2.7 million new HIV infections occurred globally. In the same year, 2 million individuals died of AIDS related diseases, and an estimated 33 million were still living with HIV. Poor regions in general, and sub-Saharan Africa in particular, continues to carry a disproportionally heavy burden in terms of HIV victims; 35 percent of all HIV infections, and 38 percent of the AIDS deaths in 2007 occurred in sub-Saharan Africa (UNAIDS, 2008; Rao et al., 2006). In recent years, the development of antiretroviral (ARV) drugs, and the combination of different ARV drugs into Highly Active Antiretroviral Treatment (HAART), has enabled HIV positive individuals to lead a relatively healthy and long life. However, although there have been important breakthroughs, HAART still only reduces mortality with 50-80 percent and ARV treatment is still associated with severe side effects (Lakdawalla, et. al., 2006; Mechoulan, 2007). In addition, although the distribution of antiretroviral drugs have improved substantially during the past decades, a large share of HIV positive individuals, especially in poor countries with soaring HIV epidemics, still have no access to HIV treatment.¹

The inefficiency of HIV/AIDS prevention programs are still puzzling researchers and policy makers. Although some signs of increased condom use and reduced number of sexual partners, the lack of behavioral change in high risk groups effectively means that the there are almost no signs of significant reductions in HIV rates² (UNAIDS, 2006; Caldwell,

¹ The percentage of pregnant women receiving antiretroviral treatment has improved substantially over that past couple of years. In many countries with reports of coverage rates, numbers are as high as 80 percent. However, although the trend in antiretroviral treatment distribution in poor countries has been positive, the number of new HIV infections each year outnumber the increase in individuals on ARVs, by 2.5 to 1 (UNAIDS, 2008).
² Lagarde et al. (2001) estimate condom coverage at 27-31 percent for men and 11-17 percent for women in a number of highly affected cities in sub-Saharan Africa. In Zambia the share of unprotected sexual acts with non-cohabiting partners actually increased during the late 1990's.
1999; Bloom et al., 2000; Hearst et al., 2004; Mwaluko et al., 2003). Indeed, although the transmission rate of HIV during vaginal intercourse is substantially higher in sub-Saharan Africa than elsewhere (c.f. Oster, 2005; Gray et al., 2001; Quinn et al., 2000), and although HIV prevalence rates can be as high as 70 percent among commercial sex workers, the price of unprotected sex has been found to be as high as four times that of safe sex (Abdool et al., 1995; Audrey et al., 2000; Morison et al., 2001; UNAIDS, 2008). Admittedly, myths and misinformation about HIV still prevail (c.f. Swart-Kruger and Richter, 1997), but HIV/AIDS knowledge is not necessarily inadequate in communities with unsafe sex practices and high HIV prevalence rates (cf. Campbell, 1997; Pettifor et al. 2000).

In this paper, I argue that we, in order to understand the persistent practice of unprotected casual sex in the presence of HIV, need to consider contextual uncertainty about the future. The tangible presence of risk and uncertainty of future prospects is a common feature among many HIV susceptible populations such as migratory workers, refugees, miners, military personnel, intravenous drug users and commercial sex workers (c.f. UNAIDS & IOM 2003; Rhodes, 1997; Wojcicki, & Malala, 2001). Indeed, it has been shown that HIV frequencies soar in countries with malfunctioning institutions and civic unrest (Benz, 2005). The basic intuition behind the theory, presented below, is that for people living under harsh conditions, HIV constitutes a significant threat but it is not dominant in daily life. In other words, if private actions only determine future health status to a minor extent, the gain in abstaining risky activities with potential future health costs is low.

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3 The high transmission rate of HIV in sub-Saharan Africa has been suggested to be a consequence of the high frequency of other untreated sexually transmitted diseases (Oster, 2005), and the practice of so called dry sex, where the female genitalia is tightened through use of herbs and synthetics widespread (cf. Campbell, 1997; Ferry et al., 2001; Morison et al., 2001; Wojcicki and Malala, 2001).

4 Demographic and Health Surveys (DHS) show that most individuals have a reasonably high level of HIV/AIDS awareness, and studies on South African miners (one of the most AIDS susceptible groups) suggest that risky sex prevails in spite of free access to condoms and a sufficient knowledge about HIV (Campbell, 1997; South African Advisory Panel Report, 2001:75).

5 According to the South African Chamber of Mines, mine workers face a 2.9 percent chance of being killed in a work related accident and a 42 percent chance of suffering a reportable injury in a 20-year working life (1984-1993, South African Chamber of Mines cited in Campbell, 1997). In Thailand soldiers have been found to visit prostitutes to a higher extent than other men and to use condoms to a lesser extent than other Thai men (Van Ladingham et al. 1993).

6 In Sierra Leone, estimated HIV prevalence surmounted to 60-70 percent among soldiers in 2002 and HIV frequencies among prostitutes increased from 26.7 percent to 70.6 during the civil war (UNAIDS/WHO: AIDS Epidemic Update 2002: 35).
Although sexual behavior is not a conventional subject within economic research, expected utility theory has the potential to provide an understanding to the mechanisms that drive both destructive and protective behavior in the presence of HIV. For example, in their seminal paper on the subject, Philipson and Posner (1993) were able to explain why HIV frequencies actually remain at relatively moderate levels in many countries. By explicitly modeling the marginal costs and benefits of unsafe sex in a sexual bargaining game, Philipson and Posner showed that, as the HIV frequency increases so does the marginal cost of engaging in unsafe sex for HIV negative individuals. Hence, at sufficiently high HIV frequencies the marginal cost of unsafe sex will surmount the marginal benefit and the epidemic will therefore subside (see also Schroeder and Rojas, 2002). However, although the predictions of models for sexual bargaining fit relatively well with the situation in the United States and Europe, they cannot fully explain the persistent presence of unprotected casual sex in regions with extreme prevalence rates of HIV. Philipson and Posner (1993) have also been criticized for not including asymmetric power in the bargaining game (see Christensen, 1998). Philipson and Posner (1995) suggest that the prevailing differences in HIV prevalence rates between rich and poor countries can be explained in terms of an inelastic supply of sexual services among commercial sex workers (CSW’s) in poor countries. A similar approach is taken by Gertler et al. (2005). However, the higher price for unsafe sex in poor countries is, in their paper, explained in terms of compensation for the risk taken by the seller of the sexual service (that is, quite to opposite of the assumption in Philipson and Posner). However, neither the fear of attracting HIV by the buyer of the sexual service, nor the asymmetric power distribution between CSW and client, is formally analyzed in Gertler et al., (2005) or in Philipson and Posner (1995).

The theoretical analysis presented in this paper relates most closely to a number of economic papers that incorporates the effect of the physical and social context on health.

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7 The engagement in risky sexual activities is naturally likely to be affected by other things than contextual uncertainty, among those the presence of hyperbolic discounting and of destructive social norms. Hyperbolic discounting of the future gives rise to myopic behavior and is likely to exist in all cultures. However, there is little evidence that sub-Saharan Africans should discount hyperbolically to a larger extent than, for example, Europeans. As for social norms, sexual preferences and gender ideals clearly differs geographically and between cultures, thus supporting the idea that social norms could hold important information for explaining risky sexual behavior. However, I will not focus on norms in this paper (this is done in a forthcoming paper). To maintain simplicity I instead turn to the economic theory of behavior under uncertainty. According to economic theory, the presence of future uncertainty reduces the value of expected future utility and thus makes it rational for a risk-averse individual to act shortsighted.

8 Early epidemiological models predicted that HIV rates would explode as the pool of potential virus hosts increased and spread the virus (Philipson and Posner, 1993)
related behavior (Dow et al. 1999; Benz, 2005; Dinkelman et al. 2007; Oster, 2005, 2007a, 2007b). Dow et al. (1999) suggest that health oriented policy in poor countries may have positive spill-over effects on private health seeking behavior. Interventions, such as immunization programs, reduce pressing mortality risks and thereby increase the marginal benefit of private health investments. Thus, health interventions that seem cost-inefficient when only direct effects are evaluated may well prove cost-efficient if indirect effects on health investments are considered. Benz (2005) follows the same line of argument and suggests that the correlation between high HIV prevalence and experiences of civil conflict can at least partly be explained by of the increase in uncertainty due to crumbling civic institutions. Dinkelman et al. (2007) are not able to show a direct link between economic shocks and sexual risk taking in South Africa, but show that youth from poor households have an earlier sexual debut and use condoms less frequently than young adults from richer households. In the paper most closely related to the one presented here, Oster’s (2007a) suggests that a reduction in expected life length or future earnings reduces the expected cost of risky sexual behavior. Using cross-sectional data on nine African countries\(^9\), Oster finds that, while HIV knowledge and HIV frequency does not have a significant effect on the number of sex partners, income levels and expected life length do affect sexual risk taking.\(^10\) Oster (2007b) further finds that an increase in the exports of a country in sub-Saharan Africa is closely correlated with HIV incidence in that country.\(^11\) The intuition behind this result is a combination of the high degree of sexual risk taking among transit populations (such as truck drivers) and the increase in the presence of these transit populations during export booms.

The above mentioned papers have contributed with important insights on potentials and pitfalls for HIV prevention. However, none of the above studies have theoretically considered the link between future uncertainty and risk on the one hand, and sexual behavior on the other. As suggested by Oster (2007b), some populations, such as truck drivers, seem to be important for the spread of HIV. The model presented below can be seen as a complement to Oster’s contribution in that it provides an analysis of the reason

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\(^9\) Demographic and Health data on Benin, Burkina Faso, Ethiopia, Ghana, Kenya, Malawi, Mali, Namibia and Zimbabwe

\(^10\) In order to deal with the obvious endogeneity between life expectancy and HIV prevalence, malaria frequency and maternal mortality is used as a proxy for life expectancy. Oster further acknowledges the link between sexual promiscuity and HIV prevalence, and therefore uses the distance to the (believed) origin of the HIV virus (the Democratic Republic of Congo) as an instrument for HIV frequency.

\(^11\) In fact, the relationship may be strong enough to explain the drastic decline in HIV rates in Uganda during the 1990s.
behind risky sexual behavior in these populations. The main contribution of my model is the explicit analysis of health related uncertainty of the future. My hypothesis is that health related uncertainty (unrelated to the individual’s own behavior) by itself holds an important key for explaining risky sex. Admittedly, the future is always, to some extent, uncertain. Indeed, few individuals know with certainty that they will be alive and healthy in the future. However, in poor countries the future is likely to be both more uncertain and less bright than in rich countries, due to for example malfunctioning economic and political systems. In such an environment, investments in health are likely to be more risky than elsewhere. It may thus be optimal for an individual to focus on the present instead of investing in the future. Hence, although individuals in poor countries appear to act myopic, they are acting completely rational within the context of uncertainty.

2. Theoretical approach

The theoretical model in this paper relates to the question of how an uncertain future affects sexual choices in the present. In order to retrieve qualitative results the analysis is kept simple. The model is based on the lifetime utility of an individual (man), who faces a trade-off between sexual pleasure and future health. Although a simplification, the model does produce interesting results, and it is possibly a relatively good approximation of the reality in, for example, sub-Saharan Africa. The assumption that the individual is a man, and that this man can make an independent choice in a sexual relation, is based on the dominating position of men in many poor countries. The model can be seen as an analysis of a price-taking man facing a supply of commercial sexual services.12

The theoretical approach used in this paper is loosely based on a modeling idea developed by Katz et al. (1982), (see also Brock et al. 1982; Johansson and Löfgren, 1985; and Koskela, 12 In other words, we assume that the individual only consumes marketed sex. This assumption is clearly a stark simplification. However, safe sex within a relationship is a complex issue. Not only do we need to include the uncertainty of the fidelity of the partner, we also need to take into consideration the desire to bring children into the world, for which unprotected sex is necessary. In addition, we would have to include bargaining between spouses and preferences for extramarital sex. However, if we disregard the complexities of bargaining and reproduction wishes, a model with marital sex would only differ with respect to unsafe sex within the marriage having zero monetary cost (for safe sex within the marriage, the monetary cost would be constituted by the cost for condoms). Hence, the qualitative results are unlikely to be altered.
1989). In the analysis presented below I use some of their ideas to model sexual behavior in a world where future health prospects are uncertain.

2.1 The Health Risk Model

Consider an individual that lives for two time periods. The preferences in period $t$ are described by the instantaneous utility function

$$U = U(c, h, x_s, x_u)$$

where $c = (c_1, c_2)$ is consumption, $h = (h_1, h_2)$ health, $x_s = (x_{s1}, x_{s2})$ safe sex, and $x_u = (x_{u1}, x_{u2})$ unsafe sex. The instantaneous utility function is assumed to be twice continuously differentiable, increasing in each argument, and strictly concave. It is also assumed that, at a given level of sexual consumption, the marginal utility from sex with a condom is lower than that of unprotected sex due to, for example, reductions in sensation and interrupted foreplay.

Health and consumption are assumed to be complements, in the sense that the marginal utility of consumption is an increasing function of health and vice versa. Likewise, the marginal utility of sex is most likely positively related to health status and possibly to the level consumption. However, with a two-period model and a purpose to analyze how changes in future prospects affect present sexual incentives, the inclusion of sex in period two and health in period one adds little to the analysis. I therefore normalize health in period one to unity and disregard sexual activities in period two. Consequently, the assumption that sex and health are complements is superfluous within the framework of this model. For simplicity I furthermore assume that consumption and sex are additively

\[\begin{align*}
&13 \text{ Katz et al. analyze a firm operating on an international market under price uncertainty, while Brock et al., Johansson and Löfgren, and Koskela investigate optimal forest harvesting under uncertainty of future prices.} \\
&14 \text{ Sex is assumed to be safe if the individual uses a condom. Now, sexual consumption is in reality a discrete variable. However, economists commonly treat consumption as a continuous variable although this is rarely the case. Thus, in order to facilitate our analysis we assume that sex, as well as other consumption goods, can be treated as a continuous variable.} \\
&15 \text{ I.e., } U_c, U_h, U_{xs}, U_{xu} > 0, \quad U_{xu}, U_{x_{s1}}, U_{x_{u1}}, U_{x_{u2}} < 0. \\
&16 \text{ In addition to the physical reduction in sensitivity caused by condoms, social norms concerning sexuality are likely to affect experienced utility of safe and unsafe sex. The analysis of social norms, sexuality and risk is analyzed in a forthcoming paper.} \\
&17 \text{ That is, health and consumption are not necessarily complements in a Hicksian sense (as derived from the cross derivatives of the Hicksian demand function).}
\]
separable. Concerning safe and unsafe sex, I assume that the marginal utility of safe sex is a decreasing function of unsafe sex.\(^{18}\)

Before we define the lifetime utility function, let us first discuss the uncertainty part of the model. Future health status is affected by two kinds of uncertainty; the risk of contracting HIV via unprotected sexual intercourse, and an exogenous stochastic health shock. The latter can, for example, be thought of as a work or traffic related accident when negative, while a positive shock may reflect improved sanitation or distribution of malaria prophylaxis.

The individual is assumed to be HIV negative in the beginning of period one. However, consumption of unsafe sex is associated with a known risk of acquiring HIV.\(^{19}\) Let us assume that a fraction \(\delta\) of HIV positive individuals receive and respond to HIV treatment. Let us for simplicity also assume that if the individual acquires HIV in period one, and does not access HIV treatment, he dies of AIDS before reaching period two. Now, each time the individual has unprotected sex with a casual contact, he is exposed to a risk of contracting HIV, given by \(\Pr(HIV|x_{us} = 1)\).\(^{20}\) Consequently, the probability of staying HIV negative after an unprotected sexual act is \(1 - \Pr(HIV|x_{us} = 1)\). Let us denote this probability of remaining HIV negative \(\varphi\). This means that, if the individual engages in \(x_{us}\) risky sexual acts, the probability of remaining HIV negative is given by \(\varphi^{x_{us}}\).

In addition to the risk of acquiring HIV, the individual also faces an exogenous risk of experiencing reduced health via a stochastic health shock. Keeping the analysis as simple as possible, I assume that there are only two possible outcomes for an individual with a given

\(^{18}\) In the literature, health and consumption has been treated both as additively separable and as complements (cf. Carbone et. al., 2005; Gjerde et al., 2001). However, I have not found any research on the form (i.e. concavity or convexity) of the relationship between health and consumption. Likewise, research regarding whether safe and unsafe sex should be treated as substitutes or complements is scarce, and the few papers that exist give conflicting advice. For example, Remien et. al (1995), in a study on Gay men, found that safe and unsafe sex was considered both as complements and substitutes. However, regardless of whether safe and unsafe sex are treated as substitutes, complements or additively separable, the analysis below holds. The only difference is that, if safe and unsafe sex are either complements or additively separable, we no longer need an assumption of the relative magnitude of the cross-derivative between safe and unsafe sex and the second order derivative of safe sex.

\(^{19}\) The individual is assumed to know the HIV frequency and the risk of transmission among commercial sex workers. This is clearly a stark assumption; it is extremely hard to find consistent data on HIV transmission rates per sexual act for sub-Saharan Africa. However, it may be reasonable to assume that the perceived risk of acquiring HIV via unprotected sex is proportional to the perceived risk of the partner being HIV infected, and that individuals have a relative good idea about the HIV frequency in a certain population group.

\(^{20}\) Formally, the probability of contracting HIV from one risky sex act is constituted by the product of the probability that the partner is HIV positive and the risk of transmission
HIV status; $h_{\text{low}}$ and $h_{\text{high}}$, where $0 < h_{\text{low}} < h_{\text{high}}$. However, unless HIV treatment is fully efficient, in terms of completely restoring the health of an HIV infected individual, both positive and negative health shocks should produce lower health levels for a HIV positive individual than for a HIV negative individual. In the analysis below, I will consequently distinguish between the health of HIV positive ($h^+$) individuals and HIV negative ($h^-$) individuals in terms of plus and minus signs.

The probability of a positive health shock is given by $\rho$, while the probability of a negative health shock is given by $(1-\rho)$, where $0 < \rho < 0$. The possible health scenarios are presented in Figure 1 below.

![Figure 1](image)

Let us define the spread between high and low health for both HIV positive and HIV negative individuals as, $\gamma = h^{\text{high}} - h^{\text{low}}$, and the expected health level in period two of an HIV negative individual as, $h^{\text{net}} = \rho \cdot h^{\text{high}} + (1-\rho) \cdot h^{\text{low}}$. Assuming that the expected health status of an HIV positive individual is a scaled version of the expected health of an

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21 We disregard from any effects of health shocks on earning capabilities in period two. Treating income as a function of health would affect the analysis marginally. However, the most striking effect is the production of relatively meaningless algebra. We thus focus on the simpler version of the model (results are available from the author upon request).

22 The conditional variance is given by,

$$\sigma^2_h = \rho \cdot (h^{\text{high}} - h^{\text{low}})^2 + (1-\rho) \cdot (h^{\text{high}} - h^{\text{low}})^2 + \delta \cdot \rho \cdot (h^{\text{high}} - h^{\text{low}})^2 + (1-\rho) \cdot (h^{\text{high}} - h^{\text{low}})^2$$

$$= \rho \cdot ((1-\rho)^2 + (1-\rho) \cdot (-\rho)^2 + \delta \cdot ((1-\rho)^2 + (1-\rho) \cdot (-\rho)^2)$$

$$= (1+\delta) \cdot \rho \cdot (1-\rho)^2$$
HIV negative individual, we can define expected health of an HIV positive individual as,

\[ E[h^*_2] = \theta \cdot \tilde{h}_2, \]

where \( \theta; 0 \leq \theta \leq 1 \) measures the efficiency of HIV treatment and This implies that if \( \theta = 1 \) HIV treatment is fully efficient in terms of providing a HIV infected individual with the same health as that of an uninfected individual. This definition enables us to write the possible outcomes of future health as,\(^{23}\)

\[
\begin{align*}
    h^-_{\text{low}} &= \tilde{h}_2 - \rho \cdot \gamma \\
    h^+_{\text{high}} &= \tilde{h}_2 + (1 - \rho) \cdot \gamma \\
\end{align*}
\]

The discussion above implies that we can define the expected utility function as

\[ E[U] = u(c_1) + g(x_{us}, x_s) + \beta \cdot E[u(c_2, h_2)] \]

where \( \beta, 0 < \beta < 1 \), is an exogenous discount factor, and where

\[ E[u(c_2, h_2)] = \varphi^{x_{us}} \cdot [\rho \cdot u(c_2, h^-_{\text{low}}) + (1 - \rho) \cdot u(c_2, h^+_{\text{high}})] + \delta \cdot \beta \cdot (1 - \varphi^{x_{us}}) \cdot [\rho \cdot u(c_2, h^+_{\text{high}}) + (1 - \rho) \cdot u(c_2, h^-_{\text{low}})] \]

is the expected utility in period two. Next, let us define the individual’s budget constraint.

\[
\begin{align*}
    Y_1 &= c_1 + q_{s_i} \cdot x_i + q_{s_u} \cdot x_{su} + S \\
    (1 + r) \cdot S + Y_2 &= c_2 \\
\end{align*}
\]

where \( q_{s_i} \) is the price of safe sex, \( q_{s_u} \) the price of unsafe sex, and \( c_i \) the numeraire good in each time period. Labor supply is assumed to be completely inelastic, and \( Y_1 \) and \( Y_2 \) represent the exogenous incomes in period one and two, respectively.\(^{24}\) Further, \( r \) is the interest rate, and \( S \) represents savings. As access to credit is severely restricted for many

\(^{23}\) In order to keep the analysis as simple as possible, I disregard the monetary cost of antiretroviral (ARV) drugs, estimated to $13,000 per patient and year (Lakdawalla, et. al., 2006). This seemingly stark simplification is partly motivated by the presence of government subsidies and partly by that the focus of this paper is on health related costs rather than monetary costs.
households in poor countries, it is assumed that saving is non-negative. For simplicity, the interest rate is set to zero. Substituting equations (7) - (9) into equation (6) and maximizing with respect to $x_s, x_w$ and $S$ produces the following first order conditions,

$$\frac{\partial U_s}{\partial x_s} = U_{x_s} = -u_{c_1} \cdot q_{x_s} + g_{x_s} = 0$$  \hspace{1cm} (10)

$$\frac{\partial U_s}{\partial x_w} = U_{x_w} = -u_{c_1} \cdot q_{x_w} + g_{x_w} + \beta \cdot \ln(\varphi) \cdot \varphi^x_s \left[ \left( \rho \cdot u(c_2, h_{\text{high}}) + (1-\rho) \cdot u(c_2, h_{\text{low}}) \right) - \delta \cdot \left( \rho \cdot u(c_2, h_{\text{high}}^+) + (1-\rho) \cdot u(c_2, h_{\text{low}}^+) \right) \right] = 0$$  \hspace{1cm} (11)

$$\frac{\partial U_s}{\partial S} = U_S = -u_{c_1} + \beta \cdot \varphi^x_s \left[ \left( \rho \cdot u_{c_2}(c_2, h_{\text{high}}) + (1-\rho) \cdot u_{c_2}(c_2, h_{\text{low}}) \right) - \delta \cdot \left( \rho \cdot u_{c_2}(c_2, h_{\text{high}}^+) + (1-\rho) \cdot u_{c_2}(c_2, h_{\text{low}}^+) \right) \right] < 0$$  \hspace{1cm} (13)

where $g_{x_w} = \partial g / \partial x_w, g_{x_s} = \partial g / \partial x_s, u_{c_1} = \partial u_1 / \partial c_1$, and $u_{c_2} = \partial u_2 / \partial c_2$.\(^{25}\)

Let us briefly interpret these first order conditions. For safe sex, equation (10) implies that the marginal benefit, in terms of sexual pleasure, is equalized to the marginal cost, in terms of forsaken consumption in period one. Turning to equation (11), we see that unsafe sex is associated with an additional marginal cost compared with equation (10). This additional cost, which is captured by the last term on the right hand side (RHS) in equation (11), reflects the reduced survival probability caused by an unprotected sexual act. As can be seen in the equation, the more efficient the HIV treatment is and the larger share of the HIV positive population that is receiving this treatment, the lower is the expected future marginal cost of unsafe sex. As for equation (12), we see that the consumption of unsafe sex scales down the marginal benefit of savings via $\varphi^x_s$. Each risky sexual act reduces the probability of surviving to period two, and therefore it also reduces the probability of consuming saved resources. Hence, the presence of HIV increases the likelihood of a corner solution in terms of zero savings. In addition, even if we have an interior solution, the consumption of unsafe sex reduces the optimal amount of savings in the presence of HIV compared to the case without HIV.

\[^{25}\varphi^x_s = \varphi^x_{x_w} \cdot \ln(\varphi) \cdot \frac{\partial \varphi^x_s}{\partial x_w} = \ln(\varphi) \cdot \varphi^x_{x_w}\]
2.2 Risky Sex in a Risky World

Let us now turn to the main question of how uncertainty and exogenous changes of future health influence sexual choices today. In order to analyze the effect of health related uncertainty on sexual incentives, we need to isolate the effects induced by changes in the variance of future health from changes in the expected level of health. In terms of equations (2)-(5), the effect of an increased variability of future health corresponds to an increase in the spread \((\gamma)\), whereas the effect of improvements in expected health corresponds to increases in \(\bar{h}_2\). The effect of changes in the direct riskiness of engaging in unprotected casual sex corresponds to changes in \(\theta, \delta, \varphi\). Let us therefore totally differentiate the first order conditions (10) and (12) with respect to the parameters, \(\gamma, \bar{h}_2, Y, \delta, \theta, \varphi\). This gives us the system

\[
\begin{pmatrix}
U_{xu_s xu_s} & U_{xu_s x_s} & U_{xu_s 2} \\
U_{xu_s x_s} & U_{xu_s 2} & U_{xu_s 3} \\
U_{xu_s x_s} & U_{xu_s 2} & U_{xu_s 3}
\end{pmatrix}
\begin{pmatrix}
dx_s \\
dx_x \\
dS
\end{pmatrix}
= \begin{pmatrix}
-U_{xu_s y} & -U_{xu_s \bar{h}_2} & -U_{xu_s 2} & -U_{xu_s \delta} & -U_{xu_s \theta} \\
-U_{x_s y} & -U_{x_s \bar{h}_2} & -U_{x_s 2} & -U_{x_s \delta} & -U_{x_s \theta} \\
-U_{S, y} & -U_{S, \bar{h}_2} & -U_{S, 2} & -U_{S, \delta} & -U_{S, \theta}
\end{pmatrix}
\begin{pmatrix}
dy \\
dh_2 \\
d\delta \\
d\theta
\end{pmatrix}
\]

(14)

where \(U_{xu_s xu_s} = \partial E^2[U]/\partial (x_{us})^2\), \(U_{xu_s y} = \partial E^2[U]/\partial x_{us} \partial y\), etc. Let us denote the determinant of the 3\times3 matrix on the left hand side (LHS) \(D\). Thus, if we have an interior solution, it follows from the second order condition for a maximum that, \(D < 0\). However, if the constraint on saving is binding, equation (12) and the last row and column in equation (14) become redundant. Hence, the determinant of the LHS reduces to a 2\times2 matrix which is positive from the second order condition for a maximum.

As stated in the introduction, a large share of HIV susceptible individuals lives environments where HIV treatment is still not available. For these individuals the decision to engage in unsafe sex or not is unlikely to depend heavily on marginal changes in the quality or distribution of HIV drugs. Let us therefore start our analysis by investigating the

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26 Expected health is, to be formal, affected by five possible outcomes, \(h_{\text{AIDS}}, h_{\text{other}} h_{\text{high}}, h_{\text{low}}, \text{dying of AIDS}\). However, since the purpose of this paper is to isolate the effects of exogenous changes in future prospects on sexual incentives, and since the individual’s own effect on health is incorporated via \(\phi^*\), expected health is conditioned on being alive.

27 It can easily be shown that an induced change in \(\bar{h}_2\) does not alter the conditional variance of future health as; \(\sigma^2_{\bar{h}_2} = (1 + \delta) \cdot \rho (\rho - 1) \gamma^2\).
effect of changes in uncertainty in a world without HIV treatment, i.e., \( \delta = \theta = 0 \) (c.f. Oster, 2005, for a similar approach).

### 2.2.1 Sexual risk taking in a world without HIV treatment

For HIV susceptible individuals living in an environment where antiretroviral drugs are not available, the last term in the first order condition for unsafe sex in equation (11) vanishes.

**Risky Sex and Risky Health**

Let us start by analyzing the effects on sexual incentives due to changes in the variability and expected value of future health. Define,

\[
D_1 = \frac{U_{x,t_0} \cdot U_{S_{x,0}} - (U_{s,t_0})^2}{D} \tag{15}
\]

\[
D_2 = \frac{U_{s_{x,0}} \cdot U_{S_{x,0}} - U_{s_{l,0}} \cdot U_{s_{l,0}}}{D} \tag{16}
\]

\[
D_3 = \frac{U_{x_{l,0}} \cdot U_{S_{x,0}} - U_{s_{l,0}} \cdot U_{x_{l,0}}}{D} \tag{17}
\]

\[
D_{23} = \frac{U_{x_{l,0}} \cdot U_{s_{l,0}} - U_{s_{l,0}} \cdot U_{x_{l,0}}}{D} \tag{18}
\]

\[
D_{33} = \frac{U_{s_{l,0}} \cdot U_{s_{l,0}} - (U_{s_{l,0}})^2}{D} \tag{19}
\]

Let us begin with a mean preserving increase in the spread. Using Cramér’s rule we obtain

\[
\frac{\partial x_w}{\partial \gamma} = -U_{x_{l,0}} \cdot D_{11} - U_{S_{x,0}} \cdot D_{13} \tag{20}
\]

\[
\frac{\partial x_s}{\partial \gamma} = U_{s_{l,0}} \cdot D_{12} + U_{S_{x,0}} \cdot D_{23} \tag{21}
\]

\[
\frac{\partial S}{\partial \gamma} = -U_{s_{l,0}} \cdot D_{13} - U_{S_{x,0}} \cdot D_{33} \tag{22}
\]

Equations (20) – (22) imply that the effect of a change in the spread of future health on \( x_w \), \( x_s \), and \( S \) can be decomposed into two parts. The first affects sexual consumption and savings via the effect on unsafe sex. This effect stems from a perceived reduction in the expected value of future health, reflected by a reduction in the last term in equation (11),

\[ -\delta \cdot \left( \rho \cdot u(\varepsilon_2, h_{high}) + (1 - \rho) \cdot u(\varepsilon_2, h_{low}) \right) \]
and is included in equations (20) – (22) via $U_{u_s}$. The second effect follows from the assumption that the marginal utility of consumption is an increasing function of health in the second period. This assumption implies that a reduction in the perceived value of future health also reduces the marginal benefit of savings in equation (12). Incentives to consume safe and unsafe sex are thus affected by changes in the budget space via the term $U_{s}$ in equations (20) – (22).

To facilitate the interpretation of the first effect, consider the special case when the restriction on savings is binding,

**Proposition 1** If $S = 0$, a mean preserving increase in the spread of future health increases the consumption of unsafe sex and reduces the consumption of safe sex, i.e.,

$$\frac{\partial x_s}{\partial \gamma} \bigg|_{S=0} = \frac{-U_{u_s} \cdot U_{u_s}}{D} > 0, \quad \text{(23)}$$

$$\frac{\partial x_s}{\partial \gamma} \bigg|_{S=0} = \frac{U_{u_s} \cdot U_{u_s}}{D} < 0. \quad \text{(24)}$$

**Proof.** See the appendix.

As can be seen in Proposition 1, if $S = 0$, the mechanism for behavioral change only relates to the effect given by $U_{u_s}$. An increase in the uncertainty about future health reduces the expected utility in period two. As a consequence, the expected return to investments in health is reduced. In other words, uncertainty about future health makes the cost of unsafe sex less salient, thereby increasing incentives to engage in pleasurable, but risky, activities in the present. Hence, if savings are already zero, the optimal response to an increase in $\gamma$ is to increase unsafe sex consumption on behalf of the consumption of safe sex.

The assumption of zero savings may seem like a stark simplification. However, with a majority of the populations living under the poverty line, and severely malfunctioning credit markets, the assumption of zero savings is not completely unrealistic for an analysis of the situation in a poor country.

Let us now turn to the situation when $S > 0$. This implies that the induced changes in consumption of unsafe sex, safe sex and savings are given by the full system given in
equations (20) – (22). In this case, the effects on $x_u$, $x_s$ and $S$, of an increase in $\gamma$, cannot in general be signed as is clear from an inspection of the term $D_{t,3}$ in equations (20) – (22), above. Now, the sign of $D_{t,3}$ to a large extent hinges on the relative magnitude of the substitutability between safe and unsafe sex. More specifically, the sign of $D_{t,3}$ depends on whether \( \left( \frac{q_s}{q_u} \right) g_{s,x_u} - g_{s,x_s} \) is positive or negative (where $g_{s,x_u}$ is the cross derivative between safe and unsafe sex and $g_{s,x_s}$ is the second order derivative of safe sex). Note that, in the special case when safe and unsafe sex are nearly perfect substitutes, $g_{s,x_u} \approx g_{s,x_s}$. This implies that, given that $q_s > q_u$ (consistent with the reality in, for example, sub-Saharan Africa) \( \left| g_{s,x_u} \right| < \left| g_{s,x_s} \right| \) and $D_{t,3}$ is thus positive. With this in mind, we obtain the following Proposition,

**Proposition 2** If $S > 0$, $u_{x,x} < 0$, and if \( \left| \frac{q_s}{q_u} \right| g_{s,x_u} \leq \left| g_{s,x_s} \right| \), a mean preserving increase in the uncertainty of future health status increases consumption of unsafe sex and reduces savings, i.e., $\partial x_u / \partial \gamma > 0$, and, $\partial S / \partial \gamma < 0$.

**Proof.** See the appendix.

When savings are positive, an increase in $\gamma$ will influence the individual’s incentives both in terms of a reduction in the perceived value of future health, and in terms of a reduction in the expected utility from future consumption. The first effect was interpreted under Proposition 1. In order to interpret the second effect, let us briefly discuss the implications of the relationship between the future health status and the marginal utility of consumption. If the marginal utility of consumption is increasing at a decreasing rate in health (i.e. if $u_{x,x} > 0$ and $u_{x,h} < 0$), then an increase in the uncertainty about future health does not only reduce the value of future health, it also has a negative effect on the incentives to save. This, in turn, means that there is now room for an increase in the consumption of sex in period one. However, the increase in budget space is not necessarily reserved for unsafe sex consumption. Hence, the magnitude of the increase in unsafe sex consumption, in part depends on how much safe sex increases and on how this affects the
marginal utility of unsafe sex (this is the reason for the condition of \( \| q_s / q_w \| g_{s,s} \| g_{s,s} \| \text{in Proposition 2}).

It should be noted that, if health and consumption are additively separable, \( u_{s,h} = 0 \). This implies that an increase in uncertainty of future health does not affect the expected marginal utility of future consumption. In this case, the effect on unsafe sex via the effect on the budget space is zero \( (U_s = 0) \). This, in turn, implies that an increase in uncertainty unambiguously increases incentives to consume unsafe sex. Likewise, if safe and unsafe sex are additively separable, a change in the incentives to consume safe sex does not affect the marginal utility of unsafe sex. Hence, the sign of equation (20) becomes unambiguously positive.

Let us also briefly discuss the effects on the consumption of safe sex under Proposition 2. As can be seen in equation (21), safe sex is not affected directly by the increase in the uncertainty of future health. However, the perceived reduction in the value of future health and consumption affects safe sex indirectly. First, an increase in the variability of future health reduces the benefit of choosing safe sex instead of unsafe sex, thus reducing incentives to consume safe sex. Second, the perceived reduction in the marginal value of future consumption reduces the marginal cost of safe sex, thus increasing incentives to consume safe sex.

Next, let us analyze the effects of an increase in the expected level of health, \( \tilde{h}_i^- \). Consider the following results,
\[
\frac{\partial x}{\partial h} = -U_{s,k} \cdot D_{11} - U_{sk} \cdot D_{31}
\]  
(25)

\[
\frac{\partial x}{\partial h} = U_{s,k} \cdot D_{12} + U_{sk} \cdot D_{23}
\]  
(26)

\[
\frac{\partial S}{\partial h} = -U_{s,k} \cdot D_{13} - U_{sk} \cdot D_{33}
\]  
(27)

As in the above, the effects on sexual incentives can be decomposed into two parts, which we can analyze by referring to the first order conditions given in equations (10) - (12); first, an improvement in future health increases the value of the future and thereby affects incentives to consume unsafe sex directly. This effect is reflected by an increase in the last term in equation (10). Second, the increase in the expected value of health also affects the expected utility from future consumption. This effect is represented by an increase in the last term of equation (12). Hence, unsafe and safe sex consumption is also affected via a change in the budget space for current consumption. In order to facilitate the interpretation of the first effect, we once again start with the special case when \( S = 0 \).

Consider the following proposition,

**Proposition 3** If \( S = 0 \), a spread preserving increase in the expected value of future health has a negative effect on the consumption of unsafe sex and a positive effect on the consumption of safe sex, i.e.,

\[
\frac{\partial x}{\partial h} \bigg|_{S=0} = \frac{-U_{s,k} \cdot U_{sk}}{D} < 0,
\]  
(28)

\[
\frac{\partial x}{\partial h} \bigg|_{S=0} = \frac{U_{s,k} \cdot U_{sk}}{D} > 0.
\]  
(29)

**Proof.** See the appendix

To interpret these results, let us again return to the first order conditions in equations (10) - (12). When \( S = 0 \), the only effect on safe and unsafe sex is via the second term in equation (10). An increase in expected health effectively raises the marginal cost of consuming unsafe sex in terms of an increase in the loss of dying. Hence, an improvement in expectations of future health unambiguously reduces incentives to consume risky sex. However, whether or not the reduction in the consumption of unsafe sex is transformed into an increase in the consumption of safe sex depends on the degree of substitutability between the two.
Next, let us once again turn to the situation when the constraint on savings in non-binding. Consider the following Proposition,

**Proposition 4**  
If $s > 0$, and if $(q_s / q_{s+1}) \leq g_{s+1}$, a spread-preserving increase in the expected value of future health reduces consumption of unsafe sex and increases savings, i.e., $c_{s+1} / c_{s+2} < 0$, and, $\frac{\partial h}{\partial h} > 0$.

**Proof.** See the appendix

When savings are positive, an increase in $h$ will influence sexual incentives, both in terms of the increase in the expected utility from future health, and in terms of the positive spillover effect on the expected utility from future consumption. As in the case of zero savings, an improvement in expected health makes the cost of unsafe sex more salient. However, as health and consumption are complements, the improvement in expected health also raises the marginal value of future consumption. Hence, there are incentives to increase saving. This, in turn, reduces the budget space for consumption in period one and thereby has a negative effect on the consumption of unsafe sex. Incentives to consume unsafe sex thus fall for two reasons, first due to the perceived increase in the marginal cost of savings and second due to increased competition for budget space. However, the reduction in budget space does not only affect unsafe sex; a part of the reduced resources are taken from safe sex and other consumption.

The reduction in budget space implies that the effect on safe sex goes in two directions; on the one hand the individual has an incentive to substitute unsafe sex for safe sex, and the closer substitutes safe and unsafe sex are, the more safe sex will increase. On the other hand, the reduction in budget space limits the increase in safe sex consumption.
Risky sex and Risky Income

A positive change in future income should, intuitively, increase the incentives to abstain from risky activities in the present to reap the benefits of the future. However, with more money as old the need for savings decline and may thus actually increase the budget space for risky activities. Consider the following results,

$$\frac{\partial x_{ux}}{\partial Y_z} = -U_{y_{sxz}y} \cdot D_{11} - U_{y_{sxz}y} \cdot D_{13}$$ (30)

$$\frac{\partial x_{x}}{\partial Y_z} = U_{y_{sxz}y} \cdot D_{12} + U_{y_{sxz}y} \cdot D_{23}$$ (31)

$$\frac{\partial S}{\partial Y_z} = -U_{y_{sxz}y} \cdot D_{13} - U_{y_{sxz}y} \cdot D_{33}$$ (32)

As in previous sections, the first term in equations (30) – (32), $U_{y_{sxz}y} < 0$, can be interpreted as a direct effect on unsafe sex incentives. In this case the direct effect on unsafe sex is an increase in the expected value of the future, in terms of an increase in future income in the last term in equation (10). The second term, $U_{y_{sxz}y} < 0$, on the other hand, reflects the reduced need to save for old age due to the increase in future earnings (i.e., a reduction in the marginal value of saving). This effect is represented by a reduction in the second term in equation (12) and affects safe and unsafe sex via the effect on current budget space. The two effects create conflicting incentives for the individual. As in the above, we interpret the first effect by an analysis of the special case of $S = 0$. Consider the following proposition,

**Proposition 5** If $S = 0$, an increase in future earnings reduces the consumption of unsafe sex and increase consumption of safe sex, i.e.,

$$\frac{\partial x_{u}}{\partial Y_z}_{S=0} = -U_{y_{sxz}y} \cdot \frac{U_{y_{sxz}y}}{D} < 0$$ (33)

$$\frac{\partial S}{\partial Y_z}_{S=0} = U_{y_{sxz}y} \cdot \frac{U_{y_{sxz}y}}{D} > 0$$ (34)

**Proof.** See the appendix

If the constraint on saving is binding, the individual only experiences the direct effect of an increase in the expected value of the future (via $U_{y_{sxz}y}$). As in proposition 3, the individual experiences an increase in the marginal cost of risky sex reflected by an increase in the last
term in the first order condition for unsafe sex (equation (10)). An increase in future income has two effects on expected utility in period two. First, a higher income implies greater consumption possibilities, second, greater consumption possibilities implies a higher utility of health. Hence, with an improvement of future earnings makes unsafe sex consumption more costly, in terms of forsaken utility as dead. Consequently, the optimal amount of unsafe sex declines.

With positive savings the individual faces somewhat of a dilemma, portrayed by the potentially different signs of the two terms in equation (30). Consider the following proposition,

**Proposition 6**  
If $s > 0$, an increase in future income creates conflicting incentives for the individual, in terms of an increase the weight of future utility, and a reduction in the marginal utility of savings.

**Proof.** See the appendix

With positive savings, the individual experiences the same increase in the value of surviving to old age as he did under proposition 5. However, the increase in future income also reduces the need to postpone consumption (reflected by a reduction of the marginal utility of saving in equation (12)), thus making a more voluminous consumption of unsafe sex possible. The first effect can, with some effort, be interpreted as a ‘substitution effect’; with an increase in the value of the future, the individual has incentives to substitute present utility, in terms of unsafe sex consumption, for future utility. The latter effect can in the same manner be interpreted as a kind of ‘income effect’; the increase in future income makes it possible for the individual to ‘borrow’ from the future in terms of reduced savings. Hence, although the value of being alive in period two increases with income, the increase in resources also tempt the individual to engage in risky behavior.

### 2.2.2 Sexual risk taking in a world with HIV treatment

If a partially or fully effective treatment to HIV exists, the last term in equation (10) is greater than zero. In other words, the presence of HIV treatment reduces the marginal cost of engaging in unsafe sex. In order to capture the main mechanism of the effect on uncertainty on risky sexual choices, we simplify our model and disregard from saving
possibilities. Let us start by briefly analyzing the direct effects of improved access to HIV treatment ($\delta$) and improved quality of HIV treatments ($\theta$). Consider the following proposition,

**Proposition 7** An increase in the coverage rate of HIV treatment ($\delta$) and/or the effectiveness of HIV treatment ($\theta$) increases the incentives to engage in unprotected casual sex, i.e., $\frac{\partial x_{us}}{\partial \theta} = \frac{-u_{x_{us}} - u_{x_{x_{us}}}}{d} > 0$, and $\frac{\partial x_{us}}{\partial \delta} = -u_{x_{us}} > 0$.

**Proof.** See the appendix

For individuals engaging in sexual risk taking, an increase in the coverage rate of ARV drugs reduces the probability of dying prematurely of AIDS, and an increase in the quality of the HIV treatment increases the quality of life given that the individual has access to this treatment. In other words, both the change in coverage and quality of ARV drugs reduce the expected health cost of an HIV infection. Consequently, the larger the share of the HIV infected population that has access to treatment, and the better this treatment is, the smaller are the incentives to abstain from unprotected casual sex. The result in proposition 7 quite intuitive; if an effective treatment is available for HIV positive individuals, risky sex is not so risky anymore. Thus, if the cost of HIV drugs is not carried by the treated individual, the private cost of engaging in unprotected sex may become negligible. However, the cost of ARV treatment is not negligible to governments (nor is it to individuals that may become infected by an HIV positive individual that continues to engage in unsafe sex). Hence, if HIV treatment is free of charge, the availability of HAART creates a type of moral hazard in terms of reducing incentives to abstain risky sex.

As long as the available HIV treatment does not fully cure the patient from HIV, it is of great policy importance to investigate how varying levels of HIV treatment coverage ($\delta$) and effectiveness ($\theta$) affects sexual risk taking. However, the moral hazard of ARV treatment is not the main interest in this paper (for an analysis of the presence of moral hazard see Lakdawalla, et. al., 2006), and will therefore not be further analyzed here. Instead, the main interest in this paper lies in how behavior under uncertainty changes.

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29 The assumption of zero savings may seem like a stark simplification. However, the analysis below mainly relates to populations in developing countries with a majority of the population living under the poverty line, and with severely malfunctioning credit markets. Consequently, the assumption of zero savings is not completely unrealistic for an analysis of the situation in a poor country.
when we introduce the possibility of surviving with HIV/AIDS. Let us therefore continue our analysis by investigating the effect of improvements in the coverage rate and quality of HIV treatment on sexual risk taking under uncertainty. Consider the following proposition,

**Proposition 8** If ARV treatment with efficiency \( \theta \) is freely available to a share \( \delta \) of the HIV positive population, the effect of a mean preserving increase in the spread of future health on unsafe sexual consumption depends on the relative size of \( \delta \) and \( \theta \).

- **a)** If \( \delta = \theta = 1 \), a mean preserving increase in the spread of future health has no effect on the choice to consume unsafe sex, i.e., \[ \frac{\partial x_u}{\partial \gamma} \bigg|_{\delta=\theta=1} = -\frac{U_{\delta x_{1x_2}}}{D} = 0 \]

- **b)** If \( \theta = 1 \), and \( 0 < \delta < 1 \), a mean preserving increase in the spread of future health increases incentives to engage in sexual risk taking, i.e., \[ \frac{\partial x_u}{\partial \gamma} \bigg|_{\theta=1,0<\delta<1} = -\frac{U_{\delta x_{1x_2}}}{D} > 0 \]

- **c)** If \( \delta = 1 \), and \( 0 < \theta < 1 \), a mean preserving increase in the spread of future health reduces incentives to engage in sexual risk taking, i.e., \[ \frac{\partial x_u}{\partial \gamma} \bigg|_{\delta=1,0<\theta<1} = -\frac{U_{\delta x_{1x_2}}}{D} < 0 \]

- **d)** If \( 0 < \theta < 1 \), and \( 0 < \delta < 1 \), a mean preserving increase in the spread of future health creates conflicting effects on the incentives to engage in sexual risk taking (the sign of the derivatives are ambiguous).

**Proof.** See the appendix

The intuition behind propositions 8a and 8b is relatively straightforward. If the treatment is freely and globally available, and if the treatment actually cures HIV (or at least provides the infected individual with the same health and longevity as an uninfected individual), unprotected casual sex does no longer have a future cost and can thus not be deemed as a risky activity in terms of HIV\(^ {30} \). Hence, with \( \delta = \theta = 1 \), the introduction of uncertainty does not change the first order conditions, and therefore not behavior, in period one. If treatment is effective but coverage is less than global, unprotected sex is still associated with an expected future cost. Hence, as in Proposition 1, an increase in future uncertainty increases incentives to engage in unsafe sex.

If coverage of HIV treatment is global but not completely effective (proposition 8c), an increase in uncertainty reduces incentives to engage in unsafe sex. As before, the increase in uncertainty reduces the expected utility of surviving to old age. However, if the health of an HIV infected individual is lower than that of an uninfected individual, the expected utility

\(^{30}\) Naturally, casual unprotected sex can have other costs in terms of other sexually transmitted infections, risk of violence etc. this is not within the scope of the paper however, so I will not dwell further on the issue.
of living with an HIV infection falls relatively more than the expected utility of living without HIV. Hence, an increase in the spread between high and low health outcomes makes it less tempting to engage in unsafe sex as the spread increases the expected cost of living with HIV in period two. The lower the average health of an HIV positive individual is (as long as he or she is still alive), the greater is the reduction in the expected utility of health due to an increase in the spread between high and low health.

Finally, if treatment is not freely available and does not fully restore health, an increase in the uncertainty of future health creates conflicting incentives concerning sexual choices. The larger the share of the HIV positive population that access treatment, the smaller is the expected cost of an unprotected sexual act, and consequently the smaller is the effect of uncertainty on the choice to engage in unsafe sex. However, the less efficient the accessed treatment is, the more will uncertainty increase the perceived health cost of living with HIV, and the greater will incentives be to remain HIV negative in an uncertain environment. Thus, if the share of untreated HIV positive individuals is relatively large and the treatment is relatively effective, the net change in the expected cost of engaging in unprotected casual sex will be negative as the reduction in the perceived cost of dying of AIDS will outweigh the reduction in utility of living with HIV.

Next, let us turn to how the presence of ARV treatment affects incentives to engage in unsafe sex when the expected health level of both HIV positive- and negative individuals changes. If ARV drugs are globally available but unable to fully restore health, the effect of improvements in the expected level of future health \((\bar{h}_x)\), on sexual risk taking in the present \((u_x)\), cannot in general be signed. The direction of the effect to a large extent depends on a measure that resembles a discrete version of the Arrow-Pratt measure of relative risk aversion. Let us denote this discrete measure of relative risk aversion \(DR\),

\[
DR = -\theta \cdot h_\theta \cdot \left( \rho \cdot u_{h_{high}} + (1 - \rho) \cdot u_{h_{low}} \right) \left( \rho \cdot u_{h_{high}} + (1 - \rho) \cdot u_{h_{low}} \right) =, \quad (35)
\]

where \(u_{h_{high}} = \partial^2 u(c_2, h_{high}) / \partial (h_{high})^2\) and \(u_{h_{high}} = \frac{\partial^2 u(c_2, h_{high})}{\partial (h_{high})^2}\). Consider proposition 9,

\[ R = -u''(x) \cdot x / u'(x) \]

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31 R = \(-u''(x) \cdot x / u'(x)\).
Proposition 9  If ARV treatment with efficiency $\theta$ is freely available to a share ($\delta$) of the HIV positive population, the effect of a spread preserving increase in the mean of future health on unsafe sexual consumption depends on the relative size of $\delta$ and $\theta$.

a) If $\delta = \theta = 1$, a spread preserving increase in the mean of future health has no effect on the choice to consume unsafe sex, i.e., $\frac{\partial x}{\partial h_1}$ = 0

b) If $\theta = 1$, and $0 < \delta < 1$, a spread preserving increase in the mean of future health reduces incentives to engage in sexual risk taking, i.e., $\frac{\partial x}{\partial h_1}$ = $\frac{-U_{\alpha h_1} \cdot U_{x_{1/1}}}{D} < 0$

c) If $\delta = 1$, and $0 < \theta < 1$, the effect of a spread preserving increase in the mean of future health depends on the degree of “discrete relative risk aversion” (DR).

a) If $DR > 1$, a spread preserving increase in future health increases incentives to consume unsafe sex, i.e., $\frac{\partial x}{\partial h_1}$ = $\frac{-U_{\alpha h_1} \cdot U_{x_{1/1}}}{D} > 0$

b) If $DR = 1$, a spread preserving increase in future health has no effect on incentives to consume unsafe sex, i.e., $\frac{\partial x}{\partial h_1}$ = 0

c) If $DR < 1$, a spread preserving increase in future health reduces incentives to consume unsafe sex, i.e., $\frac{\partial x}{\partial h_1}$ = $\frac{-U_{\alpha h_1} \cdot U_{x_{1/1}}}{D} < 0$

e) If $0 < \theta < 1$, and $0 < \delta < 1$, the effect of a spread preserving increase in the mean of future health creates conflicting effects on incentives to engage in sexual risk taking.

Proof. See the appendix.

As in Proposition 8, an effective and globally available treatment of HIV implies that unprotected casual sex has no HIV related future cost. Hence, changes in the expected level of future health do not affect incentives to engage in sexual risk taking in the present (proposition 9a). However, if the coverage rate of HIV treatment is less than global, or if ARV treatment only partially restores health, some cost of engaging in unsafe sex prevails (Propositions 9b and 9c).

With a fully effective treatment but only partial coverage of this treatment, an expected improvement in future health can easily be seen to unambiguously increase the perceived cost of engaging in unsafe sex. However, with a global coverage of a treatment that only partially restores health the effect of an improvement of future health will be affected by the discrete relative risk aversion measure in equation (35). To see this, note that an increase in $h_1$ has two effects on the incentives to engage in sexual risk taking; it increases the expected utility of remaining HIV negative, but it also reduces the cost of acquiring HIV since the expected health of HIV positive individuals increases proportionally to that of
HIV negative individuals. Hence, whether the consumption of unsafe sex increases or decreases will depend on which one of these effects that dominates. This, in turn, depends on our measure of discrete relative risk aversion (DR).

Now, for all risk averse individuals, the marginal valuation of future health diminishes as the expected health increases; the greater the risk aversions, the greater the rate at which the marginal utility is reduced. This implies that, if the relative risk aversion is “sufficiently” high (in our case DR>1), the increase in the expected value of being HIV positive will be smaller than the reduction in the expected cost of being HIV positive (i.e., the reduction in the expected marginal utility of being HIV negative is greater than the expected reduction in the marginal utility of being HIV positive). For individuals with a low relative risk aversion (DR<1), on the other hand, the increase in the expected utility of remaining HIV positive will outweigh the reduction in the expected cost of an HIV infection.

3. Discussion and policy implications

The main question asked in this paper, is whether the high degree of uncertainty and low level of general health, can help us understand the persistent presence of sexual risk taking in countries where HIV prevalence is high. The above analysis suggests that there is no such thing as “one size fits all” when it comes to AIDS policy. Instead, we need to calibrate interventions for different social and physical contexts. As showed above, the degree of uncertainty and expected level of future health, the access to and quality of ARV drugs, and the individual risk preferences, all interact and affect the outcome of AIDS interventions. Now, the model presented in this paper is very stylized and not in all respects a realistic representation of reality and it certainly needs to be complemented with empirical tests. However, I would still like to linger around some possible policy implications.

For individuals with no access to HIV treatment, uncertainty of future health contributes to low expected benefits of health investments and a low expected cost of health destructive behavior. Likewise, low levels of expected future health and income contribute to low levels of perceived cost of unsafe sex activities for individuals with low earnings and
no access to ARV drugs. In regions where access to ARV drugs is limited, interventions that improve general health and reduces uncertainty may thus constitute an important ingredient in an effective AIDS policy. It may also be beneficial to improve income prospects for poor households. Now, for a large share of the HIV susceptible population in sub-Saharan Africa, the above scenario fits the picture. Hence, interventions such as the distribution of impregnated mosquito nets and immunization programs may be candidates for a more holistic AIDS policy in these areas. Indeed, Oster (2007a), shows that the average number of sex partners in life is an increasing function of the malaria frequency. It is interesting to note that, between 2002 and 2005, a Malaria intervention program on Zanzibar reduced malaria attributed mortality in children under five with over 70 percent (Bhattarai, et. al., 2007). It may be too early to investigate, but it will be of importance to investigate whether this intervention had any effect on sexual behavior.

The above analysis also show that the implication for policy changes with the availability and quality of ARV drugs. Now, I do not attempt to measure societal costs and benefits of ARV drugs, and will therefore not dwell on the net social benefit of HIV treatment. Instead, I will settle with a brief discussion on how the implications for health related policy changes in a world where ARV drugs are increasingly available and effective.

The above results suggest that, the larger the share of the HIV positive population that is covered by HAART, the less beneficial will policy that focus on reductions in uncertainty be for sexual risk taking. Hence, while it may be major importance to focus AIDS policy on social insurance etc. in poor countries, it may be of little interest (it may even be destructive!) to focus AIDS policy on reducing uncertainty in rich countries.

As of today, we are still unable to cure HIV. However, over the years there has been a substantial improvement in the effectiveness of ARV drugs. According to proposition 5, the more effective HIV treatment becomes, the more important it will become to invest resources in improving overall health in areas where access to drugs is limited. Thus, if a cure for HIV is found in the future, and if we lack resources to distribute this cure to all, it may be of great value to intensify efforts on general health interventions such as immunization programs etc.
Proposition 8 further suggest that as long as we cannot cure HIV, risk preferences among targeted individuals will affect the outcome of AIDS policy. More specifically, the more risk prone an individual is, the more likely it is that a policy aimed to improve general health has positive spill-over effects on sexual behavior. Now, some of the most HIV susceptible populations can be suspected to be less risk averse than the average person (for example, commercial sex workers, mine workers, military workers and drug users). Hence, according to proposition 5, health focused interventions may be of great importance in these populations.

One of the main obstacles left for reducing HIV incidence in poor countries has been identified as the lack of behavioral change in high risk groups such as commercial sex sellers, mine workers, and refugees. Taken together, the above analysis suggests that policies that improves the general health and reduces uncertainty may be of particularly benefit to these populations. In the above, I have distinguished between policies that purely changes uncertainty (in terms of the spread between high and low health outcomes) and policies that only affects the expected level of health (thus leaving the variability unchanged). In reality, and based on the results above, it may be both difficult and unnecessary to divide between a mean preserving increase in the spread, and a spread preserving increase in the mean. Policies aimed to reduce uncertainty of future health are likely to also affect the expected level of future health. For example, an increase in the presence of police officers, fire brigades, more permanent houses and better sewage systems in refugee camps would both reduce uncertainty and improve health prospects. Likewise, providing miners and commercial sex workers with social insurance and improved work conditions, and providing resources for institution building in countries torn by civic unrest are likely to improve general health and to reduce uncertainty. For the high risk populations in poor countries both these effects are welcome and, as shown above, may have beneficial spill-over effects on the spread of HIV.
References


Johansson, Per-Olov, & Löfgren Karl-Gustaf, (1985), The Economics of Forestry and Natural Resources, Basil Blackwell, Oxford


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Appendix

Table A1 Parameter and variable description

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<th>Greek letter</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
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<tr>
<td>$\delta$</td>
<td>Coverage rate of ARV drugs in HIV positive population</td>
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<tr>
<td>$\varphi$</td>
<td>Probability of remaining HIV negative after one risky sexual act</td>
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<td>Spread between high and low health in period two</td>
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<td>$\rho$</td>
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<td>$\Theta$</td>
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Other variables and parameters

<table>
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<tr>
<th>$c_t$</th>
<th>Consumption</th>
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<tr>
<td>$x_{us}$</td>
<td>Unsafe sex consumption</td>
</tr>
<tr>
<td>$x_s$</td>
<td>Safe sex consumption</td>
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<tr>
<td>$\bar{h}_2$</td>
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<tr>
<td>$h_{\text{high}}$</td>
<td>High health outcome for an HIV negative individual</td>
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<td>$h_{\text{high}}^+$</td>
<td>High health outcome for an HIV positive individual</td>
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<td>$q_{x_{us}}$</td>
<td>Price of unsafe sex</td>
</tr>
<tr>
<td>$q_{x_s}$</td>
<td>Price of safe sex</td>
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</table>

By assumption in optimum we have that,

$$U_{x_{us}} = (q_{x_s})^\top \cdot u_{x_{us}} + g_{x_{us}} < 0$$  \hspace{1cm} (A1)

$$U_{x_{us}x_{us}} = (q_{x_{us}})^\top \cdot u_{x_{us}x_{us}} + \beta \cdot (\ln(\varphi))^2 \cdot \varphi^{\omega \omega} \cdot \left[ \rho \cdot u(c_{2}, h_{\text{high}}^+) + (1 - \rho) \cdot u(c_{2}, h_{\text{low}}^+) \right] < 0$$ \hspace{1cm} (A2)

$$U_{x_{s}x_{s}} = u_{x_{s}} + \beta \cdot \varphi^{\omega \omega} \cdot \left[ \rho \cdot u_{x_{s}x_{s}}(c_{2}, h_{\text{high}}^+) + (1 - \rho) \cdot u_{x_{s}x_{s}}(c_{2}, h_{\text{low}}^+) \right]$$

$$+ \beta \cdot (1 - \varphi^{\omega \omega}) \cdot \delta \cdot \left[ \rho \cdot u_{x_{s}x_{s}}(c_{2}, h_{\text{high}}^+) + (1 - \rho) \cdot u_{x_{s}x_{s}}(c_{2}, h_{\text{low}}^+) \right] < 0$$ \hspace{1cm} (A3)

by the second order condition for a maximum. Further, cross derivatives are given by,

$$U_{x_{us}x_{s}} = q_{x_{us}} \cdot q_{x_{s}} \cdot u_{x_{us}x_{s}} + g_{x_{us}x_{s}} < 0$$ \hspace{1cm} (A4)
\[ U_{s,z} = q_{s,z} \cdot u_{v,z} + \beta \cdot \ln(\phi) \cdot \varphi^r \cdot \left[ \rho \cdot u_{z_1} \left( c_2, h_{low}^+ \right) + (1 - \rho) \cdot u_{z_2} \left( c_2, h_{low}^- \right) \right] \]

\[ U_{s,z} = q_{s,z} \cdot u_{v,z} < 0 \]  (A5)

\[ U_{s,y} = 0 \]  (A6)

\[ U_{s,y} = \beta \cdot \varphi^\alpha \cdot \rho \cdot (1 - \rho) \cdot u_{z_{low}} \left( c_2, h_{high}^+ \right) - u_{z_{low}} \left( c_2, h_{low}^+ \right) \]  

\[ U_{s,y} = \beta \cdot \varphi^\alpha \cdot \rho \cdot (1 - \rho) \cdot u_{z_{low}} \left( c_2, h_{high}^+ \right) - u_{z_{low}} \left( c_2, h_{low}^- \right) \]  

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\[ U_{s,y} = \beta \cdot \varphi^\alpha \cdot \rho \cdot (1 - \rho) \cdot u_{z_{low}} \left( c_2, h_{high}^+ \right) - u_{z_{low}} \left( c_2, h_{low}^+ \right) \]  

\[ \Phi \text{ of propositions 1, 3 and 5} \]

If \( S = 0 \), then \( U_s = -u_{z_1} + \varphi^{\alpha} \cdot \beta \cdot [\rho \cdot u_{z_1} (c_2, h_{high}^+) + (1 - \rho) \cdot u_{z_1} (c_2, h_{low}^-)] < 0 \). Focusing on the changes inserted by \( \gamma, h_{low}^+ \) and \( Y_2 \), our equation system is thus reduced to,
From the second order condition for an interior maximum it follows that the determinant is now positive definite \((D > 0)\). By Cramér’s rule, the induced changes in safe and unsafe sex consumption due to changes in the parameters are given by,

\[
\begin{align*}
\frac{\partial x_u}{\partial \gamma} &= \frac{-U_{x,v} \cdot U_{u,v}}{D} \quad & (A18), \\
\frac{\partial x_s}{\partial \gamma} &= \frac{U_{x,v} \cdot U_{u,v}}{D} \quad & (A19), \\
\frac{\partial x_u}{\partial h_s} &= \frac{-U_{x,v} \cdot U_{u,v}}{D} \quad & (A20), \\
\frac{\partial x_s}{\partial h_s} &= \frac{U_{x,v} \cdot U_{u,v}}{D} \quad & (A21), \\
\frac{\partial x_u}{\partial Y_s} &= \frac{-U_{x,v} \cdot U_{u,v}}{D} \quad & (A22), \\
\frac{\partial x_s}{\partial Y_s} &= \frac{U_{x,v} \cdot U_{u,v}}{D} \quad & (A23).
\end{align*}
\]

If and \(\delta = \theta = 0\), then

\[
\begin{align*}
U_{x,v} \big|_{\theta=\delta,0} &= \phi \cdot \ln(\phi) \cdot \beta \cdot \rho \cdot (1-\rho) \cdot \left[ u_{h,\text{low}}(c_2, h_{\text{high}}) - u_{h,\text{low}}(c_2, h_{\text{low}}) \right] > 0 \quad (A9') \\
U_{x,v} \big|_{\theta=\delta,0} &= \ln(\phi) \cdot \phi \cdot \beta \cdot \rho \cdot \left[ u_{h,\text{low}}(c_2, h_{\text{high}}) - u_{h,\text{low}}(c_2, h_{\text{low}}) \right] < 0 \quad (A12') \\
U_{x,v} \big|_{\theta=\delta,0} &= \ln(\phi) \cdot \phi \cdot \beta \cdot \rho \cdot \left[ u_{h,\text{low}}(c_2, h_{\text{high}}) - u_{h,\text{low}}(c_2, h_{\text{low}}) \right] < 0 \quad (A15')
\end{align*}
\]

Since \(U_{x,v} < 0\) by assumption, the results of propositions 1, 3 and 5 follows, q. e. d.

**Proof of proposition 2**

If the individual saves a positive amount for the future, we have to analyze both effects in equations \((20) - (22)\). From equation \((A9')\) we know that if \(\delta = \theta = 0\), then \(U_{x,v} > 0\). By equation \((A8')\), the indirect effect via changes in incentives to save is given by,

\[
U_{x,v} = \phi \cdot \rho (1-\rho) \cdot \left[ u_{h,\text{low}}(c_2, h_{\text{high}}) - u_{h,\text{low}}(c_2, h_{\text{low}}) \right] \quad (A8')
\]

The sign of equation \((A8')\) clearly hinges on relationship between, \(u_{h,\text{low}}(c_2, h_{\text{high}})\) and \(u_{h,\text{low}}(c_2, h_{\text{low}})\). By assumption, \(u_{h,\text{low}}(c_2, h) > 0\). This implies that, if the marginal utility of
consumption increases at a diminishing rate in health \( u_{c_3h}(c_i, h_i) < 0 \), then \( u_{c_3h}(c_3, h_{high}) < u_{c_3h}(c_3, h_{low}) \), and hence \( U_{c_3h} < 0 \).

Next, let us turn to the terms connected to \( U_{c_3h} \) and \( U_{c_3h} \). From the second order conditions for a maximum we know that

\[
D_{11} = \frac{\left( U_{c_3h} \cdot U_{c_3h} - \left( U_{c_3h} \right)^2 \right)}{D} < 0
\]

(A24)

\[
D_{33} = \frac{\left( U_{c_3h} \cdot U_{c_3h} - \left( U_{c_3h} \right)^2 \right)}{D} < 0
\]

(A25)

where \( D < 0 \) is the 3-3 determinant of the LHS in equation (14). This implies that the first effect in equation (20) is unambiguously positive, while the second effect in equation (20) is unambiguously negative. The reduction in the expected value of future health reduces the marginal cost of unsafe sex and the marginal value of savings.

The only term left to sign is \( D_{13} \) given by

\[
D_{13} = \frac{u_{c_3h} \left( \frac{q_{c_3h}}{q_{c_3h}} \cdot g_{c_3h} - g_{c_3h} \right)}{D} - \frac{\beta \ln(\phi) \rho \left( \beta E[u(c_3, h_i)] \right) \left( q_{c_3h}^2 \cdot u_{c_3h} + g_{c_3h} \right)}{D}
\]

(A26)

Now, the second term in equation (A26) is clearly positive. However, the sign of the first term hinges on the relative magnitude of the substitutability between safe and unsafe sex and the concavity of safe sex. If safe and unsafe sex where additively separable \( (g_{c_3h} = 0) \) an increase in other sexual consumption, due to the increase in budget space, would not affect the marginal utility of unsafe sex. On the other hand, if safe and unsafe are perfect
substitutes, so that $g_{x,t} = g_{x,t}$. Then, since $q_{x}/q_{w} < 1$, $|q_{x}/q_{w}| \cdot g_{x,t} < |g_{x,t}|$ and the first
in equation (A26) term is thus still positive, q. e. d..

**Proof of proposition 4**

As in the previous proof of increased uncertainty, a change in expected health creates two
types effects on unsafe sex consumption and savings, presented in equations (25) – (27).
From the proof when $S=0$ we know that if $\delta = \theta = 0$, then $U_{xss} < 0$. With positive
savings we also need to include $U_{xss}$ which, if $\delta = \theta = 0$, is given by

$$U_{xss} = P_{xss} \beta \rho \cdot u_{x,k}(c_{z},h_{z}) + (1-\rho) \cdot u_{x,k}(c_{z},h_{z}) > 0$$

(A11')

The sign of equation (A11') follows from the assumption that $u_{x,k}(c_{z},h_{z}) > 0$. Hence, as in
the proof of proposition 2, the sign of $\partial \cdot \partial h_{x}$ and $\partial S/\partial h_{x}$ hinges on the relative
magnitude of $g_{x,t}$. If $|q_{x}/q_{w}| \cdot g_{x,t} \leq |g_{x,t}|$, $D_{13}$ is clearly positive and hence, $\partial x_{w}/\partial h_{z} < 0$,
while $\partial S/\partial h_{z} > 0$,

q. e. d..

**Proof of proposition 6**

For an increase in future income the induced changes are given by equations (30) – (32).
According to equation (A15'), $U_{xss} < 0$. When $S > 0$, we also need to analyze $U_{xss}$, which
by equation (A14) is negative. Hence, given that $|q_{x}/q_{w}| \cdot g_{x,t} \leq |g_{x,t}|$ ($D_{13} > 0$), we have
two effects with opposite signs for unsafe sex and savings. For unsafe sex the direct effect,
via the perceived increase in the value of the future $-U_{xss} \cdot D_{11}$ is clearly negative, while the
second via the increased budget space for current consumption, $-U_{xss} \cdot D_{13}$ is positive. For
savings the opposite hold. Incentives to reduce unsafe sex increase the potential budget

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share for savings \((-U_{n,s,1} \cdot D_{3,1} > 0)\), while the reduction in the marginal utility of future consumption reduce incentives to save \((-U_{s,1} \cdot D_{3,1} < 0)\), q. e. d.

**Proof of proposition 7**

The direct effects of a change in the coverage rate of HIV treatment on unsafe sex consumption are given by,

\[
U_{x, \delta} = 0
\]

\[
U_{xus, \delta} = -\beta \cdot \ln(\varphi) \cdot q_{us} \cdot [\rho \cdot u(c_2, h_{high}^+) + (1 - \rho) \cdot u(c_2, h_{low}^+)] > 0
\]  

(A27)

(A28)

Hence,

\[
\frac{\partial U_{xus, \delta}}{\partial \delta} = \frac{-U_{xus, \delta} \cdot U_{xus, \delta}}{\delta} > 0
\]

The direct effects of a change in the effectiveness of HIV treatment on unsafe sex consumption are given by,

\[
U_{x, \theta} = 0
\]

\[
U_{xus, \theta} = -\beta \cdot \ln(\varphi) \cdot q_{us} \cdot \delta \cdot \tilde{h}_2 [\rho \cdot u_{h_{high}} + (1 - \rho) \cdot u_{h_{low}}] > 0
\]  

(A29)

(A30)

Hence,

\[
\frac{\partial U_{xus, \theta}}{\partial \theta} = \frac{-U_{xus, \theta} \cdot U_{xus, \theta}}{\theta} > 0, \text{ q.e.d.}
\]

**Proof of proposition 8**

If \(0 < \delta, \theta \leq 1\), \(U_{xus, \gamma}\) is given by,

\[
U_{xus, \gamma} = \beta \cdot \ln(\varphi) \cdot q_{us} \cdot [\rho \cdot (1 - \rho) \cdot \left( u_{h_{high}} - u_{h_{low}} \right) + \delta \cdot \left( u_{h_{high}} - u_{h_{low}} \right) - \delta \cdot \left( u_{h_{high}} - u_{h_{low}} \right)]
\]  

(A31)

Hence if \(\theta = \delta = 1\), then \(|u_{h_{high}} - u_{h_{low}}| = \delta \cdot |u_{h_{high}} - u_{h_{low}}|\), and thus \(U_{xus, \gamma} = \partial x_{us}/\partial \gamma = 0\).

If \(\theta = 1\), and if \(0 < \delta < 1\), then \(|u_{h_{high}} - u_{h_{low}}| > \delta \cdot |u_{h_{high}} - u_{h_{low}}|\). Thus \(U_{xus, \gamma} > 0\) and we thereby have that \(\partial x_{us}/\partial \gamma > 0\).

If \(\delta = 1\), and if \(0 < \theta < 1\), then \(|u_{h_{high}} - u_{h_{low}}| < \delta \cdot |u_{h_{high}} - u_{h_{low}}|\) in absolute value. Thus \(U_{xus, \gamma} < 0\) and we thereby have that \(\partial x_{us}/\partial \gamma < 0\).
If $0 < \delta, \theta < 1$, the sign of equation (A31) remains ambiguous. A differentiation of equation (A9) with respect to $\delta$ and $\theta$ produces the following expressions,

$$\frac{\partial u_{xus}}{\partial \delta} = -\beta \cdot \ln(\phi) \cdot \varphi^{xus} \cdot \rho \cdot (1 - \rho) \cdot \left( u_{h_{high}}^+ - u_{h_{low}}^+ \right) < 0 \quad (A32)$$

$$\frac{\partial u_{xus}}{\partial \theta} = -\beta \cdot \ln(\phi) \cdot \varphi^{xus} \cdot \rho \cdot (1 - \rho) \cdot \delta \cdot \bar{h}_Z \cdot \left( u_{h_{high}}^+ h_{high}^+ - u_{h_{low}}^+ h_{low}^+ \right) \quad (A33)$$

Equation (A32) implies that the smaller $\delta$ is, the greater is the value of $\varphi^{xus}$. Consequently, if $0 < \theta < 1$, then the smaller $\delta$ is, the more likely it is that $\partial x_{us}/\partial \gamma > 0$. The sign of equation (A43) depends on the third order derivative of health related utility. If the third order derivative of future utility with respect to health is greater or equal to zero, then equation (A43) implies that the larger $\theta$ is, the larger is $U_{xus}$. Consequently, the probability that $\partial x_{us}/\partial \gamma > 0$ increases with the magnitude of $\theta$, q. e. d.

**Proof of proposition 9**

If $0 < \delta, \theta \leq 1$, $U_{xus}$ $h_2$ is given by,

$$U_{xus} h_2 = \beta \cdot \ln(\phi) \cdot \varphi^{xus} \cdot \left[ (\rho \cdot u_{h_{high}}^- + (1 - \rho) \cdot u_{h_{low}}^-) - \delta \cdot \theta \cdot (\rho \cdot u_{h_{high}}^+ + (1 - \rho) \cdot u_{h_{low}}^+) \right]$$

$$A34$$

Thus, if $\theta = \delta = 1$, then $(\rho \cdot u_{h_{high}}^+ + (1 - \rho) \cdot u_{h_{low}}^+) = \delta \cdot \theta \cdot (\rho \cdot u_{h_{high}}^+ + (1 - \rho) \cdot u_{h_{low}}^+)$, and

$$U_{xus} h_2 = \partial x_{us}/\partial \bar{h}_Z = 0.$$

If $\theta = 1$, and $0 < \delta < 1$, then $|\rho \cdot u_{h_{high}}^+ + (1 - \rho) \cdot u_{h_{low}}^-| > \delta \cdot \theta \cdot |\rho \cdot u_{h_{high}}^+ + (1 - \rho) \cdot u_{h_{low}}^+|$.

Hence $U_{xus} h_2 < 0$, and we thereby have that $\partial x_{us}/\partial \bar{h}_Z < 0$.

If $\delta = 1$, and $0 < \theta < 1$ equation (A34) is not signed. However, by differentiating the expression $\theta \cdot (\rho \cdot u_{h_{high}}^+ + (1 - \rho) \cdot u_{h_{low}}^+)$ in equation (A42) with respect to $\theta$ and rearranging terms we get the expression,
\[
\frac{\partial}{\partial \theta} \left[ \theta \cdot \left( \rho \cdot u_{h_{\text{high}}}^* + (1 - \rho) \cdot u_{h_{\text{low}}}^* \right) \right] = \left[ 1 + \frac{\theta \cdot \bar{h}_2 \cdot \left( \rho \cdot u_{h_{\text{high}}}^* + (1 - \rho) \cdot u_{h_{\text{low}}}^* \right)}{\left( \rho \cdot u_{h_{\text{high}}}^* + (1 - \rho) \cdot u_{h_{\text{low}}}^* \right)^2} \right] \]

\[\text{(A35)}\]

The expression on the RHS of equation (A35) is positive if the second term within the square brackets is smaller than one in absolute value and negative if the term is larger than one in absolute value. It is tempting to describe the second term within square brackets as some form of discrete measure of relative risk aversion.\(^{32}\) Let us denote this measure DR. As \(\rho \cdot u_{h_{\text{high}}}^* + (1 - \rho) \cdot u_{h_{\text{low}}}^*\), we have the following results;

i. If \(\delta = 1, \ 0 < \theta < 1, \text{ and } DR > 1, \text{ then } \partial x_{us}/\partial \bar{h}_2 > 0\)

ii. If \(\delta = 1, \ 0 < \theta < 1, \text{ and } DR < 1 \text{ then } \partial x_{us}/\partial \bar{h}_2 < 0\)

iii. If \(\delta = 1, \ 0 < \theta < 1, \text{ and } DR = 1 \text{ then } \partial x_{us}/\partial \bar{h}_2 = 0\) \(^{33}\), q. e. d.

If \(0 < \theta, \delta < 1\), the sign of equation (A33) depends both on the magnitude of DR and on the size of \(\delta\). By differentiating equation (A35) with respect to \(\delta\) we see that,

\[
\frac{\partial u_{x_{us}}}{\partial \delta} = -\beta \cdot \ln(\phi) \cdot \phi \cdot \left( \rho \cdot u_{h_{\text{high}}}^* + (1 - \rho) \cdot u_{h_{\text{low}}}^* \right) > 0
\]

\[\text{(A36)}\]

Hence, if \(0 < \theta < 1\) the larger \(\delta\) is, the more likely it is that \(U_{x_{us}} > 0\) and therefore that \(\partial x_{us}/\partial \bar{h}_2 > 0\) q. e. d.

\(^{32}\) This measure should be compared to the continuous measure of relative risk aversion; \(-u''(x) \cdot x/u'(x)\)

\(^{33}\) Empirical estimates concerning the size of the relative risk aversion coefficient vary substantially, but estimates are commonly larger than one (cf. Meyer, D. J., & Meyer, J., 2005). However, most empirical estimates have mostly been based on data from financial economics. These estimates have been shown to be inconsistent with the income elasticity of the value of statistical life (VSL), which is commonly estimated at 0.5-0.6 (Kaplow, L., 2005), consequently, we know little of the size of the relative risk aversion concerning future health, and even less of expression depicted in equation (A43).