

Optimal Taxation and Risk-Sharing Arrangements in an Economic Federation*

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Abstract

This paper analyzes optimal taxation and risk-sharing arrangements in an economy with two levels of government. Both levels provide public goods and finance their expenditures via labor income taxation, where the tax base is responsive to the private agents' labor supply decisions. The localities are assumed to experience different random productivity shocks, meaning that the private labor supply decision as well as the choices of income tax rates are carried out under uncertainty. Part of the central government's decision problem is then to provide tax revenue sharing between the local governments. The optimal degree of revenue sharing depends on whether or not the localities/regions differ with respect to labor supply incentives.

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1 Introduction

This paper concerns optimal taxation and risk-sharing arrangements in an economic federation, where tax and expenditure decisions are being made by both local and central levels of government. Each locality (or region) is assumed to experience a random productivity shock, implying that the private agents as well as the policy makers optimize under uncertainty. The main purpose is to examine how the central government can improve the resource allocation by means of policies designed to affect the behavior of local governments.

The paper relates to (and tries to combine) earlier literature in primarily two fields: (i) optimal taxation and provision of public goods under fiscal externalities and (ii) fiscal arrangements for risk-sharing. In the literature on fiscal externalities, it has been recognized that if the local (lower level) governments act as Nash competitors to one another, then the resulting outcome in terms of taxes and publicly provided goods is generally suboptimal from society's point of view. One reason is the presence of horizontal fiscal externalities: the decisions made by one locality affect the residents in other localities either because of mobility across localities or benefit spillover from local public goods¹. A second reason is the presence of vertical externalities, which arise from co-occupancy of a common tax base. Typically, the local authorities do not recognize that their policies affect the central authority's tax base. This was pointed out by e.g. Hansson and Stuart (1987) and Johnson (1988). To internalize the vertical externality, Boadway and Keen (1996) and Boadway et al. (1998) propose that the power of taxation be assigned to one level only (i.e. the optimal tax rate for the central government is zero), whereas Aronsson and Wikström (1999) show that taxation at the central and local levels of government can be combined with an intergovernmental transfer scheme inducing the correct incentives.

Fiscal arrangements for risk-sharing have been examined both theoretically and empirically. Asdrubali, Sorensen and Yosha (1996) analyze risk-sharing among the

¹The standard reference here is Oates (1972). Wildasin (1991) shows that horizontal externalities arising from mobility can be internalized by means of a system of matching grants from the central to the local governments.

states in the U.S. by means of econometric methods. They find that, even if the capital and credit markets appear to be the most important mechanisms for interstate risk-sharing, some 13 per cent of shocks to the 'gross state product' are smoothed by the federal government. The figures presented by Sala - I - Martin and Sachs (1992) suggest that the federal government plays a much more important role in the context of risk-sharing: a negative shock in state income of one dollar leads the federal government to decrease taxes by 35 cents and increase transfer payments by approximately 30 cents. In any case, these and other empirical studies suggest that the tax-transfer system is an important mechanism for risk-sharing² in an economic federation and, therefore, worth further research.

At a theoretical level, Persson and Tabellini (1996a) study risk-sharing as part of the fiscal policy in an economic federation with two levels of government. Their results suggest a tradeoff between federal risk-sharing and moral hazard in the sense that federal risk-sharing may induce local governments to undertake policies that increase the local risk. Similarly, Persson and Tabellini (1996b) point at a tradeoff between risk-sharing and redistribution, and show that a federal social insurance scheme (chosen by federation-wide voting) will provide overinsurance, whereas an intergovernmental transfer scheme (chosen by bargaining between the regions/localities) provides underinsurance. Lockwood (1999) analyzes the decision problem of a central government, which can use intergovernmental grants to insure local governments against locality specific random shocks. The central government then faces the problem of trading off insurance of the localities against offering correct incentives for local public good provision. Depending on the source of the shocks, the grant program may either induce oversupply or undersupply of local public goods relative to the Samuelson rule.

The previous studies on fiscal arrangements for risk-sharing commonly treat private incomes as exogenous conditional on the state of nature, meaning that the effects on the tax base arising from labor supply behavior and/or mobility are being

²Sorensen, Wu and Yosha (2001) find that state budget surpluses are procyclical in the sense that an increase in the gross state product increases the budget surplus. A possible interpretation is that state and local governments use the budget surplus to smooth the disposable income of the state residents.

neglected³. In this paper, we assume that the labor supply is endogenous, and that both levels of government finance their expenditures via labor income taxation. The localities are assumed to experience different random productivity shocks, implying that the private labor supply decision and the governments' choices of income tax rates are carried out before the real wage rates have become realized. To be able to study the consequences of endogenous labor supply in the context of risk-sharing, we shall disregard any horizontal externalities. We also refrain from discussing moral-hazard problems, which are addressed elsewhere in this literature. The paper contributes to the literature in at least two ways. The first is by analyzing risk-sharing as part of the optimal tax policy, which enables us to address vertical fiscal externalities and risk-sharing simultaneously. The second contribution is to show that the optimal arrangement for risk-sharing within the public organization depends on whether or not the localities differ with respect to labor supply incentives.

Section 2 analyzes a 'benchmark' version of the model, where all localities are identical before the real wage rates have become realized, i.e. all differences between the localities with respect to the realized real wage rates are due to the assumption that they experience different random shocks. We show that the socially optimal (second best) resource allocation involves 'full insurance' in the sense that the outcome of the productivity shocks in terms of tax revenues is shared equally between the local governments. This result is interesting in the sense of providing an efficiency argument in favor of tax revenue-sharing across localities. Such revenue-sharing is common in the Nordic countries and motivated primarily on the basis of distributional objectives. To implement the second best resource allocation, the central government announces a policy rule which simultaneously internalizes the vertical externality and redistributes tax revenues between the local governments. Section 2

³An interesting exception is Lockwood (1999), who briefly addresses distortionary taxation in the context of intergovernmental grants and risk-sharing. By assuming Cobb-Douglas preferences, he finds that the structure of taxation (i.e. whether taxes are lump-sum or distortionary) may be important for the qualitative properties of the optimal intergovernmental grant. See also Lee (1998), who analyzes income redistribution under uncertainty. Lee considers the question of whether the federal government or the local governments should redistribute income, when the localities are subject to different random shocks and the labor force is mobile across localities.

also addresses the conditions under which a centralized resource allocation system gives rise to a higher income tax rate than would be chosen, if all tax revenues are collected by the local governments.

In Section 3, we extend the analysis by assuming that part of the differences in labor productivity (or realized real wage rates) between the localities is deterministic and observed before the random shocks have become realized. This means that the labor supply incentives will differ across localities. The main result here is that, even if the central government can improve the resource allocation by means of revenue-sharing, full insurance is no longer optimal. The intuition is that, if the labor supply behavior differs across localities, the outcome of the productivity shocks in terms of tax revenues will not cancel out at an aggregate level. Section 4 concludes the paper.

2 The Benchmark Model

The federation consists of N localities, each of which is populated by one immobile resident. The utility function of the resident in locality i is written⁴

$$u^i = u(c^i, l^i) + \phi(x^i) + \zeta(G) \quad (1)$$

where c is private consumption, l labor supply, x a local public good provided by the local government and G a federal public good provided by the central (or federal) government. We assume that $u(c, l)$ is increasing in c , decreasing in l and strictly concave. The other parts of the utility function are assumed to obey the conditions $\phi_x > 0$, $\phi_{xx} < 0$, $\zeta_G > 0$ and $\zeta_{GG} < 0$, where the subindices denote partial derivatives. The budget constraint of the local resident takes the form

$$w^i(1 - \tau_l^i - \tau_c)l^i = c^i \quad (2)$$

where w is the real wage rate, τ_l the local income tax rate and τ_c the income tax rate chosen by the central government.

⁴To simplify the analysis, we follow Boadway and Keen (1996) and Boadway et al. (1998) by assuming that public goods are additively separable from other goods.

The real wage rate contains a deterministic part, which is identical across localities, and a stochastic part. We define the realized real wage rate as

$$w^i = \bar{w} + \gamma \varepsilon^i \quad (3)$$

with $E[\varepsilon^i] = 0$ and $\sigma_\varepsilon^2 = 1$. To rule out the possibility of negative real wage rates, we assume that $\Pr(\varepsilon^i > -\bar{w}/\gamma) = 1$. We can, therefore, interpret w^i as a positive stochastic variable with mean \bar{w} and standard deviation γ .

We shall make two assumptions about the order in which the decisions are being made. First, the choices of income tax rates as well as the private agent's labor supply decision are made before the random shocks have become realized. Second, following the convention in the optimal tax literature, we also assume that the income tax rates are chosen before the private agents determine their labor supply, meaning that the private agent in each locality optimizes subject to a set of fixed tax parameters.

The resident of locality i chooses l_i to maximize

$$E[u(w^i(1 - \tau_l^i - \tau_c)l^i, l^i)] + \phi(x^i) + \zeta(G)$$

in which case the supply of labor can be written as $l^i = l(\bar{w}, \gamma, \tau_l^i + \tau_c)$. The realized consumption can then be calculated by substituting the labor supply into equation (2).

To simplify the notations as much as possible, we shall be using

$$v(\bar{w}, \gamma, \tau_l^i + \tau_c) = E[u(w^i(1 - \tau_l^i - \tau_c)l(\bar{w}, \gamma, \tau_l^i + \tau_c), l(\bar{w}, \gamma, \tau_l^i + \tau_c))]$$

as a short notation for the 'private' part of the expected indirect utility function. We can then write the expected indirect utility function as follows

$$V^i = v(\bar{w}, \gamma, \tau_l^i + \tau_c) + \phi(x^i) + \zeta(G) \quad (4)$$

which is assumed to be the objective function of the local government. In a similar way, we assume that the objective of the central government is the sum of the expected indirect utilities taken over all localities.

2.1 Centralized Policy Decisions

Suppose, to begin with, that all decisions regarding taxation and provision of public goods are being made by the central government. To facilitate the comparison between centralized and decentralized decision making (the latter is to be examined in the next subsection), we would like to make two additional assumptions. First, there is no 'aggregate productivity shock', which is here taken to imply that $(1/N) \sum_{i=1}^N \varepsilon^i = 0$. This means that there is a sufficient number of localities to ensure that the influences of shocks on the real wage rates cancel out on an aggregate level. Second, risk-sharing at the private agent level is not feasible⁵. The reason for making the second assumption is that such risk-sharing would in principle require lump-sum taxation, which has already been ruled out by the assumption that the tax revenues are collected via distortionary taxes. The second assumption will imply that our model is comparable with those in previous studies on vertical fiscal externalities mentioned in the introduction.

Note that the central government's decision problem will, in this case, be to choose taxation and provision of public goods to maximize the sum of expected utilities subject to the public resource constraint for the economy as a whole, meaning that the decision problem of a centralized policy maker coincides with the social optimization problem. The resource constraint facing the centralized policy maker is deterministic, since the outcome of productivity shocks in terms of tax revenues cancel out at an aggregate level. When all policy decisions are being made by the central government, there is no need to distinguish between local and central (or federal) income tax rates, so we can define $\tau^i = \tau_l^i + \tau_c$ to be the income tax rate facing the resident in locality i . In addition, since there are no observed differences between the localities before the real wage rates have become realized, the optimal tax and provision of public goods will be the same in all localities. We can then drop the superindex i (for locality) and write the social optimization problem as follows;

⁵As was pointed out to us by Oved Yosha, the parameter γ may be interpreted as a measure of the degree of market insurance, provided that this insurance is exogenous in the model. This interpretation suggests that if γ is small (large), most (almost none) of the variability of ε has been insured by the market institution.

$$\underset{\tau, x, G}{Max} N[v(\bar{w}, \gamma, \tau) + \phi(x) + \zeta(G)] \quad (5)$$

subject to

$$N[\tau \bar{w} l(\bar{w}, \gamma, \tau) - x] - G = 0 \quad (6)$$

In addition to equation (6), which is the resource constraint of the optimization problem, the necessary conditions become

$$v_\tau + \lambda[\bar{w} l + \tau \bar{w} l_\tau] = 0 \quad (7)$$

$$\phi_x - \lambda = 0 \quad (8)$$

$$N\zeta_G - \lambda = 0 \quad (9)$$

where $l_\tau = \partial l / \partial \tau$, $v_\tau = \partial v / \partial \tau$, $\phi_x = \partial \phi / \partial x$ and $\zeta_G = \partial \zeta / \partial G$. The variable λ is the Lagrange multiplier associated with the resource constraint. Note that the term within brackets in equation (7) represents the slope of the so called 'Laffer curve', i.e. the relationship between the total tax revenues and the income tax rate. Since $v_\tau \leq 0$ by the properties of an indirect utility function, equation (7) implies that the tax revenue is a nondecreasing function of the tax rate at the equilibrium. Throughout this paper, we will assume that the tax revenue is a strictly increasing function of the income tax rate; $l + \tau l_\tau > 0$. For further reference, denote the outcome of the social utility maximization problem by (τ^*, x^*, G^*) .

2.2 Decentralized Policy Decisions

In this subsection, we shall contrast the centralized resource allocation system with a decentralized system, where all tax revenues are being collected by the local governments. To be able to make this comparison, suppose that the central government collects a fee from each locality in order to finance its expenditures on the federal public good. Since the localities are identical before the real wage rates have become

realized, we can concentrate on the representative locality and drop the superindex i . The budget constraint facing the local government is written

$$\tau w l(\bar{w}, \gamma, \tau) - x - \Gamma = 0 \quad (10)$$

in which Γ is the fee collected by the central government. To be able to compare the outcome of decentralized policy decisions with the choices made by the centralized policy maker in the previous subsection, it is convenient to define the decentralized decision problem conditional on the optimal allocation for the federal public good. We assume that the central government chooses $\Gamma = G^*/N$ (which is feasible, since the central government is able to solve the hypothetical second best problem of subsection 2.1 in order to determine G^*). Therefore, the only difference between the centralized and decentralized decision problems will be that the budget constraint is stochastic from the point of view of the local government.

Each local government chooses its income tax rate before the random productivity shock has become realized. More specifically, the local government's decision problem will be to choose income tax rate in order to maximize the expected utility of the local resident. The provision of the local public good is then determined residually from the budget constraint when the real wage rate has become realized. The optimal tax problem can be written as

$$Max_{\tau} v(\bar{w}, \gamma, \tau) + E[\phi(\tau w l(\bar{w}, \gamma, \tau) - G^*/N)] - \zeta(G^*)$$

where $w = \bar{w} + \gamma\varepsilon$, and the expectations operator on the second term is due to the fact that the budget constraint is stochastic from the local government's point of view. The first order condition can be written

$$v_{\tau} + E[\phi_x]\bar{w}(l + \tau l_{\tau}) + cov(\phi_x, \varepsilon)\gamma(l + \tau l_{\tau}) = 0 \quad (11)$$

Note that the covariance between ϕ_x and ε is negative as a result of risk-aversion. This is seen because $sign cov(\phi_x, \varepsilon) = sign [d\phi_x/d\varepsilon] < 0$ due to the assumption that $\phi_{xx} < 0$.

Given that the choice of income tax rate in each locality obeys equation (11), is it possible for the central government to improve the resource allocation by imposing

a national (or federal) income tax on top of τ ? Suppose that the additional tax revenues are distributed as lump-sum grants of equal size to each local government. Each local government can then respond by changing its income tax rate and/or expenditures on the local public good. The additional tax rate imposed by the central government (which is small by assumption) will be denoted by α . The value function facing the central government is given by

$$V = \sum_{i=1}^N [v(\bar{w}, \gamma, \tau + \alpha) + E[\phi(x^i)] + \zeta(G^*)] \quad (12)$$

where the 'local policy variables' τ and x^i , $i = 1, \dots, N$, are functions of α . The public good provided by the central government is not affected by the reform. In line with the assumption that the tax revenue increases with the tax rate, we shall also require that the 'effective tax rate' facing a local resident, $\alpha + \tau^i$, is (locally) increasing in the rate chosen by the central government, so $\partial \tau^i / \partial \alpha = \tau_\alpha^i > -1$ around the equilibrium point defined by equation (11). The cost benefit rule can be derived by differentiating equation (12) with respect to α and then evaluating the resulting derivative at the point where $\alpha = 0$ (which represents the prereform resource allocation). Since the localities are identical before the real wage rates have become realized, and since the risk associated with this marginal project is shared equally among them, the cost benefit rule for α will be

$$\frac{\partial V}{\partial \alpha} = -N \text{cov}(\phi_x, \varepsilon)[l + \tau l_\tau][1 + \tau_\alpha] > 0 \quad (13)$$

Equation (13) is formally derived in the Appendix. We shall here be concerned with the interpretation of equation (13);

Proposition 1 *Suppose that all revenues from distortionary taxation are being collected by the lower level of government and that the localities are identical before the real wage rates have become realized. It is then possible for the central government to improve the resource allocation by introducing a federal income tax on top of the local rates and redistribute the additional tax revenues equally among the local governments in the form of a lump-sum grant.*

Proposition 1 means that the central government can improve the resource allocation, when all prereform tax revenues are being collected by the lower level of

government. An interpretation is that risk-sharing is welfare improving at the margin in an economy with decentralized policy decisions.

If the central government can improve the resource allocation by adding an income tax on top of the local rates, a natural next question is whether the centralized policy decisions of subsection 2.1 give rise to a higher income tax rate than the decentralized framework set out here. Consider Proposition 2, which provides a sufficient condition for a centralized resource allocation system to give rise to a higher income tax rate than would be chosen by the local governments in the decentralized system;

Proposition 2 *If the localities are identical before the real wage rates have become realized, and if the preferences for local public goods are such that $E[\phi_x(x)] \leq \phi_x(E[x])$ for all x , the centralized system will always give rise to a higher income tax rate than the decentralized system.*

Proof. Let τ^0 be the income tax rate chosen by the identical localities and introduce the short notations $E[\phi_x(\tau^0)] = E[\phi_x(\tau^0 w l^0 - G^*/N)]$ and $\bar{\phi}_x(\tau^0) = \phi_x(\tau^0 \bar{w} l^0 - G^*/N)$. Then, by adding and subtracting $\bar{\phi}_x(\tau^0)[\bar{w} l^0 + \tau^0 \bar{w} l_\tau^0]$, equation (11) can be rewritten as

$$v_\tau(\tau^0) + \bar{\phi}_x(\tau^0) \bar{w} [l^0 + \tau^0 l_\tau^0] + \{E[\phi_x(\tau^0)] - \bar{\phi}_x(\tau^0)\} \bar{w} [l^0 + \tau^0 \bar{w} l_\tau^0] + \text{cov}(\phi_x(\tau^0), \varepsilon) \gamma [l^0 + \tau^0 l_\tau^0] = 0$$

Therefore, if $E[\phi_x(\tau^0)] \leq \bar{\phi}_x(\tau^0)$, and since $\text{cov}(\phi_x(\tau^0), \varepsilon) < 0$, it holds that

$$v_\tau(\tau^0) + \bar{\phi}_x(\tau^0) \bar{w} [l^0 + \tau^0 l_\tau^0] > 0$$

which is the central government's first order condition evaluated at the tax rate chosen by the local governments. The second order sufficient conditions for maximization will then imply $\tau^* > \tau^0$. ■

In technical terms, this condition means that the marginal utility of local public goods, $\phi_x(x)$, is a globally concave function. This will effectively rule out the possibility of decreasing absolute risk aversion and, therefore, the possibility of reducing the disutility of risk by increasing the consumption of local public goods. In the absence of such incentives - or more generally, if they are not 'too strong' - risk aversion will induce the local governments to choose a lower income tax rate than

would be chosen by a central government, which is able to provide full insurance. However, if $\phi_x(x)$ is not a globally concave function (a situation that may arise as a consequence of decreasing absolute risk aversion), the comparison between the two policy regimes with respect to the income tax rate will, in general, remain inconclusive. Finally, notice the connection between Propositions 1 and 2. If $\phi_x(x)$ is a globally concave function, this will be sufficient to ensure that $\tau_\alpha > -1$, which is an assumption underlying Proposition 1. However, the converse result is not true: if $\tau_\alpha > -1$ around the equilibrium point defined by equation (11), it does not follow that the condition in Proposition 2 is valid.

2.3 Implementation of Second Best in a Decentralized Economy

Let us now return to the framework set out in the beginning of Section 2, where tax and expenditure decisions are made by both levels of government. Our concern will be to study how the central government must act in order to make the local governments behave in an optimal way from society's point of view. There are two issues involved: internalization of vertical fiscal externalities and risk-sharing.

By reintroducing the superindex i for local government, we can write the local budget constraint as

$$\tau_l^i w^i l(\bar{w}, \gamma, \tau_l^i + \tau_c) + T^i - x^i = 0 \quad (14)$$

where T is a subsidy (positive or negative) from the central government to the local government. Similarly, the budget constraint facing the central government takes the form

$$\sum_{i=1}^N [\tau_c \bar{w} l(\bar{w}, \gamma, \tau_l^i + \tau_c) - T^i] - G = 0 \quad (15)$$

which means that the central government has three policy instruments at its disposal; τ_c , G and T .

Clearly, when the central government has perfect information about the preferences of local residents, and since the central government has the means to provide

risk-sharing, it is actually able to solve the (hypothetical) social optimization problem described in subsection 2.1 in order to determine G^* . It is also able to design the subsidy to each local government in such a way, that the distortions that would otherwise arise from vertical externalities and risk-aversion are offset. This is described in Proposition 3;

Proposition 3 *If the central government announces that the subsidy to local government i ($i = 1, \dots, N$) will take the form*

$$T^i = \tau_c \bar{w} l(\bar{w}, \gamma, \tau_l^i + \tau_c) - G^*/N + \tau_l^i (\bar{w} - w^i) l(\bar{w}, \gamma, \tau_l^i + \tau_c) \quad (16)$$

where τ_c is any income tax rate imposed by the central government, then $\tau_l^i = \tau^* - \tau_c$ and $x^i = x^*$ will solve the local governments' optimization problems.

Proof. By substituting $T^i = \tau_c \bar{w} l(\bar{w}, \gamma, \tau_l^i + \tau_c) - G^*/N + \tau_l^i (\bar{w} - w^i) l(\bar{w}, \gamma, \tau_l^i + \tau_c)$ into equation (14), the local optimization problem can be written as

$$\underset{\tau, x}{Max} \ v(\bar{w}, \gamma, \tau) + \phi(x) + \zeta(G^*)$$

subject to

$$\tau \bar{w} l(\bar{w}, \gamma, \tau) - x - G^*/N$$

where $\tau = \tau_l + \tau_c$ (and we have used that the local tax rates will be identical). This is equivalent to the social optimization problem of subsection 2.1. ■

Note that the central government announces this policy before the real wage rates have become realized. The final term on the right hand side of equation (16) means that the local governments are fully insured against risk in the social optimum, since they will behave as if they face the average labor income (or tax base). This pure revenue sharing between the local governments will not affect the central government's budget constraint, since $\sum_{i=1}^N [\bar{w} - w^i] = 0$. Knowing this, the central government can announce the revenue sharing part without information about what the realized real wage rate will be for each locality. Since the revenue sharing between the local governments is motivated by the desire to avoid risk, one can interpret the third term on the right hand side of equation (16) as if it provides

an efficiency argument in favor of 'tax revenue equalization' within the local public sector. Such systems (with more or less revenue equalization) have been common in the Nordic countries and imply that the localities share the tax revenues or tax bases within a nation-wide system.

The first two terms on the right hand side of equation (16) reflect (in principle) what the optimal policy rule would be in the absence of uncertainty and has been derived by Aronsson and Wikström (1999). Note that the central government's choice of τ_c will not affect the nature of the policy rule as such; only the size of the subsidy. In other words, when the central government is able to solve the (hypothetical) social optimization problem, and is able to transfer resources between the two levels of government, the central government's income tax will become a redundant policy instrument.

3 Deterministic Wage Differentials

An important assumption in the previous section is that all differences between the localities with respect to the realized real wage rates are due to different random shocks. Even if this assumption is convenient and may provide a suitable starting point, it is by no means realistic. We shall here relax this assumption and, instead, assume that part of the 'locality specific' productivity is known before the random shock has become realized.

The realized real wage rate of locality i will be written as

$$w^i = \bar{w} + \beta^i + \gamma\varepsilon^i \quad (17)$$

where β^i represents the deterministic part of the locality specific productivity. The local resident will still be assumed to choose his/her labor supply before the random shock has become realized. The utility maximization problem is given by

$$Max_l E[u((\bar{w} + \beta^i + \gamma\varepsilon^i)l^i(1 - \tau_l^i - \tau_c), l^i)] + \phi(x^i) + \zeta(G)$$

in which case the labor supply becomes $l^i = l(\bar{w} + \beta^i, \gamma, \tau_l^i + \tau_c)$, and consumption is determined residually from the budget constraint when the random shock has become realized.

3.1 Marginal Risk-Sharing in a Decentralized Framework

The purpose of this subsection is to study whether tax revenue-sharing is welfare superior to a decentralized policy regime. Recall from the previous section that the central government's income tax rate is a redundant policy instrument from the point of view of internalization of the vertical fiscal externalities. To begin with, therefore, we assume that the central government's income tax rate is equal to zero. In addition, we would like to simplify by conditioning the policy analysis on the optimal allocation of the federal public good (as we did in subsection 2.2). A convenient way of doing this without first solving the hypothetical second best problem (which we do not intend to do here) is by introducing 'local provision' towards the federal public good, which will be appropriately subsidized by the central government. With these assumptions, the central government's budget constraint can be written

$$\sum_{i=1}^N [S^i - s^i g^i] = 0 \quad (18)$$

where g^i is local government i 's contribution towards the federal public good, so $\sum_{i=1}^N g^i = G$. The central government subsidizes local government i 's contribution towards the federal public good at the rate s^i and collects a lump-sum tax or fee from local government i , S^i . The convenience of introducing subsidized local provision of the federal public good is that, by choosing this subsidy in a particular way (to be described below) the federal public good will be allocated in an optimal way from society's point of view conditional on the choice of tax rate and provision of the local public good. This makes it easy to concentrate the analysis on tax revenue-sharing.

The budget constraint facing local government i takes the form

$$\tau^i w^i l(\bar{w} + \beta^i, \gamma, \tau^i) - x^i - (1 - s^i)g^i - S^i = 0 \quad (19)$$

where $w^i = \bar{w} + \beta^i + \gamma \varepsilon^i$, and the subindex on the local income tax rate has been dropped. The local government chooses income tax rate and local provision towards the federal public good in order to maximize the expected utility, and the provision of the local public good is then determined residually from the budget constraint

when the real wage rate has become realized. The utility maximization problem can be written as

$$Max_{\tau^i, g^i} v(\bar{w} + \beta^i, \gamma, \tau^i) + E[\phi(\tau^i w^i l(\bar{w} + \beta^i, \gamma, \tau^i) - (1 - s^i)g^i - S^i)] + \zeta(G)$$

The first order conditions are given by

$$v_{\tau^i} + E[\phi_{x^i}][\bar{w} + \beta^i][l^i + \tau^i l_{\tau^i}] + \gamma cov(\phi_{x^i}, \varepsilon^i)[l^i + \tau^i l_{\tau^i}] = 0 \quad (20)$$

$$-(1 - s^i)E[\phi_{x^i}] + \zeta_G = 0 \quad (21)$$

in which we have used the short notations $l^i = l(\bar{w} + \beta^i, \gamma, \tau^i)$ and $l_{\tau^i} = \partial l^i / \partial \tau^i$. Clearly, if the central government chooses the subsidy such that $s^i = (N - 1)/N$, equation (21) will be equivalent to the Samuelson condition, $-E[\phi_{x^i}] + N\zeta_G = 0$. In what follows, we shall assume that the central government imposes this subsidy on each local government, meaning that all of them will contribute to the federal public good in accordance with the Samuelson condition.

Our concern will be to study whether the central government can improve the resource allocation by introducing a small income tax on top of the local rates and then redistribute the additional tax revenues between the local governments in the form of lump-sum grants. To define the resource constraint for the economy as a whole, start by aggregating the local budget constraints given by equation (19). Then, by using the fact that the central government's budget constraint changes to read $\sum_{i=1}^N [\alpha w^i l^i + S^i - s^i g^i] = 0$, where α (which is assumed to be small) is the tax rate imposed by the central government, we can write the public resource constraint as

$$\sum_{i=1}^N [(\tau^i + \alpha)w^i l^i - x^i - g^i] = 0 \quad (22)$$

where the local policy variables τ^i , x^i and g^i , for $i = 1, \dots, N$, are functions of α , since the local governments will respond to the policy reform conducted by the central government.

To be able to focus on the efficiency aspects of risk-sharing, and since there are differences between the localities which are observed before the real wage rates have

become realized, only part of the additional tax revenues will be used for purposes of revenue-sharing. More specifically, we assume that the deterministic part of the additional tax revenues collected from the resident of locality i is returned to local government i , whereas the part of the additional tax revenues that depend on the realization of the random productivity shocks is distributed equally between the local governments. With these assumptions, the reform will have the following effect on the resource constraint relevant for locality i ;

$$(\bar{w} + \beta^i)(l^i + \tau^i l_{\tau^i})(1 + \tau_{\alpha}^i) + \gamma \frac{1}{N} \sum_{j=1}^N \varepsilon^j (l^j + \tau^j l_{\tau^j})(1 + \tau_{\alpha}^j) - x_{\alpha}^i - g_{\alpha}^i = 0 \quad (23)$$

which has been evaluated at the point where $\alpha = 0$, and the subindices denote partial derivatives.

In a way similar to Section 2, the value function facing the central government is written as

$$V = \sum_{i=1}^N [v(\bar{w} + \beta^i, \gamma, \tau^i + \alpha) + E[\phi(x^i)] + \zeta(G)] \quad (24)$$

in which τ^i , x^i and g^i , for $i = 1, \dots, N$, are functions of α , and $G = \sum_{i=1}^N g^i$. The welfare change measure can now be derived by differentiating the value function in equation (24) with respect to α , evaluating the derivative at the point where $\alpha = 0$ and then using equation (23). We show in the Appendix that

$$\frac{\partial V}{\partial \alpha} = \gamma \sum_{i=1}^N \{-cov(\phi_{x^i}, \varepsilon^i)[l^i + \tau^i l_{\tau^i}][1 + \tau_{\alpha}^i] + \frac{1}{N} \sum_{j=1}^N E[\phi_{x^i} \varepsilon^j][l^j + \tau^j l_{\tau^j}][1 + \tau_{\alpha}^j]\} \quad (25)$$

For each locality, i , the cost benefit rule consists of two terms: first, the welfare gain of local risk reduction and, second, the cost facing the locality from having to carry part of the risk associated with productivity shocks in all N localities. In this particular case, however, there is no risk-sharing at all in the prereform equilibrium, which means $E[\phi_{x^i} \varepsilon^j] = E[\phi_{x^i}]E[\varepsilon^j] = 0$ for $j \neq i$. The intuition is that, when there are no risk-sharing arrangements in the prereform equilibrium, the marginal utility of local public goods in locality i does not depend on the tax revenues collected by the other localities. As a consequence, equation (25) reduces to read

$$\frac{\partial V}{\partial \alpha} = -[1 - \frac{1}{N}]\gamma \sum_{i=1}^N \text{cov}(\phi_{x^i}, \varepsilon^i)[l^i + \tau^i l_{\tau^i}][1 + \tau_{\alpha}^i] > 0 \quad (26)$$

We can interpret equation (26) as follows;

Proposition 4 *Suppose that part of the differences in productivity between the localities is observed before the random shocks have become realized, and that all revenues from distortionary taxation are being collected by the local governments. The central government can then improve the resource allocation by introducing an income tax on top of the local rates, and distribute the part of the additional tax revenues that depends on the random productivity shocks equally between the local governments in the form of lump-sum grants.*

It is important to emphasize that Proposition 4 only applies when there is no risk-sharing at all in the prereform equilibrium. In this case, there are no costs associated with a 'preexisting' arrangement for risk-sharing. If the prereform equilibrium is characterized by risk-sharing, the welfare effect can go in either direction, which is seen from equation (25).

3.2 Can 'full insurance' be optimal?

In Section 2, where all localities are identical before the random shocks have become realized, we found that full insurance against the risk associated with these shocks is, indeed, the optimal policy. An interesting question then arises: is it possible to rule out that full insurance is optimal, when the local governments differ before the random productivity shocks are realized? We shall interpret full insurance to mean that the outcome of the productivity shocks in terms of tax revenues is shared equally between the local governments. In other words, they will share the part of the aggregate tax revenues that is determined by these productivity shocks. Therefore, the 'effective budget constraint' facing local government i can be written as

$$\tau^i[\bar{w} + \beta^i]l^i + \gamma \frac{1}{N} \sum_{j=1}^N \tau^j \varepsilon^j l^j - x^i - (1 - s^i)g^i - S^i = 0 \quad (27)$$

where $l^i = l(\bar{w} + \beta^i, \gamma, \tau^i)$.

Let us, for the moment, disregard how this risk sharing arrangement is implemented, and simply assume that local government i chooses τ^i and g^i to maximize the expected utility subject to equation (27), while x^i is determined residually from the budget constraint when the stochastic part of the tax revenues are realized. It is easy to show that the first order condition for g^i will take the form of equation (21). As in the previous subsection, we assume that the central government subsidizes the local provision towards the federal public good in such a way, that the Samuelson condition is fulfilled. The first order condition for τ^i changes to read

$$v_{\tau^i} + E[\phi_{x^i}][\bar{w} + \beta^i][l^i + \tau^i l_{\tau^i}] + \gamma \frac{1}{N} \text{cov}(\phi_{x^i}, \varepsilon^i)[l^i + \tau^i l_{\tau^i}] = 0 \quad (28)$$

Now, suppose that the policies chosen by each local government, i , obey equations (21) and (28). We shall refer to this outcome by the term 'full insurance equilibrium'. Consider next what happens if we introduce a small labor subsidy in each locality, which is equivalent to a small uniform decrease of the income tax rates. In addition, the subsidy given to local resident i is financed by a lump-sum fee paid by local government i to the central government. If the labor subsidy is denoted by θ , and by assuming that the reform is budget neutral at the local government level, the change in local government i 's provision of the local public good is implicitly defined by

$$-[\bar{w} + \beta^i][l^i + \tau^i l_{\tau^i}][1 + \tau_{\theta}^i] - \gamma \frac{1}{N} \sum_{j=1}^N \varepsilon^j [l^j + \tau^j l_{\tau^j}][1 + \tau_{\theta}^j] - x_{\theta}^i - g_{\theta}^i = 0 \quad (29)$$

which has been evaluated at the point where $\theta = 0$. The value function looks like equation (24) with the exceptions that (i) α is replaced by $-\theta$, and (ii) the value function is here evaluated in the full insurance equilibrium. By analogy to the previous subsection, if we differentiate the value function with respect to θ , evaluate the resulting derivative at the point where $\theta = 0$, and then use equations (21), (28) and (29), we obtain

$$\frac{\partial V}{\partial \theta} = -\gamma \frac{1}{N} \sum_{i=1}^N \sum_{j \neq i} \text{cov}(\phi_{x^i}, \varepsilon^j)[l^j + \tau^j l_{\tau^j}][1 + \tau_{\theta}^j] > 0 \quad (30)$$

The sign of equation (30) is explained by the character of the initial equilibrium. Since all local income tax rates were optimally chosen subject to a 'full insurance constraint' prior to the reform, the local net benefits associated with risk-sharing are zero at the margin. Instead, the welfare effect arises because the aggregate tax burden on labor is being slightly reduced and, as a consequence, each local government will be carrying a smaller fraction of the outcome of productivity shocks in the other localities.

We have derived the following result;

Proposition 5 *The full insurance equilibrium is suboptimal from society's point of view. If the economy is in the full insurance equilibrium, the central government can increase the welfare level by a policy designed to imply less risk-sharing.*

4 Summary and Discussion

This paper concerns optimal taxation and risk-sharing arrangements in an economy, where tax and expenditure decisions are being made by both central and local governments. One important assumption is that the localities experience different random productivity shocks, meaning that the governments' choices of income tax rates and the private labor supply decision are carried out before the real wage rates have become realized. Another is that risk-sharing at the private agent level is not feasible, and we concentrate the analysis on risk-sharing arrangements within the public organization. The main purpose of the paper is to study how the central government can improve the resource allocation by means of policies designed to affect the decisions made by the local governments.

When the localities are identical before the random productivity shocks have become realized, the socially optimal resource allocation will imply that the local governments are fully insured against risk. Since the labor supply is endogenous, implementation of this resource allocation in a decentralized setting implies that the central government simultaneously internalizes vertical fiscal externalities and designs a system for tax revenue-sharing within the local public sector. The latter is particularly interesting, since it provides an efficiency argument in favor of 'revenue-

equalization' among local governments. Such systems are common in the Nordic countries and are mainly motivated on the basis of distributional objectives.

If, on the other hand, part of the differences in productivity between the localities is deterministic and observed before the random productivity shocks have become realized, the localities will differ with respect to how the local residents choose their labor supply. As a consequence, even if the locality specific, stochastic components of the real wage rates cancel out on an aggregate level, differences in labor supply incentives across localities will introduce what resembles 'aggregate risk'. The main result here is that, even if the central government can improve the resource allocation by introducing risk-sharing in an otherwise decentralized policy regime, full insurance at the local level of government is not optimal. This is explained by the costs arising, when each local government has to carry part of the outcome of random productivity shocks in the other localities.

5 Appendix

Derivation of equation (13)

By differentiating equation (12) with respect to α , while using that G^* is predetermined, we obtain

$$\frac{\partial V}{\partial \alpha} = N\{v_\tau[1 + \tau_\alpha] + E[\phi_x x_\alpha]\} \quad (\text{A1})$$

Since the localities are identical before the random productivity shocks have become realized, and since the additional tax revenues are shared equally between the local governments in the form of lump-sum grants, the change in the provision of the local public good will be the same in all localities. The change in the provision of the local public good is implicitly defined by

$$[1 + \tau_\alpha]\bar{w}l + \tau\bar{w}l_\tau[1 + \tau_\alpha] - x_\alpha = 0 \quad (\text{A2})$$

which has been evaluated at the point where $\alpha = 0$, and where $l = l(\bar{w}, \gamma, \tau + \alpha)$. Solving equation (A2) for x_α , substituting into equation (A1), while using $v_\tau +$

$E[\phi_x]\bar{w}(l + \tau l_\tau) = -cov(\phi_x, \varepsilon)\gamma(l + \tau l_\tau)$ according to equation (11), gives equation (13).

Derivation of equation (25)

Differentiating equation (24) with respect to α gives

$$\frac{\partial V}{\partial \alpha} = \sum_{i=1}^N \{v_{\tau^i} [1 + \tau_\alpha^i] + E[\phi_{x^i} x_\alpha^i] + \zeta_G G_\alpha\} \quad (\text{A3})$$

By solving equation (23) for x_α^i , substituting into equation (A3) and rearranging, we can use equations (A3), (20) and (21) to derive equation (25). Note that the use of equation (21) is based on the assumption that each local government's contribution towards the federal public good obeys the Samuelson condition, so $s^i = (N - 1)/N$ for $i = 1, \dots, N$.

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