General Income Taxation, Public Goods and Decentralized Leadership*

Thomas Aronsson
Department of Economics, Umeå University
SE - 901 87 Umeå, Sweden

August 2007

Abstract

This paper concerns redistribution and public good provision under asymmetric information, which are here ingredients of a policy-problem facing each member state (nation) of an economic federation with decentralized leadership. Each member state is assumed to have its own redistributive policy and pattern of public consumption, whereas the federal level redistributes (ex-post) between the member states. The results show how and why federal ex-post redistribution may modify the use of income taxation and public good provision at the national level, relative to the policy outcome in the absence of a federal government, as well as how the national policy incentives depend on whether or not the federal government uses distortionary taxes.

Keywords: Income taxation, redistribution, public goods, fiscal federalism, decentralized leadership.

JEL classification: D31, D60, D82, H21

*The author would like to thank Olof Johansson-Stenman, Tomas Sjögren and Magnus Wikström for helpful comments and suggestions. A research grant from Jan Wallander’s and Tom Hedelius’ Foundation is also gratefully acknowledged.
1 Introduction

There is a growing literature dealing with optimal nonlinear taxation and public good provision under asymmetric information\(^1\). Earlier studies are typically based on models of a single jurisdiction with one single government. Real world economies, on the other hand, are characterized by several levels of government, where each such level has responsibility for its own activities and funds. Therefore, although earlier research gives considerable insight into the incentives underlying tax and expenditure policies, it tends to abstract from the institutional structure within which public policies are decided upon. In a European context, this fiscal federalism argument is probably even stronger than before, as part of the redistributive policy is now formally decided upon by a supranational level. An important question, therefore, is how the redistribution carried out by a supranational authority affects the use of income tax and expenditure policies at the national level. The present paper concerns the incentive structure underlying public policy at the national (member state) level, by analyzing redistributive nonlinear income taxation and public good provision as ingredients of a policy-problem facing each member state of an economic federation with decentralized leadership.

Why is this particular extension of the optimal income tax model interesting to consider? It has been argued in earlier research (see below) that the European Union (EU) exemplifies an economic federation with decentralized leadership. This is so for at least two reasons. First, the EU is still in the

\(^1\)Seminal contributions to the theory of nonlinear income taxation and/or mixed taxation are Mirrlees (1971), Phelps (1973), Atkinson and Stiglitz (1976), Mirrlees (1976), Sadka (1976), Atkinson (1977), Stern (1982), Stiglitz (1982) and Edwards et al. (1994). See also the related studies dealing with public good provision under nonlinear taxation; e.g. Christiansen (1981) and Boadway and Keen (1993).
process of being developed, and the national (member state) governments may already have precommitted to policies based on their own objectives. Second, the decision-structure of the EU implies that the member states have a significant influence over union policy via the Council of Ministers, which is comprised of ministers from the member state governments.

The division of responsibilities that this paper attempts to capture is that, while each member state has its own revenue collection, redistributive policy and pattern of public consumption, there is also redistribution between the member states that is decided upon by the federal level. Again, the EU serves as a raw model in the sense that the structural funds offer a broad spectrum of means for redistribution between countries. I will take the idea of decentralized leadership to its extreme point by analyzing how 'ex-post' redistribution carried out by a federal government affects the incentives underlying the use of redistributive nonlinear income taxation and public good provision at the national level, meaning that the national level is assumed to act as first mover vis-à-vis the federal level. The decision-problem facing each national government is based on the two-type optimal income tax model developed by Stern (1982) and Stiglitz (1982), which is here extended in such a way that the national government recognizes (and incorporates into its decision-problem) how its contributions to, and benefits from, federal redistribution are affected by its own policies.

The study of optimal taxation and public expenditures in the context of economic federations has so far mainly focused on centralized leadership, where the federal government is able to commit to its policies, whereas the lower level governments are not. A major issue dealt with in earlier literature is how the federal government should behave in order to implement a socially optimal resource allocation, where correction for fiscal externalities and/or
redistribution are important ingredients of the federal decision-problem\textsuperscript{2}. Only a few earlier studies have examined the incentive structure underlying public policy in economic federations with decentralized leadership, and most of them use the EU as a starting point for the analysis. An important question here is how federal ex-post redistribution affects the incentives for public good provision at the lower (e.g. national) level. Caplan et al. (2000) consider an economy with federal lump-sum redistribution between the local jurisdictions and spillover effects of local public goods. In their study, the redistribution carried out by the federal government is directed towards the private sector (i.e. private consumption), and they find that decentralized leadership gives rise to an efficient outcome. On the other hand, as shown by Köthenburger (2007), if the federal (lump-sum) redistribution is, instead, directed towards the public funds at the lower level of government and the public good is impure, then this result no longer applies, meaning that decentralized leadership might lead to inefficient provision of public goods\textsuperscript{3,4}. Other issues dealt with in earlier literature on decentralized leadership are tax competition\textsuperscript{5} and environmental policy\textsuperscript{6}.

To my knowledge, there are no earlier studies dealing with redistributive nonlinear taxation in an economic federation with decentralized leadership, where each lower level government redistributes income among its own res-


\textsuperscript{3}See also, e.g., Broadway et al. (2003).

\textsuperscript{4}The main issue in the study by Köthenburger (2007) is how federal ex-post redistribution affects the incentives underlying federal (ex-ante) corrective policies.

\textsuperscript{5}See Köthenburger (2004).

idents as well as rationally anticipates how its own policy-choices affect the transfer payment it receives from (or the contribution it gives to) the federal level. The present paper considers an economy comprising a number of lower-level jurisdictions, to be called 'countries' in what follows, and a federal government which redistributes resources between the lower level jurisdictions. To be able to focus on how the combination of federal redistribution and decentralized leadership affects the policies decided upon at the national level, I will disregard any horizontal interaction between lower level jurisdictions such as policy induced factor mobility or spillover effects of national public goods. This does not reflect a belief that horizontal interaction is unimportant; only that it is well understood from earlier literature, whereas the aspects of public policy addressed here are not.

The major questions to be examined are (i) how does the combination of federal redistribution and decentralized leadership modify the use of income taxation and public good provision at the national level relative to the use of these instruments without federal redistribution, and (ii) how does the use of distortionary finance at the federal level affect the policy rules for marginal income taxation and public provision chosen by the national governments? I will consider two versions of the model; Cases I and II, respectively. Case I means that the federal government redistributes lump-sum (from an ex-post perspective) between the national governments, whereas Case II implies that the federal government raises revenue for redistributive purposes by imposing a tax (or fee) on each country, which is proportional to labor income. This distinction is reasonable in the sense that, while the assumption of lump-sum redistribution at the federal level provides a simpler model and seems to be in accordance with most earlier comparable studies, distortionary finance is a more realistic description of the options for funding available at the
supranational level.

The paper contributes to the literature in three ways. First, it explains how and why the combination of federal redistribution and decentralized leadership may modify the use of nonlinear income taxation and public good provision at the national level relative to the outcome that would be chosen by each country in the absence of federal redistribution. Second, it exemplifies how the set of policy instruments at the federal level is important from the perspective of national public policy. Third, there are relatively few earlier studies dealing with redistribution both within and between countries, and the paper contributes to this literature by considering the role of general income taxation in combination with intergovernmental transfers.

The outline of the paper is as follows. Section 2 presents the model and the outcome of private optimization. The optimal tax and expenditure policies are analyzed in Section 3. The results are summarized and discussed in Section 4.

2 The Model

Consider an economic federation comprising a given number of lower-level jurisdictions, whose governments behave as Nash competitors to one another. As the number of lower level jurisdictions is, itself, of no concern, it will be normalized to two. Each lower level jurisdiction will be referred to as a ‘country’ in what follows and consists of two types of immobile residents; a low-ability type (denoted by superindex 1) and a high-ability type (denoted by superindex 2). The distinction between ability-types refers to productivity, which is interpreted to mean that the high-ability type faces a higher

\footnote{Note that, although the EU is not formally equipped with the power of taxing its residents, the major parts of the national contributions are proportional to the national income and to the tax base for the value added tax.}
gross wage rate than the low-ability type. As the number of individuals of each ability-type in each country is not important for the qualitative results derived below, it will be normalized to one.

The utility function facing ability-type \( i \) in country \( j \) is written

\[
    u^i_j = u(c^i_j, z^i_j, g_j) = \tilde{u}(c^i_j, z^i_j) + v(g_j)
\]

where \( c \) is the consumption of a private good, \( z \) leisure and \( g \) the consumption of a (national) public good. Leisure is, in turn, defined as a time endowment, \( H \), less the time spent in market work, \( l \). The function \( u(\cdot) \) is increasing in each argument and strictly quasi-concave. I also assume that the consumer treats \( g \) as exogenous. The expression after the second equality means that the public good is additively separable from the other goods in the utility function; this assumption simplifies the analysis and allows me to derive unambiguous results (in Case I). However, although separable utility functions are fairly common in the literature dealing with public policy in economic federations, I will comment more on the consequences of this assumption below.

The budget constraint facing the consumer is given by

\[
    w^i_j l^i_j - T_j(w^i_j l^i_j) - c^i_j = 0
\]

where \( w \) is the gross wage rate; as indicated above, \( w^2_j > w^1_j \). The function \( T_j(\cdot) \) is the income tax, which is decided upon by the national government. The price of the private consumption good has been normalized to one for notational convenience.

By substituting the budget constraint into the objective function, the first order condition for the hours of work can be written as

\[
\frac{\partial u^i_j}{\partial c^i_j} w^i_j [1 - T'_j(w^i_j l^i_j)] - \frac{\partial u^i_j}{\partial z^i_j} = 0
\]
where \( T'_j(w^j_l^i) = \partial T_j(w^j_l^i)/\partial(w^j_l^i) \) is the marginal income tax rate. This (standard) labor supply condition will be slightly modified in Case II below, where the federal government also collects revenue by using distortionary taxation. For further use, note also that the assumption of additive separability means that \( l_j^i \) does not depend directly on \( g_j \).

Turning to the production side, we follow much of the earlier literature on optimal income taxation in assuming that output is produced by a linear technology, such that the producer price and the wage rates are fixed.

3 Tax and Expenditure Policies

The objective function facing each national government is assumed to be a generalized Utilitarian social welfare function\(^8\) with distributional weights attached to the utility of each ability-type. The social welfare function in country \( j \) is written as

\[
U_j = \sum_i \alpha^i_j u^i_j
\]

(4)

where \( \alpha^i_j \) is the distributional weight that the national government in country \( j \) attaches to ability-type \( i \) and \( \sum_i \alpha^i_j = 1 \). In a similar way, the objective function facing the federal government is also a generalized Utilitarian social welfare function.

---

\(^8\)An alternative formulation would be to assume that each national government maximizes the utility of one of the ability-types subject to a minimum utility restriction for the other. This formulation would give the same results in terms of expressions for the marginal income tax rates and policy rule for public provision as those derived below. I have chosen to use a social welfare function because this approach is slightly more convenient in the context of a multi-level government structure. This is also in accordance with most earlier studies on public policy in economic federations referred to in the introduction.
\[ U = \sum_j \beta_j U_j \]  

in which \( \beta_j \) is the distributional weight that the federal government attaches to the welfare of country \( j \) and \( \sum_j \beta_j = 1 \).

As mentioned in the introduction, two versions of the optimal tax and expenditure problem will be discussed, where the distinction refers to the set of policy instruments faced by the federal government. Case I means that the federal government redistributes lump-sum between the national governments (ex-post), whereas Case II implies that the federal government raises revenue for purposes of redistribution by imposing a tax (or fee) on each country, which is proportional to labor income.

### 3.1 Case I: The Federal Government Redistributes Lump-Sum

I begin by discussing the policy chosen by the federal government, which is follower, and then continue by analyzing the policies chosen by the national governments.

#### 3.1.1 The Federal Government

The federal government chooses the (positive or negative) transfer payments, \( s_1 \) and \( s_2 \), in order to maximize the objective function given by equation (5) subject to

\[ \sum_j s_j = 0 \]  

\[ \sum_i T_j(w_j^i l_j^i) + s_j - g_j = 0 \]  

The federal government treats the parameters of \( T_j(\cdot) \) as exogenous. In addition, since \( T_j(w_j^i l_j^i) = w_j^i l_j^i - c_j^i \), and \( l_j^i \) does not depend directly on \( s_j \)
according to equation (3), this means that the federal government treats $l_j^i$ and $c_j^i$ as exogenous. The first order condition can be written

$$
\beta_j \sum_i \alpha_j^i \frac{\partial u_j^i}{\partial g_j} - \beta_k \sum_i \alpha_k^i \frac{\partial u_k^i}{\partial g_k} = 0 \quad (8)
$$

for $k \neq j$, meaning that the federal government equalizes a weighted average of marginal utilities of public consumption. Equation (8) implicitly defines the transfer payment to country $j$, $s_j$, as a function of variables which are controlled by each national government (via the general income tax). By using equations (7) and (8) together with $T_j(w_j^i l_j^i) = w_j^i l_j^i - c_j^i$, the reaction function can be written as

$$
s_j = s_j(l_j^1, c_j^1, l_j^2, c_j^2, l_k^1, c_k^1, l_k^2, c_k^2) \quad (9)
$$

where the constant terms have been suppressed for notational convenience. It is straightforward to show that $\partial s_j / \partial l_j^i < 0$ and $\partial s_j / \partial c_j^i > 0$ for $i = 1, 2$.

### 3.1.2 The National Government

The national government in country $j$ maximizes the objective function given by equation (4) subject to a self-selection constraint, a budget constraint and the reaction function defined by equation (9).

The government is assumed to observe the gross income of each individual, although ability is private information. The latter means that the government does not know whether any given individual is a low-ability type or a high-ability type. Following the convention in much of the earlier literature on optimal income taxation, I assume that the purpose of redistribution is to redistribute from the high income to the low income earners. This means that the most interesting aspect of self-selection will be to prevent the high-ability type from mimicking the low-ability type. The utility facing the mimic becomes
\[ \hat{u}^2_j = u(c^1_j, H - \phi_j l^1_j, g_j) \]  

where \( \phi_j = w^1_j / w^2_j \) is the wage ratio (relative wage rate). One may interpret \( \phi_j l^1_j \) as the labor that the mimicker needs to supply in order to reach the same income as the low-ability type. Therefore, the self-selection constraint that may bind becomes

\[ u^2_j - \hat{u}^2_j \geq 0 \]  

implying that the high-ability type weakly prefers the allocation intended for him/her over the gross labor income and consumption intended for the low-ability type. Note also that, since the population in each country is immobile by assumption, there is no need for other self-selection constraints than that referring to the incentives of the high-ability type to mimic the low-ability type in the same country.

To define the budget constraint facing the government, note once again that \( T_j(\cdot) \) is a general income tax; it can be used to implement any desired combination of work hours and private consumption for each ability-type. As a consequence, it is convenient to use \( l^1_j, c^1_j, l^2_j \) and \( c^2_j \), instead of the parameters of \( T_j(\cdot) \), as direct decision variables of the government in country \( j \). Therefore, the budget constraint will be written as follows:

\[ \sum_i (w^i_j l^i_j - c^i_j) + s_j - g_j = 0 \]  

in which \( T_j(w^i_j l^i_j) = w^i_j l^i_j - c^i_j \) has been used.

The Lagrangean is given by

\[ L_j = U_j + \lambda_j [u^2_j - \hat{u}^2_j] + \gamma_j \sum_i (w^i_j l^i_j - c^i_j) + s_j - g_j \]  

where \( s_j \) is given by equation (9). The first order conditions are presented
in the Appendix. Let me here discuss what these conditions imply in terms of tax and expenditure policies.

**Income Taxation**

The income tax structure is implicitly defined by equations (9) and (A1)-(A4). Let

\[
MRS_{x,c}^{i,j} = \frac{\partial u_j^i}{\partial z_j^i} \quad \text{for } i = 1, 2 \quad \text{and} \quad M\hat{R}S_{x,c}^{i,j} = \frac{\partial \hat{u}_j^2}{\partial z_j^2}
\]

de note the marginal rate of substitution between leisure and private consumption for ability-type \(i\) and the mimicker, respectively, where \(z_j^2 = H - \phi_j l_j^1\). By using the short notations

\[
A_j = \beta_j \sum \alpha_j^i \frac{[\partial^2 u_j^i/\partial (g_j)^2]}{A} < 0, \quad A = \sum A_j < 0 \quad \text{and} \quad \sigma_j = 1 - A_j / A \in (0, 1)
\]

which are further discussed in the Appendix, the marginal income tax structure is characterized as follows;

**Proposition 1** The combination of federal lump-sum redistribution and decentralized leadership means that the national marginal income tax rates can be written as

\[
T'_j(w_j^1 l_j^1) = \frac{1}{\sigma_j} \left\{ \frac{\lambda_j^*}{w_j^1} [MRS_{x,c}^{i,j} - \phi_j M\hat{R}S_{x,c}^{i,j}] \right\}
\]

\[
T'_j(w_j^2 l_j^2) = 0
\]

where \(\lambda_j^* = \lambda_j (\partial \hat{u}_j^2 / \partial c_j^1) / \gamma_j\).

Proof: see the Appendix.

To understand how federal ex-post redistribution affects the marginal income tax structure, it is useful to compare the tax formulas in Proposition
1 with those that would apply if there were no federal government. If \( s_j \equiv 0 \), and within the given framework, the marginal income tax rate of the high-ability type would be equal to zero (as in the Proposition), whereas the marginal income tax rate for the low-ability type would become \( T^*_j(w^1_j) = \lambda_j^*[MRS^{i,j}_z - \phi_j^*MRS^{i,2}_z]/w^1_j > 0 \). The special case with \( s_j \equiv 0 \) reproduces the results derived in the conventional optimal income model with two ability-types; see Stiglitz (1982).

With this ‘benchmark case’ in mind, note that federal ex-post redistribution provides an incentive for the national government to reduce its own tax revenues, ceteris paribus, which is due to the desire to increase the transfer payment from the federal government. This may, in turn, be accomplished by altering the lump-sum components of the income tax. As a consequence, there is still no need to distort the labor supply decision made by the high-ability type\(^9\).

On the other hand, the expression for the marginal income tax rate of the low-ability type does not take the same form as in the conventional optimal income tax model. This is seen by the appearance of scale term \( 1/\sigma_j^1 > 1 \) in the proposition, which contributes to increase the marginal income tax rate of the low-ability type via the self-selection constraint. The intuition is that, as the national government attempts to attract federal resources via lower tax revenues (increased private consumption and/or fewer hours of work), it also changes the consumption-leisure point for the low-ability type in such a way that mimicking becomes more attractive (i.e. the self-selection constraint tightens for any given consumption-leisure point of the

\(^9\)The zero marginal income tax rate for the high-ability type in Proposition 1 would not carry over to a more general, nonseparable, utility function. In this case, the marginal income tax rate facing the high-ability type can be either positive or negative, depending on how the public good affects the marginal utility of private consumption and leisure, respectively.
high-ability type); as effect which the higher marginal income tax rate serves to avoid.

Public Good Provision

A cost benefit rule for the provision of the public good comparable to those described in earlier literature can be derived by using equations (9), (A2), (A4) and (A5). Let

\[
MRS_{g,c}^{j,i} = \frac{\partial u_j^i}{\partial g_j} \quad \text{for } i = 1, 2 \quad \text{and} \quad M\hat{R}S_{g,c}^{j,2} = \frac{\partial \hat{u}_j^2}{\partial c_j^i}
\]

denote the marginal rate of substitution between the public good and private consumption for ability-type \(i\) and the mimicker, respectively. Consider Proposition 2;

**Proposition 2** The combination of federal lump-sum redistribution and decentralized leadership means that the provision of the national public good is characterized by

\[
\sum_i MRS_{g,c}^{j,i} + \lambda_j^i (MRS_{g,c}^{i,1} - M\hat{R}S_{g,c}^{i,2}) = 1 + \sum_i MRS_{g,c}^{j,i} \frac{\partial s_j}{\partial c_j^i}
\]

Proof: see the Appendix.

The first term on the left hand side is the sum of marginal rates of substitution between the public good and private consumption, and the second is the additional effect created by the self-selection constraint. The first term on the right hand side is the marginal rate of transformation between the public good and the private consumption good (which is here equal to one). If the low-ability type values the public good more (less) at the margin than does the mimicker - and in the absence of federal redistribution (i.e. \(s_j \equiv 0\)) - this means overprovision (underprovision) of the public good relative to the Samuelson rule. These effects are well understood from earlier research; see e.g. Broadway and Keen (1993).
The new aspect emphasized here is the additional incentive for public good provision created by federal redistribution, which is summarized by the second term on the right hand side\textsuperscript{10}. As increased private consumption contributes to increase the transfer payment from the federal government (which captures that federal redistribution undermines the motive to collect revenues), i.e. \( \partial s_j / \partial c_j > 0 \) for \( i = 1, 2 \), there is an incentive for the national government to increase the private consumption and, therefore, provide the public good such that the left hand side of the formula in the proposition exceeds the marginal rate of transformation. In other words, the second term on the right hand side provides an incentive for the national government to provide a smaller public good than it would otherwise have done.

3.2 Case II: Distortionary Finance at the Federal Level

In the model set out in Section 2, there is no distinction between production and income. I will, therefore, describe the fee imposed by the federal government as a country-specific income tax\textsuperscript{11}, meaning that the budget constraint facing ability-type \( i \) in country \( j \) changes to read

\[
 w_j^i l_j^i - T_j (w_j^i l_j^i) - t_j w_j^i l_j^i - c_j^i = 0
\]

where \( t_j \) is the federal tax rate. This means, in turn, that the first order condition for the hours of work can be rewritten as

\textsuperscript{10}A similar policy rule for a public good under federal lump-sum redistribution and decentralized leadership - although in the absence of redistributive nonlinear taxation - has been derived by Köthenburger (2007).

\textsuperscript{11}Even if the assumption of distortionary finance at the federal level adds realism by comparison with lump-sum redistribution, the purpose here is not to provide a description of member state contributions to the EU. The idea is, instead, to contrast the analysis carried out in the previous subsection with another extreme case (possibly with the interpretation of European federalism extended with federal power of taxation).
\[
\frac{\partial w^i_j}{\partial c^i_j} w^i_j [1 - T^i_j (w^i_j t^i_j) - t_j] - \frac{\partial w^i_j}{\partial z^i_j} = 0 \tag{15}
\]

I show in the Appendix that the partial derivative of \( l^i_j \) with respect to \( t_j \) can be written as a function of \( c^i_j \), \( l^i_j \), and \( T^i_{j,\text{sec}} \), i.e.

\[
\frac{\partial l^i_j}{\partial t_j} = \kappa^i_j = \Upsilon_j (c^i_j, l^i_j, T^i_{j,\text{sec}}) \tag{16}
\]

Equation (16) contains the variables that determine how the labor supply responds to a change in the federal tax rate: given that the national government controls the consumption and hours of work via the intercept and slope of its own income tax function, it can control the labor supply response to change in the federal tax rate via the second derivative of its own income tax function. As long as the labor supply is monotonous in \( t_j \), which we assume here for simplicity, the function \( \Upsilon_j (\cdot) \) will be monotonous in \( T^i_{j,\text{sec}} \).

In this case, therefore, \( \kappa^i_j \) is the direct instrument\(^{12} \) corresponding to the tax instrument \( T^i_{j,\text{sec}} \).

3.2.1 The Federal Government

Let me once again begin by characterizing the decision-problem of the follower. The federal government decides upon the tax rates, \( t_1 \) and \( t_2 \), and the transfer payments, \( s_1 \) and \( s_2 \). The optimization problem can be written as

\(^{12}\)The idea that the first mover government - if it has access to a general income tax - uses the second derivative of its income tax function in order to control how the labor supply responds to variables formally decided upon by other decision-makers is not new in the theory of optimal income taxation. A similar approach, although in a different context, is taken by Aronsson and Sjögren (2004) in their study of optimal income taxation and union wage formation, and by Aronsson and Blomquist (2007) in their study of optimal income taxation and public goods in an economic federation with centralized leadership.
\[ \text{Max}_{t_1, t_2, s_1, s_2} \beta_j \sum_i \alpha_j u(w^{i,j}_{j} - T_j(w^{i,j}_{j}) - t_j w^{i,j}_{j}, z^{i,j}_j, \sum_i T_j(w^{i,j}_{j}) + s_j) \]  

subject to the budget constraint\(^{13}\)

\[ \sum_j [t_j \sum_i w^{i,j}_{j} - s_j] = G \]  

the nonnegativity constraints, \( t_1, t_2, s_1, s_2 \geq 0 \) as well as subject to the labor supply defined by equation (15). Note that equation (17) includes the private budget constraints as well as captures that the federal government can influence the public consumption at the national level via the national budget constraints, i.e. I have used \( c^i_j = w^{i,j}_{j} - T_j(w^{i,j}_{j}) - t_j w^{i,j}_{j} \) and \( g_j = \sum_i T_j(w^{i,j}_{j}) + s_j \). The federal government is assumed to treat the parameters of each national income tax function (intercept, slope, second derivative, etc.) as exogenous.

Consider now the policies directed to any country, \( j \). Suppose, to begin with, that we have an interior solution for the federal policy variables facing country \( j \), i.e. \( t_j > 0 \) and \( s_j > 0 \). The first order conditions are presented in the Appendix. From the first order conditions, and by using \( b^i_j = w^{i,j}_{j} - T_j(w^{i,j}_{j}) \) to denote the private income net of the national income tax payment, one can define reaction functions\(^{14}\) for the federal tax rate and transfer payment facing country \( j \);

\(^{13}\)The variable \( G \geq 0 \) is interpretable in terms of an exogenous revenue requirement, which is intended to capture the possibility that the federal government uses part of its tax (or fee) revenues for other purposes than lump-sum subsidies to the national governments.

\(^{14}\)Equations (19) and (20) presuppose an interior solution for the tax rate imposed on the other country by the federal government, i.e. \( t_k > 0 \). If, instead, \( t_k = 0 \) and \( g_k > 0 \), the variables \( \kappa^1_k \) and \( \kappa^2_k \) (which are related to the second derivatives of the income tax function facing the two ability-types in country \( k \)) would not appear in the reaction functions relevant for country \( j \). However, this distinction is not important for the characterization of the policies chosen by country \( j \) below.
\[ t_j = t_j(l^1_j, b^1_j, l^2_j, b^2_j, k_j, \kappa^1_j, \kappa^2_j) \quad (19) \]
\[ s_j = s_j(l^1_j, b^1_j, l^2_j, b^2_j, k_j, \kappa^1_j, \kappa^2_j) \quad (20) \]

for \( k \neq j \). Therefore, in a way similar to the analysis carried out in subsection 3.1, the choice made by the federal government to tax and subsidize country \( j \) depends on variables decided upon by both national governments\(^{15}\).

### 3.2.2 The National Government

By analogy to the analysis carried out in subsection 3.1, the national government in country \( j \) maximizes the objective function given by equation (4) subject to the self-selection constraint and the budget constraint, as well as subject to the reaction functions discussed above. Now, rewrite the utility function in such a way that it reflects the policy instruments of the national government, i.e.

\[ u^i_j = u(b^i_j - t_j w^i_j l^i_j, z^i_j, g_j), \quad (21) \]

and recall that the objective function of the national government is given by \( U_j = \sum_i \alpha^i_j u^i_j \). We can write the decision-problem facing the government in country \( j \) as if it chooses \( l^1_j, b^1_j, l^2_j, b^2_j, \kappa^1_j, \kappa^2_j \) and \( g_j \) to maximize the Lagrangean

\[ L_j = U_j + \lambda_j[\bar{u}^2_j - \bar{u}^2_j] + \gamma_j[\sum_i\{w^i_j l^i_j - b^i_j\} + s_j - g_j] \quad (22) \]

subject to the reaction functions given by equations (19) and (20).

\(^{15}\)This means that, although each national government has a flexible tax instrument (by comparison with the federal level), it is not able to perfectly control the domestic welfare effects of the tax rate imposed on the domestic residents by the federal government.
The first order conditions are presented in the Appendix. I will here discuss what these conditions mean in terms of optimal tax and expenditure policies. Let

\[
\Delta_{l_j}^i = \frac{\partial L_j}{\partial t_j} \frac{\partial t_j}{\partial l_j^i} \gamma_j \frac{\partial s_j}{\partial l_j^i} \\
\Delta_{b_j}^i = \frac{\partial L_j}{\partial b_j} \frac{\partial b_j}{\partial l_j^i} \gamma_j \frac{\partial s_j}{\partial b_j^i}
\]

denote the welfare effects of an increase \(l_j^i\) and \(b_j^i\), respectively, that arise via the reaction functions for the federal decision-variables, i.e. via equations (19) and (20). In other words, \(\Delta_{l_j}^i\) and \(\Delta_{b_j}^i\) summarize the domestic welfare effects of \(l_j^i\) and \(b_j^i\), respectively, that the national government anticipates from the induced responses in the federal decision-variables. The choices made by the national government in country \(j\) are characterized as in Proposition 3:

**Proposition 3** The combination of distortionary finance at the federal level and decentralized leadership means that the national marginal income tax structure and public good provision are characterized by

\[
T_j'(w_j^1 l_j^1) = \frac{1}{\Gamma_j} \frac{\lambda_j^i}{w_j^1} \{MRS_{z_c} - \phi_j MRS_{z_c}^1 - \frac{1}{\gamma_j} \{ \Delta_{l_j}^i + w_j^1 \Delta_{b_j}^i \}}
\]

\[
T_j'(w_j^2 l_j^2) = -\frac{1}{\Gamma_j^2 \gamma_j} [\Delta_{l_j}^i + w_j^2 \Delta_{b_j}^i]
\]

\[
\sum_i MRS_{g,c}^{i,j} + \lambda_j^* [MRS_{g,c}^{i,j,1} - \hat{R}S_{g,c}^{i,j,2}] = 1 + \frac{1}{\gamma_j} \sum_i MRS_{g,c}^{i,j} \Delta_{b_j}^i
\]

where \(\Gamma_j = 1 - \Delta_{b_j}^i / \gamma_j\).

Proof: see the Appendix.
For purposes of interpretation, I add the assumption\(^{16}\) that \(\Gamma^i_j > 0\). Consider first the expression for the national marginal income tax rate facing the low-ability type. The terms within the first curly bracket on the right hand side originate from the self-selection constraint (with the same interpretation as in the absence of federal redistribution), whereas the terms within the second curly bracket reflect an incentive to adjust the marginal income tax rate in accordance with the anticipated responses in federal policy. If \(\Delta^j_{ij} < 0\), this means that an increase in the hours of work supplied by the low-ability type leads to an indirect welfare loss for country \(j\) via the reaction functions for the federal decision-variables. As a consequence, there is an additional incentive for the national government to reduce the hours of work, which it accomplishes by choosing a higher marginal income tax rate than it would otherwise have done. The opposite argument applies in the case where \(\Delta^j_{ij} > 0\).

Note also that the incentive effect caused by \(\Delta^j_{ij}\) is either reinforced or counteracted by a budget effect, \(\Delta^j_{bj}\), as a change in the hours of work will affect the public revenues. To exemplify, suppose first that \(\Delta^j_{ij} < 0\). As pointed out above, this is interpretable as an additional incentive for the national government to reduce the hours of work via a higher marginal income tax rate which, in turn, contributes to lower tax revenues with \(b^j\) held constant. Therefore, to retain budget balance, one may reduce \(b^j\); if \(\Delta^j_{bj} > 0\) \((< 0)\), the federal policy response to a decrease in \(b^j\) leads to lower (higher) welfare in country \(j\), ceteris paribus, which the national government tries to counteract (further explore) by choosing a lower (higher) marginal income tax rate than it would have done in the absence of this budget effect.

\(^{16}\)If \(\Gamma^i_j < 0\), the effect of the self-selection constraint on the marginal income tax rate of the low-ability type would be qualitatively the opposite by comparison with the second best (i.e. in the absence of federal redistribution).
If $\Delta_{t_{ij}}^j > 0$, on the other hand, the budget effect gives rise to incentives opposite to those just described. The expression for the national marginal income tax rate of the high-ability type is analogous to, and has the same interpretation as, the corresponding expression in the formula for the low-ability type discussed above.

Turning to the formula for public provision, the novel aspect is again the second term on the right hand side. In a way similar to Proposition 2, if $\Delta_{s_{ij}}^j > 0$ (which is the most likely outcome), it follows that increased private income (or decreased tax collection at the national level) contributes to an indirect welfare increase via the reaction functions for the federal decision-variables. This again exemplifies that federal redistribution may undermine the motive to collect tax revenue which, in turn, leads to less provision of the public good.

Note finally that the characterization of the marginal income tax rates and public good provision in Proposition 3 is also applicable to the possible corner solutions for the federal decision-variables, i.e. either $t_j \equiv 0$ or $s_j \equiv 0$, if we modify the expressions for $\Delta_{t_{ij}}^j$ and $\Delta_{s_{ij}}^j$ accordingly.

4 Summary and Discussion

This paper considers redistribution and public good provision as part of a policy-problem facing each member state of an economic federation with decentralized leadership, where the lower level (national) governments behave as if they are first movers vis-à-vis the federal government. Each national government collects tax revenues and redistributes by means of a general income tax as well as provides a public good, whereas the role of the federal government is to redistribute between the countries. Therefore, the paper contributes to earlier literature on nonlinear taxation by adding a fiscal federalism dimension of particular relevance for Europe, and to the literature on
fiscal federalism by considering other policy instruments and another order of
decision-making than is common in earlier literature.

As the policy instruments available to the federal government are im-
portant for the policy incentives facing the national governments, I consider
two versions of the decision-problem facing the federal government. Case I
means that the federal government redistributes lump-sum between the
national governments, whereas Case II implies that the federal government
raises revenue for redistributive purposes by imposing a tax on each country,
which is proportional to the labor income. Starting with Case I, federal ex-
post redistribution contributes to modify the income tax structure as well as
the policy rule for public good provision at the national level by comparison
with the policies that would be chosen in the absence of a federal govern-
ment; the results show, in particular, that federal ex-post redistribution will
contribute to increase the marginal income tax rate facing the low-ability
type and decrease the provision of the public good. The intuition is that the
national government partly uses its own tax policy in order to gain from fed-
eral redistribution, which undermines the motive to collect tax revenues as
well as tightens the self-selection constraint. The national policies derived in
Case II are more complex, as the indirect effects of decentralized leadership
are associated with two reaction functions. However, the general intuition
still remains; the national governments adjust their policy instruments in
order to gain from federal redistribution.

From the perspective of European fiscal federalism, the model set out
above is clearly a simplification as it focuses on two polar cases, neither of
which is a completely realistic description of the options for redistribution
available at the supranational level. In future research, it would be interest-
ing to connect the study of optimal income taxation more closely to actual
structures of fiscal federalism. Furthermore, European federalism is still in
its infancy, suggesting that it may not be entirely clear how the behavior at the 'federal level' ought to be described; another option might be to derive the redistributive policies chosen by the supranational level using a bargaining framework. I leave these and other extensions for future research.

5 Appendix

The National Government in Case I

The first order conditions are

\[
\frac{\partial L_j}{\partial l^1_j} = -\alpha_j \frac{\partial u^1_j}{\partial z^1_j} + \lambda_j \frac{\partial u^2_j}{\partial \xi_j^2} + \gamma_j w^1_j + \frac{\partial s_j}{\partial l^1_j} = 0 \quad (A.1)
\]

\[
\frac{\partial L_j}{\partial c^1_j} = \alpha_j \frac{\partial u^1_j}{\partial c^1_j} - \lambda_j \frac{\partial u^2_j}{\partial c^2_j} + \gamma_j [-1 + \frac{\partial s_j}{\partial c^1_j}] = 0 \quad (A.2)
\]

\[
\frac{\partial L_j}{\partial l^2_j} = -[\alpha^2_j + \lambda^2_j] \frac{\partial u^2_j}{\partial z^2_j} + \gamma_j [w^2_j + \frac{\partial s_j}{\partial l^2_j}] = 0 \quad (A.3)
\]

\[
\frac{\partial L_j}{\partial c^2_j} = [\alpha^2_j + \lambda^2_j] \frac{\partial u^2_j}{\partial c^2_j} + \gamma_j [-1 + \frac{\partial s_j}{\partial c^2_j}] = 0 \quad (A.4)
\]

\[
\frac{\partial L_j}{\partial g_j} = \sum_i \alpha^i_j \frac{\partial u^i_j}{\partial g_j} - \gamma_j = 0 \quad (A.5)
\]

where \( \xi^2_j = H - \phi_j l^1_j \).

Proof of Proposition 1

Consider first the tax formula for the low-ability type. By combining equations (A1) and (A2), we have

\[
MRS^1_{x,c}[\lambda_j \frac{\partial u^2_j}{\partial c^1_j} + \gamma_j (n^1_j - \frac{\partial s_j}{\partial c^1_j})] = \lambda_j \frac{\partial u^2_j}{\partial \xi^2_j} \phi + \gamma_j [n^1_j w^1_j + \frac{\partial s_j}{\partial l^1_j}] \quad (A.6)
\]

Using the first order condition for the hours of work, i.e. \( T^j_j (w^1_j l^1_j) w^1_j = w^1_j - MRS^1_{x,c} \), and rearranging gives
\[ T_j(w_j^{1j}) = \lambda_j \frac{\partial u^2}{\partial c_j} \frac{1}{\gamma_j w_j^1} [MRS_{z,c}^{1j} - MRS_{z,c}^{2j}] - \frac{1}{w_j^1} \left[ \frac{\partial s_j}{\partial l_j} + MRS_{z,c}^{1j} \frac{\partial s_j}{\partial c_j} \right] \]  
\[ \text{(A.7)} \]

Define \( A_j = \beta_j \sum_i \alpha_j^i \frac{\partial^2 u_j^i}{\partial (g_j)^2} < 0 \) and \( A = \sum_j A_j < 0. \) Use equation (8) to derive

\[
\frac{\partial s_j}{\partial l_j} = -\frac{A_j w_j^1}{A} < 0 \quad \text{(A.8)}
\]

\[
\frac{\partial s_j}{\partial c_j} = \frac{A_j}{A} > 0 \quad \text{(A.9)}
\]

Substitute equations (A8) and (A9) into equation (A7). Then, using the short notation

\[ \sigma_j = 1 - \frac{A_j}{A} \in (0, 1) \]

and rearranging, we obtain the expression for the marginal income tax rate of the low-ability type in Proposition 1. The expression for the marginal income tax rate of the high-ability type can be derived in a similar way.

**Proof of Proposition 2**

By using the definition of \( MRS_{g,c}^{ij} \) and then substituting equations (A2) and (A4) into equation (A5), we obtain

\[ 0 = MRS_{g,c}^{1j} \left[ \lambda_j \frac{\partial u^2}{\partial c_j} + \gamma_j (1 - \frac{\partial s_j}{\partial c_j}) \right] \quad \text{(A10)} \]

By rearrangements, one obtains the result in Proposition 2.

*The Labor Supply Response to a Change in The Federal Tax Rate*
By using the short notation

\[
\Upsilon_j^i = \frac{\partial^2 u_j^i}{(\partial c_j^i)^2} [w_j^i(1 - T_j^i(w_j^i l_j^i) - t_j)] - 2 \frac{\partial^2 u_j^i}{\partial c_j^i \partial z_j^i} w_j^i(1 - T_j^i(w_j^i l_j^i) - t_j) \\
- \frac{\partial u_j^i}{\partial c_j^i} (w_j^i)^2 T_j^i(w_j^i l_j^i) + \frac{\partial^2 u_j^i}{(\partial z_j^i)^2}
\]

the labor supply response by ability-type \( i \) in country \( j \) to a change in \( t_j \) can be written as

\[
\frac{\partial l_j^i}{\partial t_j} = - \frac{\partial^2 u_j^i}{(\partial c_j^i)^2} [w_j^i(1 - T_j^i(w_j^i l_j^i) - t_j)]w_j^i l_j^i - \frac{\partial u_j^i}{\partial c_j^i} w_j^i + \frac{\partial^2 u_j^i}{\partial z_j^i} w_j^i l_j^i
\]

(A11)

Then, use \( w_j^i(1 - T_j^i(w_j^i l_j^i) - t_j) = MRS_{z,c}^j,i \) and substitute into equation (A11). By observing that the derivatives of \( u_j^i \) with respect to \( c_j^i \) and \( z_j^i \) are functions of \( c_j^i \) and \( z_j^i \), we obtain equation (16).

The Federal Government in Case II

The first order conditions for an interior optimum for \( t_j \) and \( s_j \), respectively, are

\[
0 = \beta_j \sum_i \alpha_j^i [ - \frac{\partial u_j^i}{\partial c_j^i} w_j^i l_j^i + \frac{\partial u_j^i}{\partial g_j} \sum_i w_j^i T_j^i(w_j^i l_j^i) \frac{\partial l_j^i}{\partial t_j}] + \zeta \sum_i w_j^i l_j^i + t_j \sum_i w_j^i \frac{\partial l_j^i}{\partial t_j}
\]

(A12)

\[
0 = \beta_j \sum_i \alpha_j^i \frac{\partial u_j^i}{\partial g_j} - \zeta
\]

(A13)

where \( \zeta \) is the Lagrange multiplier associated with the budget constraint facing the federal government. By using \( w_j^i T_j^i(w_j^i l_j^i) = w_j^i - t_j w_j^i - MRS_{z,c}^j,i \), \( g_j = \sum_i [w_j^i l_j^i - b_j^i] + s_j \), the corresponding first order conditions for country \( k \neq j \) and the federal budget constraint, we can derive equations (19) and (20).
The national policy problem in Case II

The first order conditions can be written as

\[
\frac{\partial L_j}{\partial l_j} = -\alpha_j^1 \frac{\partial u_j^1}{\partial z_j^1} + \lambda_j \frac{\partial \hat{u}_j^2}{\partial z_j^2} + \gamma_j w_j^1 + \Delta_{ij}^1 + D_j^1 = 0 \tag{A.14}
\]

\[
\frac{\partial L_j}{\partial b_j} = \alpha_j^1 \frac{\partial u_j^1}{\partial c_j^1} - \lambda_j \frac{\partial \hat{u}_j^2}{\partial c_j^2} - \gamma_j + \Delta_{ij}^1 = 0 \tag{A.15}
\]

\[
\frac{\partial L_j}{\partial l_j} = -(\alpha_j^2 + \lambda_j) \frac{\partial u_j^2}{\partial z_j^2} + \gamma_j w_j^2 + \Delta_{ij}^2 + D_j^2 = 0 \tag{A.16}
\]

\[
\frac{\partial L_j}{\partial b_j} = [\alpha_j^2 + \lambda_j] \frac{\partial u_j^2}{\partial c_j^2} - \gamma_j + \Delta_{ij}^2 = 0 \tag{A.17}
\]

\[
\frac{\partial L_j}{\partial q_j} = \sum_i \alpha_i^j \frac{\partial u_i^j}{\partial q_i^j} - \gamma_j = 0 \tag{A.18}
\]

\[
\frac{\partial L_j}{\partial \kappa_j} = \frac{\partial L_j}{\partial t_j} \frac{\partial t_j}{\partial \kappa_j^i} + \gamma_j \frac{\partial s_j}{\partial \kappa_j^i} = 0 \text{ for } i = 1, 2 \tag{A.19}
\]

where

\[
D_j^1 = -\alpha_j^1 \frac{\partial u_j^1}{\partial z_j^1} + \lambda_j \frac{\partial \hat{u}_j^2}{\partial z_j^2} t_j w_j^1
\]

\[
D_j^2 = -(\alpha_j^2 + \lambda_j) \frac{\partial u_j^2}{\partial c_j^2} t_j w_j^2
\]

**Proof of Proposition 3**

Given equations (A14)-(A18), the expressions for the marginal income tax rates and provision of the public good can be derived in the same general way as in Propositions 1 and 2, respectively.

**References**


