Asymmetry with respect to the memory in stock market volatilities

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Abstract

The empirically most relevant stylized facts when it comes to modeling time varying financial volatility are the asymmetric response to return shocks and the long memory property. Up till now, these have largely been modeled in isolation though. To more flexibly capture asymmetry also with respect to the memory structure we introduce a new model and apply it to stock market index data. We find that, although the effect on volatility of negative return shocks is higher than for positive ones, the latter are more persistent and relatively quickly dominate negative ones.

Key Words: Financial econometrics, GARCH, news impact, nonlinear, risk prediction, time series.

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1 Introduction

To this date, the ARCH and GARCH framework of Engle (1982) and Bollerslev (1986) stands out as the single most important tool when it comes to understanding and modeling the dynamics of financial volatility. Since the birth of the basic models the literature has exploded with different extensions most often designed to cope with the stylized facts of asymmetry and long memory (see Andersen, Bollerslev, Christoffersen, and Diebold, 2006, and references therein for an overview).

The asymmetry property is most notable for equity returns and it refers to the fact that return volatility tends to rise more following negative return shocks than positive ones. This was first noted by Black (1976), who argued that negative return shocks increase financial leverage implying a riskier return on equity given an unchanged stream of cash flows. Several alternatives for incorporating asymmetric effects have been proposed in the literature. The most popular one appears to be the asymmetric threshold GARCH (TGARCH) of Glosten, Jagannathan, and Runkle (1993) and Zakoïan (1994). Other commonly employed alternatives include the Asymmetric GARCH (AGARCH) model of Engle and Ng (1993), the exponential GARCH (EGARCH) model of Nelson (1990) and the quadratic GARCH model (QGARCH) of Sentana (1995). A more recent extension is the dynamic asymmetric GARCH (DAGARCH) of Caporin and McAleer (2006) that generalizes the TGARCH to include multiple and time-varying thresholds.

When it comes to the long memory property Nelson (1990), Ding and Granger (1996) and Davidson (2004) provide insightful discussions. In particular, they note that long memory may have quite different meanings. On the one hand, motivated by the typical estimation results obtained for the basic GARCH model Engle and Bollerslev (1986) proposed the integrated GARCH (IGARCH). In this model shocks to the squared return process persists in the sense that they affect the prediction of, respectively, the squared return process and future volatility for all time. Two other and closely related ways (see Ding and Granger, 1996) of thinking about memory is in terms of the autocorrelation function (acf) of squared returns and the rate of decay of the coefficients in the infinite ARCH representations. The basic GARCH implies exponentially decaying structures. In fact, as shown in Ding and Granger (1996) this is also the case for the IGARCH model. However, empirical acf's of the squared returns are often found to die out at a slower (hyperbolic) rate (e.g. Karanasos, Psaradakis, and Sola, 2004). To provide an intermediate case between the conventional GARCH and the IGARCH Baillie, Bollerslev, and Mikkelsen (1996) suggested replacing the first difference operator in the IGARCH model with the fractional one to obtain the fractionally integrated GARCH (FIGARCH) model. With the first interpretation of memory the FIGARCH model indeed serves as an intermediate case. However, the model has also long memory in the sense that the coefficients in the infinite ARCH representation die out hyperbolically. In fact, Karanasos et al. (2004) derive autocorrelation functions for the related class of long memory models in Robinson (1991) and argue that the FIGARCH model have the same hyperbolically decaying second order structure. In this paper we view memory in terms of the rate of decay of the coefficients in the infinite ARCH representation.

Now, a tempting next step is to combine the two features discussed above. Indeed, volatility models in this direction have been proposed. For example, Bollerslev and Mikkelsen (1996) proposed the fractionally integrated EGARCH (FIEGARCH) (see also Ruiz and Veiga, 2008). Tse (1998) extends the asymmetric power ARCH (APARCH) of (Ding, Granger, and Engle, 1993). Hwang (2001) proposes a quite general class of asymmetric and fractionally integrated GARCH models (see also Ruiz and Perez, 2003). More recently, Asai, McAleer, and Medeiros (2012) extended the FIEGARCH to capture asymmetric effects in a more flexible way. In these papers asymmetry with respect to the effect of the most recent return shocks on current volatility is captured. However, shocks are essentially treated symmetrically with respect their effect over time on future volatilities since there is only one fractional difference operator involved. The difference over time occurs merely with respect to the asymmetry implied from the immediate response. Here, the focus will be on the allowance for asymmetry in this conventional sense as well as with respect to the memory property. In fact, in a related paper Lönnbark (2012) detects the occurrence of the latter. However, the proposed model does not allow for long memory. To achieve this we combine features of the TGARCH and the FIGARCH models and propose a new model: The fractionally integrated threshold GARCH, or the FITGARCH.

The remainder of the paper is organized as follows. Section 2 presents the index data used in the empirical study. In Section 3 we introduce the model and discuss some of its properties. Section 4 presents the estimator of the model parameters and empirical results are given in Section 5. In particular, to illustrate the asymmetry embedded in the model we extend the news impact curves of Engle and Ng (1993) to surface versions. The final section concludes.

2 The Data

The data considered in this paper consists of index data for seven major stock markets: CAC 40 (France), DAX (Germany), FTSE 100 (United Kingdom), Hang Seng (Hong Kong), NIKKEI 225 (Japan), S&P 500 (United States) and Straits Times (Singapore). Ten years of daily index data was downloaded from Yahoo Finance covering the period May 16, 2001 to May 16, 2011. We calculate returns as $r_t = 100 \ln(I_t / I_{i-1})$, where I_t is the value of the index at time point t. In Table 2 we provide some descriptive statistics for the return series.

All series exhibit skewness and leptokurticity. Indeed, the Jarque-Bera test

Table 1: Descriptive statistics for the return series. Vce, Skew and Kurt are the sample variance, skewness and kurtosis respectively. JB is the p-value in the Jarque-Bera test of normality. LB_{10} and LB_{10} are p-values in the Ljung-Box test of autocorrelation in returns and squared returns, respectively. The tests were computed using ten lags. Asy. is the p-value of the t-statistic of $r_{t-1}min(r_{t-1}, 0)$ in the regression of r_t^2 on a constant, $r_{t-1}^2, r_{t-2}^2, ..., r_{t-10}^2$ and $r_{t-1}min(r_{t-1}, 0)$.

Index	Obs.	Mean	Vce	Min	Max	Skew	Kurt	JB	LB_{10}	LB_{10}^2	Asy.
CAC40	2557	-0.012	2.450	-9.472	10.595	0.090	5.615	0.000	0.000	0.000	0.000
DAX	2547	0.007	2.652	-7.433	10.797	0.062	4.742	0.000	0.002	0.000	0.000
FTSE 100	2523	0.000	1.743	-9.265	9.384	-0.094	6.638	0.000	0.000	0.000	0.000
Hang Seng	2497	0.022	2.585	-13.582	13.407	0.018	9.328	0.000	0.026	0.000	0.000
NIKKEI 225	2450	-0.014	2.655	-12.111	13.235	-0.452	7.028	0.000	0.100	0.000	0.000
S&P 500	2513	0.002	1.825	-9.470	10.957	-0.152	8.819	0.000	0.000	0.000	0.000
Straits Times	2518	0.026	1.599	-9.216	7.531	-0.341	6.174	0.000	0.096	0.000	0.000

strongly rejects normality throughout. The Ljung-Box tests on the return series suggest that autocorrelation is present in many cases. The Ljung-Box test on the squared return series and the asymmetry check clearly signal that ARCH effects with asymmetry are present in all indices. Noteworthy is also that the daily swings can be quite substantial as manifested by the values on the min. and max. observations.

To further scrutinize on the properties of the autocorrelation structures we computed autocorrelation functions for the squared return series as well as cross-correlations between squared returns and lagged values on $r_t \max(r_t, 0)$ and $r_t \min(r_t, 0)$, respectively. In Figure 1 we give the corresponding plots.

The plots in Figure 1 reveal that the correlations are considerable even at high lags (cf. long memory). Noteworthy is also that the structure of the decay is quite different for positive and negative shocks.

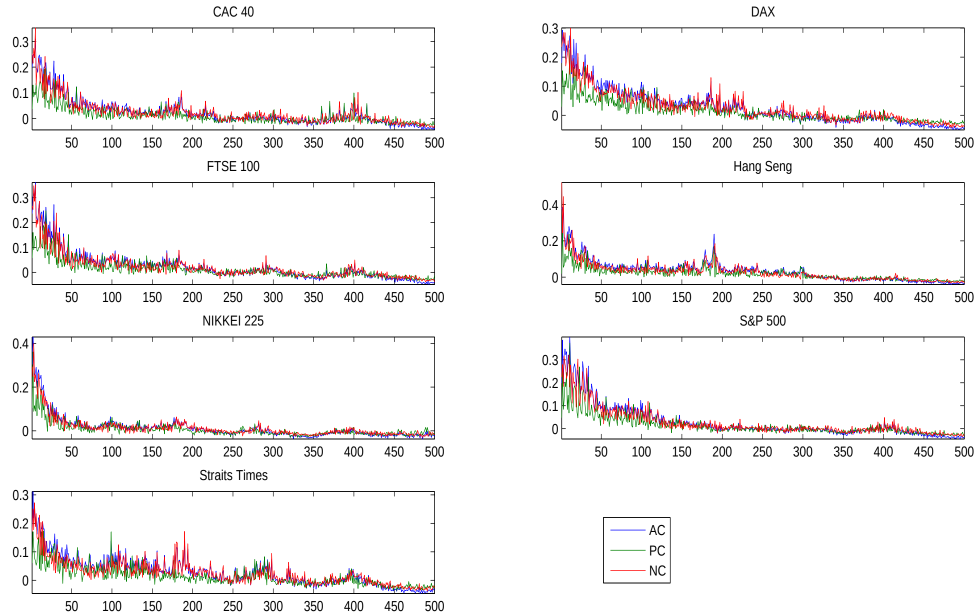

Figure 1: Empirical correlations. AC is the autocorrelation function of r_t^2 and PC and NC are the cross correlations between the r_t^2 and lagged values of $r_t \max(r_t, 0)$ and $r_t \min(r_t, 0)$, respectively.

3 Fractionally integrated threshold GARCH

To evoke our volatility model we define a return shock process $\{u_t\}$ that started in the infinite past and that is generated in discrete time by

$$u_t = \sqrt{h_t}\varepsilon_t,$$

where $\{\varepsilon_t\} \sim iid(0,1)$. We define the time series process of index returns as $r_t = \mu_t + u_t$ and with $\mathcal{F}_t$ denoting the history up to and including time t we have the conditional mean $\mu_t = E(r_t|\mathcal{F}_{t-1})$ and variance $h_t = V(u_t|\mathcal{F}_{t-1}) = V(r_t|\mathcal{F}_{t-1})$, respectively. To allow for asymmetric effects in the specification of h_t we define the two time series processes $u_t^{2+} = u_t \max(0, u_t)$ and $u_t^{2-} = u_t \min(0, u_t)$. We focus attention to a class of volatility models that may be represented as (cf. Davidson, 2004)

$$h_t = \omega + \lambda^+(L)u_t^{2+} + \lambda^-(L)u_t^{2-}, \tag{1}$$

where L is the lag operator, i.e. $Lx_t = x_{t-1}$, and $\lambda^s(L) = \lambda_1^s L + \lambda_2^s L^2 + ...$, $s = +, -$. For the variance to be positive at all times we require that $\lambda_i^+, \lambda_i^- \geq 0$ for all i.

Now, consider the GARCH(p,q) model

$$h_t = \omega + \alpha(L)u_t^2 + \beta(L)h_t, \tag{2}$$

where $\alpha(L) = \alpha_1 L + ... + \alpha_q L^q$ and $\beta(L) = \beta_1 L + ... + \beta_p L^q$. For stationarity we require that $\alpha(1)+ \beta(1) < 1$ and to guarantee a positive variance at all times it is typically assumed that $\omega > 0$ and that all the α's and β's are larger than zero. Indeed, this condition is sufficient although positivity may be ensured with less restrictive assumptions as showed in Nelson and Cao (1992) and Tsai and Chan

(2008). Rewriting (2) in the form of (1) gives

$$h_t = [1 - \beta(L)]^{-1}\omega + [1 - \beta(L)]^{-1}[\alpha(L)]u_t^2.$$

In this case $\lambda^+(L) = \lambda^-(L)$ and the rate of decay is exponential. Upon defining the prediction error $v_t = u_t^2 - h_t$ the model (2) may be written as an ARMA process for u_t^2

$$[1 - \alpha(L) - \beta(L)]u_t^2 = \omega + [1 - \beta(L)]v_t. \quad (3)$$

The IGARCH model of Engle and Bollerslev (1986) arises when the $[1-\alpha(L)-\beta(L)]$ polynomial in eq. (3) contains a unit root, i.e. $[1 - \alpha(L) - \beta(L)] = (1 - L)\phi(L)$, where $\phi(L) = 1 - \phi L - ... - \phi_q L^q$ and has all the roots outside the unit circle. We may then write

$$(1 - L)u_t^2 = [\phi(L)]^{-1}\omega + [\phi(L)]^{-1}[1 - \beta(L)]v_t. \quad (4)$$

The IGARCH model implies persistence in the sense that the effect of the forecast errors, v_t, on the squared return shock process as well as on the prediction of future conditional volatility persists. The effect on the actual volatility dies out exponentially though, which is easily seen from the infinite ARCH representation of (4)

$$h_t = [1 - \beta(L)]^{-1}\omega + [1 - \beta(L)]^{-1}[1 - \beta(L) - (1 - L)\phi(L)]u_t^2.$$

For example, for the GARCH$(1,1)$ and the IGARCH$(1,1)$ we have $\lambda_i = \alpha\beta^i$ and $\lambda_i = \beta^{i+1}$, respectively. Hence, in this sense the IGARCH is fundamentally different from a random walk type of model.

To obtain an intermediate case between the GARCH and the IGARCH Baillie et al. (1996) replaced the difference operator in (4) with the fractional one defined as

$$(1-L)^d = 1 - \sum\nolimits_{k=1}^{\infty} [\Gamma(k-d)]/[\Gamma(k+1)\Gamma(-d)] L^k,$$

where $\Gamma(\cdot)$ is the gamma function and $0 < d < 1$, to obtain the FIGARCH(p, d, q) model

$$(1-L)^d \phi(L) u_t^2 = \omega + [1 - \beta(L)] v_t. \tag{5}$$

From the infinite ARCH representation the specification is seen to imply a long (hyperbolic) memory

$$h_t = [1-\beta(L)]^{-1}\omega + [1 - [(1-\beta(L)]^{-1}(1-L)^d \phi(L)] u_t^2. \tag{6}$$

With notation from eq. (1) we have $\lambda^+(L) = \lambda^-(L) = 1 - (1-\beta L)^{-1}(1-L)^d$. This polynomial consists of the multiplication of an exponentially decaying polynomial and a hyperbolically decaying one. For large lags the latter will dominate and it is possible to show that $\lambda_k \approx [(1-\beta)\Gamma(d)^{-1}]k^{d-1}$. In practical estimation the form typically used is

$$h_t = \omega + \beta(L) h_t + [1 - (1-L)^d \phi(L)] u_t^2, \tag{7}$$

The FIGARCH is not weakly stationary, but as shown in Zaffaroni (2004) the model is strictly stationary and ergodic for the case of Gaussian innovations. Note that the FIGARCH model reduces to, respectively, the GARCH model for $d = 0$ and to the IGARCH for $d = 1$.

Turning now to the incorporation of asymmetric effects we depart from the TGARCH(p, q^+, q^-)-specification

$$h_t = \omega + \alpha^+(L) u_t^{2+} + \alpha^-(L) u_t^{2-} + \beta(L) h_t, \tag{8}$$

where $\alpha^s(L) = \sum_{i=1}^{q^s} \alpha_i^s L^i$, $s = +, -$. This specification has the same exponential memory structure as the GARCH and the difference between the effect on future volatility of positive and negative return shocks occurs merely with respect to the polynomials $\alpha^+(L)$ and $\alpha^-(L)$. To allow for more flexibility with respect to the asymmetry in the memory structure we again use the prediction error, v_t, and upon noting that $u_t^2 = u_t^{2+} + u_t^{2-}$ eq. (8) may be written as

$$[1-\alpha^+(L)-\beta(L)]u_t^{2+} + [1-\alpha^-(L)-\beta(L)]u_t^{2-} = \omega + [1-\beta(L)]v_t.$$

Then, on using $[1-\alpha^s(L)-\beta(L)] = (1-L)^{d^s}\phi^s(L)$, where $\phi^s(L) = 1-\phi_1^s L^i - ... - \phi_{q^s}^s L^{q^s}$, $s=+,-$, has all roots outside the unit circle we define the fractionally integrated threshold GARCH model, or, the FITGARCH(p, d^+, q^+, d^-, q^-), as

$$(1-L)^{d+}\phi^+(L)u_t^{2+} + (1-L)^{d-}\phi^-(L)u_t^{2-} = \omega + [1-\beta(L)]v_t, \tag{9}$$

The infinite ARCH representation is given by

$$\begin{aligned} h_t &= [1-\beta(L)]^{-1}\omega + \{1-(1-L)^{d^+}\phi^+(L)\}[1-\beta(L)]^{-1}u_t^{2+} \\ &\quad + \{1-(1-L)^{d-}\phi^-(L)\}[1-\beta(L)]^{-1}u_t^{2-}. \end{aligned} \tag{10}$$

In order for the variance to be positive at all times we require that $\lambda_i^s \geq 0$, $s=+,-$. To guarantee this for the FIGARCH$(1,d,1)$ model Baillie et al. (1996) proposed the necessary and sufficient conditions $\omega > 0, 0 \leq \beta \leq \phi + d$ and $0 \leq d \leq 1-2\phi$. The direct extension to our model is $\omega > 0, 0 \leq \beta \leq \phi^+ + d^+$, $0 \leq d^+ \leq 1-2\phi^+$ and $0 \leq d^+ \leq 1-2\phi^+$ and $0 \leq d^+ \leq \beta \leq \phi^- + d^-$. Adapting a result in (Karanasos, Psaradakis, and Sola, 2004) the λ-polynomials may for the $(1, d^+, 1, d^-, 1)$-case be shown to have the form

$$\lambda_i^s = -\frac{\Gamma(i-d^s)}{\Gamma(-d^s)\Gamma(i+1)} - \sum_{k=1}^{i}\frac{\Gamma(i-k-d^s)}{\Gamma(-d^s)\Gamma(i-k+1)}(\beta^k - \phi\beta^{k-1}),$$

where $s = +, -$. When $d^- = d^+$ and $\phi^+(L) = \phi^-(L)$ the model is simply the basic FIGARCH. With $d^- = d^+$ but $\phi^+(L) \neq \phi^-(L)$ the model becomes a long memory version of the TGARCH, which is interesting in it's on right and may serve as useful alternative to the FIEGARCH model. We say that there is asymmetry with respect to the memory when $d^- \neq d^+$. In particular, when $d^+ = 0$ ($d^- = 0$) but $0 < d^- < 1$ ($0 < d^+ < 1$) there is long memory only with respect to positive (negative) returns shocks.

4 Estimation

To estimate the model parameters we employ the QML estimator. Thus, given a normality assumption on $\{\varepsilon_t\}$ the prediction error

$$r_t - E(r_t|\mathcal{F}_{t-1}) = u_t = \sqrt{h_t}\varepsilon_t$$

is conditionally $N(0, h_t)$ and with observations up till time T, the log-likelihood function takes the form

$$\ln L \propto -\frac{1}{2}\sum_{t=s}^{T} \ln(h_t) - \frac{1}{2}\sum_{t=s}^{T} u_t^2/h_t. \tag{11}$$

where s is determined by the number of lags in the mean and variance specifications. The form of the volatility specification that appears most useful for the purpose of estimation is

$$h_t = \omega + \beta(L)h_t + \{1 - (1-L)^{d^+}\phi^+(L)\}u_t^{2+} + \{1 - (1-L)^{d-}\phi^-(L)\}u_t^{2-}.$$

For practical estimation we use the RATS 7.3 package employing robust standard errors throughout. When it comes to computing the infinite lag polynomials $(1 -$

$L)^{d+}$ and $(1-L)^{d-}$ we used the built-in frequency domain filtering procedure with sample means as presample values. Maximization of (11) was carried out with the BFGS algorithm. Each iteration in the maximization is initiated with moves of the fractional difference parameters $d+$ and $d-$ and the search for a maximum then proceeds over the remaining parameters. The model is quite rich and for feasibility we will have to be restrictive in terms of the number of lags to include in $\beta(L)$, $\phi^+(L)$ and $\phi^-(L)$. For most practical purposes the $(1, d^+, 1, d^-, 1)$-version appears sufficient. Indeed it contains the special cases GARCH$(1,1)$, TGARCH$(1,1,1)$ and FIGARCH$(1,d,1)$.

5 Empirical Results

We apply our modeling framework to the data presented in Section 2. To cope with the autocorrelation in returns we follow the suggestion in Bollerslev and Mikkelsen (1996) and consider a third order autoregressive specification for the mean function. Thus, the estimated specification for all series is

$$\begin{aligned} r_t &= \mu_0 + \mu_1 r_{t-1} + \mu_2 r_{t-2} + \mu_3 r_{t-3} + u_t, \quad u_t = \varepsilon_t \sqrt{h_t}, \quad \varepsilon_t \sim N(0,1), \\ h_t &= \omega + \beta h_{t-1} + \{1-(1-L)^{d^+}(1-\phi^+ L)\} u_t^{2+} + \{1-(1-L)^{d-}(1-\phi^- L)\} u_t^{2-}. \end{aligned}$$

In Table 5 we give QML estimates along with some diagnostics checks.

The estimates of ϕ^+, ϕ^-, $d^+$and d^- indicate the presence of asymmetric effects in all series. Indeed, in the formal Wald testing of the joint hypothesis $\phi^+ = \phi^-$ and $d^+ = d^-$ we obtained highly significant rejections. Interestingly, the estimates of β, d^+ and d^- are remarkably similar across the series. In particular, the parameter d^+ that governs the long run response of volatility to positive return shocks is slightly below 0.5 for all series, whereas d^-, the parameter governing the long run effect of

Table 2: Estimation results with t-statistics in italics. L is the value on the log-likelihood function. JB is the p-value in the Jarque-Bera test. LB_{10} and LB_{10} are p-values in the Ljung-Box test evaluated at ten lags for standardized return shocks and squared return shocks, respectively.

	CAC 40	DAX	FTSE 100	Hang Seng	Nikkei 225	S&P 500	Straits Times
μ_0	0.041 *1.838*	0.063 *2.732*	0.039 *2.115*	0.036 *1.555*	0.023 *0.850*	0.028 *1.422*	0.045 *2.276*
μ_1	-0.048 *-2.104*	-0.024 *-1.044*	-0.067 *-2.891*	0.024 *1.039*	-0.011 *-0.443*	-0.070 *-3.026*	0.005 *0.220*
μ_2	-0.046 *-2.125*	-0.017 *-0.794*	-0.028 *-1.256*	-0.004 *-0.162*	-0.012 *-0.557*	-0.049 *-2.128*	0.022 *0.989*
μ_3	-0.048 *-2.192*	-0.030 *-1.364*	-0.042 *-1.903*	0.013 *0.633*	0.014 *0.645*	-0.003 *-0.135*	0.019 *0.905*
ω	0.000 *0.000*	0.000 *0.000*	0.000 *0.000*	0.000 *0.000*	0.000 *0.000*	0.002 *0.122*	0.000 *0.000*
ϕ^+	0.257 *3.924*	0.258 *3.860*	0.253 *3.568*	0.287 *4.354*	0.299 *4.570*	0.260 *4.297*	0.264 *4.987*
ϕ^-	-0.162 *-2.390*	-0.131 *-1.915*	-0.080 *-1.101*	-0.029 *-0.469*	0.022 *0.313*	-0.145 *-2.549*	-0.051 *-1.260*
β	0.743 *10.226*	0.742 *10.931*	0.747 *10.476*	0.713 *10.279*	0.701 *9.055*	0.740 *10.848*	0.693 *12.595*
d^+	0.486 *4.621*	0.483 *4.732*	0.495 *4.846*	0.426 *4.546*	0.402 *3.933*	0.480 *5.486*	0.429 *5.558*
d^-	0.991 *8.037*	0.980 *8.339*	0.959 *7.576*	0.827 *7.806*	0.846 *6.913*	0.888 *8.241*	0.798 *9.948*
L	-4156.256	-4239.988	-3593.788	-4121.977	-4248.919	-3600.330	-3673.262
JB	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LB_{10}	0.394	0.455	0.807	0.811	0.558	0.915	0.813
LB_{10}^2	0.409	0.744	0.486	0.020	0.987	0.112	0.408

negative return shocks, is considerably higher. In fact, for none of the European indices we could reject the hypothesis $d^- = 1$.

In Figure 2 we plot the coefficients in the implied infinite ARCH representation of the model, i.e. the $\lambda^{+\prime}$s and the $\lambda^{-\prime}$s in eq. (1), and compare these to those implied from the GARCH(1,1), the TGARCH(1,1,1) and the FIGARCH(1,d,1) models. Of course, being symmetric these coincide for the GARCH and the FIGARCH models. Most notably, according to the TGARCH positive return shocks are irrelevant for the prediction of future volatility, which can hardly be said to be the case for our model. The initial effects on volatility of a negative return shock is much larger then that of a positive one of the same size. However, the latter is more persistent and dominates already after approximately ten lags.

In Figure 3 we further illustrate the dynamic properties of the estimated model for the S&P 50 with a news impact surface. Rather than just giving the response of volatility to the most recent return shock as is the case for conventional news impact curves, the surface version gives the responses to past shocks as well. For the first lags the tilt towards negative shocks, i.e. the conventional asymmetry, is clear. However, already after a few lags it becomes more symmetric and eventually starts to tilt towards positive shocks.

6 Conclusion

To more flexibly capture asymmetry with respect to the memory in volatilities we introduced a new model that takes features from the asymmetric TGARCH model and the long memory FIGARCH model. In applying the model to stock market index data we found that the initial effect on volatility of negative return shocks are larger than that of positive shocks. However, the latter are more persistent and dominate negative ones relatively quickly.

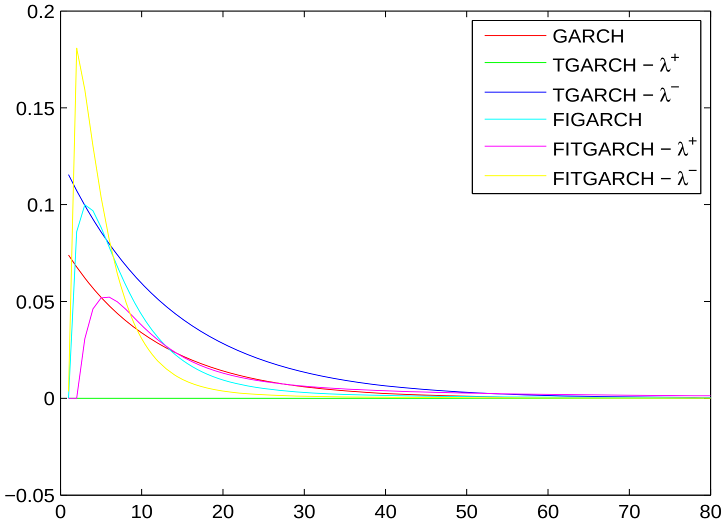

Figure 2: The coefficients in the infinite ARCH polynomials for S&P 500.

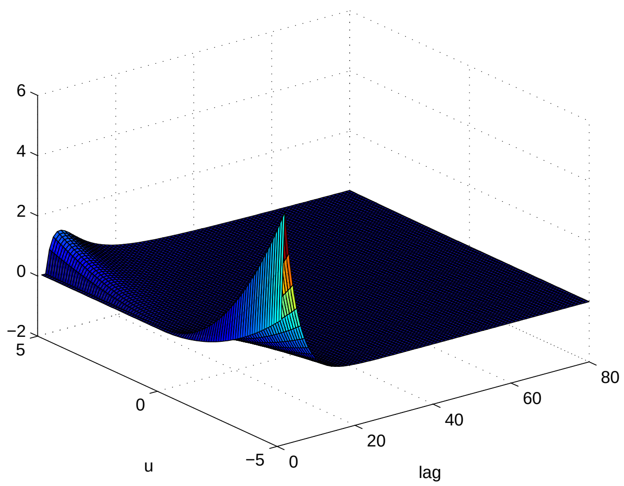

Figure 3: News impact surface for S&P 500.

An unfortunate feature of the model is that the effect on volatility of return shocks actually increases for the very first lags. This feature is shared with other models employing the fractional difference operator and we leave for future research to explore alternative routes.

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