# Occurrence of long and short term asymmetry in stock market volatilities

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#### Abstract

We introduce the notions of short and long term asymmetric effects in volatilities. With short term asymmetry we mean the conventional one, i.e. the asymmetric response of current volatility to the most recent return shocks. However, there may be asymmetries in the way the effect of past return shocks propagate over time as well. We refer to this as long term asymmetry. We propose a model that enables the study of such a feature. In an empirical application using stock market index data we found evidence of the joint presence of short and long term asymmetric effects.

Key Words: Financial econometrics, GARCH, memory, nonlinear, risk prediction, time series.JEL Classification: C22, C51, C58, G17, G15.

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### 1 Introduction

Understanding the dynamics of financial return volatility is crucially important in financial contexts such as risk management and portfolio selection. To this end the ARCH/GARCH framework of Engle (1982) and Bollerslev (1986) stands out as the single most important tool, and since the birth of the basic models the literature has exploded with different extensions (see Andersen, Bollerslev, Christoffersen, and Diebold, 2006, for an overview). Seemingly, the most popular and empirically relevant ones are those attempting to cope with the stylized fact of asymmetry. This property is most notable for equity returns and it refers to the fact that return volatility tends to rise more following negative return shocks than positive ones. This was first noted by Black (1976), who argued that negative return shocks increase financial leverage implying a riskier return on equity given an unchanged stream of cash flows.

This feature has generated a substantial amount of research and several alternatives for how to best cope with it have been proposed in the literature. The most popular one appears to be the asymmetric GARCH (GJR-GARCH) of Glosten, Jagannathan, and Runkle (1993). Other commonly employed alternatives include the exponential GARCH (EGARCH) model of Nelson (1990) and the quadratic GARCH model of (QGARCH) of Sentana (1995). A more recent extension is the dynamic asymmetric GARCH (DAGARCH) of Caporin and McAleer (2006) that generalizes the GJR-GARCH to include multiple and time-varying thresholds. Up till now the effort in terms of modeling has focused on how to best capture the response of current volatility to the most recent return shocks. Thus, the perspective is largely short term. However, it does not appear too far fetched to expect asymmetries in the way the effect of past return shocks propagate over time as well. We refer to this as long term asymmetry. In this paper we propose a simple extension of the GJR-GARCH model to allow for the potential occurrence of such a feature.

### 2 Model

We define a return shock process  $\{u_t\}$  that is generated in discrete time by

$$u_t = \sqrt{h_t} \varepsilon_t,\tag{1}$$

where  $\{\varepsilon_t\} \sim iid(0, 1)$ . Returns are given by  $r_t = \mu_t + u_t$  and with  $\mathcal{F}_t$  denoting the history up to and including time t we have the conditional mean  $\mu_t = E(r_t|\mathcal{F}_{t-1})$  and variance  $h_t = V(u_t|\mathcal{F}_{t-1}) = V(r_t|\mathcal{F}_{t-1})$ , respectively. To allow for asymmetric effects in the specification of  $h_t$  we define  $u_t^{2+} = u_t^2 \mathbf{1}(u_t > 0)$  and  $u_t^{2-} = u_t^2 \mathbf{1}(u_t \le 0)$ , where  $\mathbf{1}(\cdot)$  is the indicator function.

The basic GJR-GARCH specification for the conditional variance may then be defined as

$$h_t = \omega + \alpha^+ u_{t-1}^{2+} + \alpha^- u_{t-1}^{2-} + \beta h_{t-1}.$$
(2)

To guarantee a positive variance at all times we require that  $\omega > 0$  and  $\alpha^+, \alpha^-, \beta \ge 0$ . For a non-explosive behavior we add the restriction  $\beta < 1$ . Ling and McAleer (2002) establishes conditions for stationarity and ergodicity and the existence of moments for a family of GARCH models including the GJR-GARCH. We say that the model is asymmetric in the short term sense but not in the long term since the rate of decay of the effect of past positive and negative return shocks is the same. The difference merely occurs with respect to the levels of  $\alpha^+$  and  $\alpha^-$ . This is most easily seen from the corresponding infinite ARCH representation of (2)

$$h_{t} = \frac{\omega}{1-\beta} + \frac{\alpha^{+}}{1-\beta L} u_{t-1}^{2+} + \frac{\alpha^{-}}{1-\beta L} u_{t-1}^{2-}$$
  
$$= \omega^{*} + \alpha^{+} (1+\beta L + (\beta L)^{2} + ...) u_{t-1}^{2+} + \alpha^{-} (1+\beta L + (\beta L)^{2} + ...) u_{t-1}^{2-}, \qquad (3)$$

where  $\omega^* = \omega/(1-\beta)$  and L is the lag operator, i.e.  $Lx_t = x_{t-1}$ . Now, to parsimoniously accommodate asymmetry also in the long term sense our simple idea is to extend the model to

have one  $\beta$  for positive return shocks and one for negative ones, i.e.

$$h_{t} = \omega^{*} + \alpha^{+} (1 + \beta^{+}L + (\beta^{+}L)^{2} + ...)u_{t-1}^{2+} + \alpha^{-} (1 + \beta^{-}L + (\beta^{-}L)^{2} + ...)u_{t-1}^{2-}$$
  
$$= \omega^{*} + \frac{\alpha^{+}}{1 - \beta^{+}L}u_{t-1}^{2+} + \frac{\alpha^{-}}{1 - \beta^{-}L}u_{t-1}^{2-}.$$
 (4)

Note that upon multiplying both sides of eq. (4) by  $(1 - \beta^+ L)(1 - \beta^- L)$  and re-arranging we obtain

$$h_{t} = \omega + \alpha^{+} u_{t-1}^{2+} + \alpha^{-} u_{t-1}^{2-} + \alpha^{+} \beta^{-} u_{t-2}^{2+} + \alpha^{-} \beta^{+} u_{t-2}^{2-} + (\beta^{+} + \beta^{-}) h_{t-1} + \beta^{+} \beta^{-} h_{t-2}$$
(5)

Thus, our proposed specification may be viewed as a restricted version of a GJR-GARCH(2,2). As such, we expect results for ergodicity and stationarity for the GJR-GARCH to continue to hold. Here, we considered a single threshold set at zero and a relatively simple lag structure. Being a first development in this direction this seems to be a natural choice. However, a richer lag structure as well as extensions towards multiple and even time-varying thresholds as in Caporin and McAleer (2006) are in principle possible.

### 3 Estimation

To estimate the model parameters we employ the standard quasi maximum likelihood estimator. Thus, with a normality assumption on the innovation,  $\varepsilon_t$ , in (1) and with observations on  $r_t$  up to time T, the likelihood function takes the form

$$\ln L \propto -\frac{1}{2} \sum_{t=s}^{T} \ln(h_t) - \frac{1}{2} \sum_{t=s}^{T} u_t^2 / h_t,$$
(6)

where s is determined by the number of lags in the mean and the variance specifications. All estimations are carried out in the RATS 7.3 package using the built in BFGS algorithm for maximization of (6). We use robust standard errors throughout. To fulfill parameter restrictions we may estimate the model parameters through re-parameterizations. In particular, we may set  $\alpha^+ = \exp(\alpha^{+*})$  and  $\beta^+ = 1/[1 + \exp(\beta^{+*})]$ .

Table 1: Descriptive statistics for the return series. JB is the *p*-value in the Jarque-Bera test of normality.  $LB_{10}$  and  $LB_{10}^2$  are the *p*-values in the Ljung-Box test evaluated at ten lags for returns and squared returns, respectively. Asy. is the *p*-value for the *t*-statistic of  $\mathbf{1}(r_{t-1} < 0)r_{t-1}^2$  in the regression of  $r_t^2$  on a constant,  $r_{t-1}^2, r_{t-2}^2, ..., r_{t-10}^2$  and  $\mathbf{1}(r_{t-1} < 0)r_{t-1}^2$ .

Index	Obs	Mean	Variance	Min	Max	Skewness	Kurtosis	JB	$LB_{10}$	$LB_{10}^2$	Asy
CAC40	1305	-0.045	3.155	-9.472	10.595	0.131	5.033	0.000	0.000	0.000	0.000
DAX	1305	-0.006	2.850	-7.433	10.797	0.132	5.453	0.000	0.042	0.000	0.000
FTSE 100	1305	-0.009	2.328	-9.266	9.384	-0.081	5.775	0.000	0.000	0.000	0.000
Hang Seng	1305	0.000	4.023	-13.582	13.407	0.094	6.381	0.000	0.163	0.000	0.000
NIKKEI 225	1305	-0.046	3.181	-12.111	13.235	-0.532	8.671	0.000	0.120	0.000	0.000
S&P 500	1305	-0.005	2.726	-9.470	10.957	-0.248	6.902	0.000	0.000	0.000	0.000
Straits Times	1305	-0.008	2.163	-8.696	7.531	-0.107	4.141	0.000	0.478	0.000	0.000

Given the likelihood framework the trinity is readily available when it comes to testing of hypothesis. We say that there are short term asymmetric effects when  $\alpha^+ \neq \alpha^-$ , while there is long term asymmetry when  $\beta^+ \neq \beta^-$ . Obviously, the cases can occur simultaneously.

### 4 Application

We apply our model to seven stock market indices: CAC 40 (France), DAX (Germany), FTSE 100 (United Kingdom), Hang Seng (Hong Kong), Nikkei 225 (Japan), S&P 500 (United States) and Straits Times (Singapore). Five years of daily index data was downloaded from Datastream covering the period April 18, 2007 to April 18, 2012. We calculate returns as  $r_t = 100 \ln(I_t/I_{i-1})$ , where  $I_t$  is the value of the index at time t.

In Table (1) we give some descriptives for the return series.

ARCH effects with potential asymmetry appears to be present in all series. There is also autocorrelation in returns for some indices. As argued by Lo and MacKinlay (1990) and others this may be due to non-synchronous trading in the constituents of the indices. To cope with it Bollerslev and Mikkelsen (1996) advocate the use of a third order autoregressive specification for the mean function. We adopt this suggestion and the estimated specification for all series

Table 2: Estimation results. *t*-statistics in italics. *L* is the value of the log-likelihood function. JB is the *p*-value in the Jarque-Bera test.  $LB_{10}$  and  $LB_{10}$  are *p*-values in the Ljung-Box test evaluated at ten legs for standardized return shocks and squared shocks, respectively.

	CLC10	DAV		TT C	N'II : 005	00 D F00	a m.
	CAC40	DAX	FTSE 100	Hang Seng	Nikkei 225	S&P 500	Straits Times
$\mu_0$	-0.038 -1.023	0.014 0.416	$0.002 \ 0.061$	-0.008 -0.203	-0.024 -0.673	$0.030 \ 0.954$	0.006 0.201
$\mu_1$	-0.017 $-0.503$	$0.007 \ 0.223$	-0.036 -1.104	$0.009 \ 0.264$	-0.012 -0.394	-0.088 -2.501	$0.020 \ 0.628$
$\mu_2$	-0.022 $-0.762$	-0.020 -0.693	-0.029 -1.012	-0.012 -0.398	-0.005 -0.165	-0.029 -0.978	-0.007 -0.242
$\mu_3$	-0.046 $-1.533$	-0.023 -0.762	-0.058 $-2.003$	0.040 1.471	$0.021 \ 0.706$	-0.031 -1.033	0.003 0.120
ω	$0.061 \ 0.151$	$0.044 \ 0.142$	$0.039 \ 0.123$	$0.003 \ 2.068$	$0.067 \ 0.125$	$0.028 \ 0.144$	$0.001 \ 1.522$
$\alpha^+$	0.000 0.000	$0.000 \ \theta.\theta\theta\theta$	$0.000 \ \theta.\theta\theta\theta$	0.030 2.926	$0.000 \ \theta.\theta\theta\theta$	$0.000 \ \theta.\theta\theta\theta$	$0.021 \ 2.165$
$\alpha^-$	$0.205 \ 8.462$	0.178 8.888	0.170 8.234	0.213 6.463	$0.184 \ 9.863$	0.166 8.321	$0.180 \ 6.559$
$\beta^+$	0.000 0.000	$0.000 \ \theta.\theta\theta\theta$	$0.000 \ \theta.\theta\theta\theta$	0.950 71.506	$0.000 \ \theta.\theta\theta\theta$	$0.000 \ \theta.\theta\theta\theta$	0.961 64.936
$\beta^{-}$	0.879 74.170	0.894 93.914	0.894 82.746	0.822 26.307	0.879 84.501	0.899 88.498	$0.868 \ 36.255$
L	-2357.556	-2276.757	-2133.681	-2471.312	-2316.503	-2137.573	-2076.927
JB	0.000	0.000	0.000	0.013	0.000	0.000	0.038
$LB_{10}$	0.944	0.864	0.951	0.915	0.774	0.894	0.794
$LB_{10}^{2}$	0.679	0.973	0.861	0.032	0.299	0.319	0.962

 $\mathbf{is}$ 

$$\begin{aligned} r_t &= \mu_0 + \mu_1 r_{t-1} + \mu_2 r_{t-2} + \mu_3 r_{t-3} + u_t, \quad u_t = \varepsilon_t \sqrt{h_t}, \quad \varepsilon_t \sim N(0, 1), \\ h_t &= \omega + \alpha^+ u_{t-1}^{2+} + \alpha^- u_{t-1}^{2-} + \alpha^+ \beta^- u_{t-2}^{2+} + \alpha^- \beta^+ u_{t-2}^{2-} + (\beta^+ + \beta^-) h_{t-1} + \beta^+ \beta^- h_{t-2} \end{aligned}$$

In Table 2 we give estimation results along with some diagnostic  $checks^1$ 

The estimates of  $\alpha^+$  and  $\alpha^-$  indicates short term asymmetries. Indeed, in the formal Wald testing of the null  $\alpha^+ = \alpha^-$  we obtained very strong rejections for all series. Interestingly, the estimates of  $\beta^+$  and  $\beta^-$  suggest the presence of asymmetry in the long term sense as well. Again, the formal Wald test rejected very strongly. For the European indices, the Nikkei 225 and the S&P 500 it is difficult to discriminate between long and short term asymmetry though, since both  $\alpha^+$  and  $\beta^+$  are insignificantly estimated close to zero. It should be noted that  $\beta^+$ is unidentified under the null  $\alpha^+ = 0$  and vice versa. Thus, to further scrutinize on the role of positive return shocks for these indices we estimated our model imposing the restrictions  $\beta^+ = \beta^-$  (the conventional GJR-GARCH) and  $\alpha^+ = \alpha^-$ , respectively. Unreported estimation results confirm that positive return shocks appears to be irrelevant for the prediction of future

<sup>&</sup>lt;sup>1</sup>The Ljung-Box tests of no autocorrelation up to lag 10 in the squared standardized residuals were computed as in Li and Mak (1994).



Figure 1: The effect (y-axis) of a 5% return shock at time t on the volatility of the Hang Seng index at time t-the x-axis.

volatility for these indices. In this respect the parameter estimates for the Hang Seng and the Straits Times indices are more interesting. The estimates of the  $\alpha$ 's implies that the initial effect on volatility of a negative return shock is much larger than that of a positive one of the same size (short term asymmetry). However, the estimates of the  $\beta$ 's suggest that the effect of it dies out at a faster rate (long term asymmetry). In Figure 1 the effects of a fairly large return shock of 5% for the Hang Seng index illustrates. The effect of a negative shock is seen to be substantially higher for the first days. However, owing to a higher persistence the effect on volatility of the corresponding positive shock is larger already 15 days ahead.

## 5 Conclusion

In this paper we introduced the notions of long and short term asymmetry in volatility. To test for their joint occurrence we proposed a simple extension of the GJR-GARCH model. In an empirical application using stock market index data we found evidence of the joint presence of short and long term asymmetric effects. Much in line with previous research we find that the initial effect of negative return shocks are substantially higher than positive ones. However, for the indices where positive return shocks are relevant in explaining future volatilities we found a substantially higher persistence for positive shocks. This finding is interesting and to the best of our knowledge new to the literature.

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