Tax Policy and Present-Biased Preferences: Paternalism under International Capital Mobility*

Thomas Aronsson and Tomas Sjögren
Department of Economics, Umeå University
SE - 901 87 Umeå, Sweden

September 2011

Abstract

This paper deals with tax-policy responses to quasi-hyperbolic discounting. Earlier research on optimal paternalism typically abstracts from capital mobility. If capital is mobile between countries, it may no longer be possible for national governments to control domestic savings via capital taxation (as in a closed economy). In this paper, we take a broad perspective on public policy responses to self-control problems by showing how these responses vary (i) between closed and open economies, (ii) between small open and large open economies, and (iii) depending on whether or not both source based and residence based capital taxes can be used.
Keywords: Quasi-hyperbolic discounting, capital mobility, source based taxation, residence based taxation, labor income taxation.

JEL Classification: D61, D91, H21, H23.

*The authors would like to thank David Granlund and Magnus Wikström for helpful comments and suggestions. Research grants from The Bank of Sweden Tercentenary Foundation, The Swedish Council for Working Life and Social Research and The National Tax Board are also gratefully acknowledged.

1 Introduction

Much research effort has been put into studying savings behavior as well as the effects of tax policy on the incentives to save.[1] A major reason is, of course, that savings play a crucial role for economic growth and, therefore, ultimately also for future welfare. Concerns have also been raised about the level of savings, where a frequent argument is that the savings rates may be "too low" in many countries and, in particular, in the U.S., where the savings rates have been quite low for a long time (by historical comparison).[2] One argument emphasized in earlier research as to why individuals may save too little is that they suffer from bounded rationality in the sense of having "present-biased" preferences, i.e. a time-inconsistent preference for immediate gratification. A mechanism that generates this behavior is quasi-hyperbolic discounting, where the individual, at any time t, attaches a higher utility discount rate to tradeoffs between periods t and $t+1$ than to similar tradeoffs in the more distant future.[3,4]

The behavioral failure that quasi-hyperbolic discounting gives rise to is a self-control problem, where the preference for immediate gratification

[1] See Bernheim (2002) for a literature review.

[2] See, e.g., Guidolin and Jeunesse (2007) and Feldstein (2008).

[3] Experimental evidence pointing in this direction is presented by, e.g., Thaler (1981), Kirby and Marakovic (1997), Kirby (1997), Viscusi, Huber and Bell (2008) and Brown, Chua and Camerer (2009). See also Fredrick, Loewenstein and O'Donoghue (2002) for a review of empirical research on intertemporal choice, and Rubenstein (2003) for a critical view of the evidence for hyperbolic discounting.

[4] Bernheim, Skinner and Weinberg (2001) use data from the Panel Study of Income Dynamics and Consumer Expenditure Survey, and find that the conventional life-cycle model is unable to explain observed variation in retirement wealth in the U.S. They argue, instead, that their data is consistent with rules of thumb, mental accounting or hyperbolic discounting. A similar argument is presented by Mastrobouni and Weinberg (2009), who find (on the basis of data from the Continuing Survey of Food Intake) that retirees with little pension savings, whose income mainly comes from social security, consume much less the week before they receive the paycheck than the week after.

makes the individual's current self impose an externality on his/her future selves (sometimes referred to as "internality") which, in turn, provides an argument for policy intervention by a paternalistic government. A capital subsidy to correct the incentives to save was considered by Laibson (1996),[5] who assumed that the government aims at implementing a savings-target. This policy response is interpretable as being designed for a closed economy, since Laibson did not consider the possibility that capital is mobile between tax-jurisdictions. To our knowledge, there are no studies analyzing the corresponding policy problem under international capital mobility. Such an extension of the literature is potentially very important because if the consumers can invest their savings both at home and abroad, then domestic capital taxes/subsidies may no longer constitute perfect instruments for influencing the incentives to save faced by the domestic residents. The reason is that international capital mobility may imply restrictions on the domestic post-tax interest rate, which render capital taxes ineffective; or at least less effective than in a closed economy. This will be described in greater detail below. Therefore, the optimal policy response to quasi-hyperbolic discounting derived for a paternalistic government in a closed economy may actually be misleading if applied to an open economy. The present paper examines how a paternalistic government can use the income tax instruments available in an open economy framework to address the undersavings-problem caused by quasi-hyperbolic discounting, and the analysis is based on a general equilibrium model.

To further explain why capital mobility is important in this particular context, it is useful to distinguish between a small open economy whose government treats the world-market interest rate as exogenous, and a large open economy where the government recognizes that it may influence the

[5]Other literature on public policy responses to quasi-hyperbolic discounting includes sin taxes attached to unhealthy commodities (e.g., Gruber and Köszegi, 2004; O'Donoghue and Rabin, 2003, 2006), health capital subsidies (Aronsson and Thunström, 2008) and public investment (Aronsson and Granlund, 2011).

world-market interest rate through public policy, as well as between the source-principle and residence-principle for capital taxation. According to the former principle, capital income is taxed at source irrespective of whether it accrues to domestic or foreign residents, whereas the latter principle means that the government taxes the domestic residents irrespective of whether they earn their capital income at home or abroad. In a small open economy, a source based capital income tax would be completely ineffective as a means of influencing the incentives to save: a change in the tax rate just leads to an inflow or outflow of capital until the domestic post-tax interest rate returns to the equilibrium level given by the ("exogenous") foreign rate. Similarly, in a large open economy, neither the source based nor the residence based tax alone constitutes a perfect instrument for influencing the incentives to save, since the capital tax is also a strategic instrument for influencing the world-market interest rate. As such, to exercise perfect control over the savings behavior, both an unrestricted source based tax and an unrestricted residence based tax are needed; otherwise, the optimal tax policy may also feature adjustments of other broad-based taxes.

We take a broad perspective on optimal income taxation under quasi-hyperbolic discounting by (i) distinguishing between a closed economy and open economies with mobile capital, (ii) addressing the policy implications of time-consistent (sophisticated) versus time-inconsistent (naive) consumers, and (iii) focusing on the simultaneous use of two tax instruments that governments typically have at their disposal; labor and capital income taxes. Furthermore, since countries typically differ quite much in terms of resources and size, we examine the tax policy responses to quasi-hyperbolic discounting both in the context of small open economies (applicable to a number of European countries) and large open economies (such as the U.S.). To do so, we develop an overlapping generations (OLG) model with endogenous labor supply and savings, where each consumer lives for three periods (the minimum number of periods required to address the implications of quasi-hyperbolic discounting). The purpose is to analyze how a paternalistic government - which does not share the consumer-preference for immediate gratification - uses the capital and labor income taxes to correct for the behavioral failure that quasi-hyperbolic discounting gives rise to.

The income tax system is assumed to be nonlinear, which gives a reasonably realistic description of the tax instruments that many countries have at their disposal. This also means that the use of distortionary taxes is a consequence of optimization by the government and not due to the necessity to raise revenue per se. Regarding capital taxes, we consider both the source-principle and residence-principle: both these options are practically plausible, although not equivalent from the point of view of influencing the private incentives to save. This is also motivated by realism, simply because many real world tax systems feature elements of both source based and residence based taxation.

Earlier research shows that the behavioral implications of quasi-hyperbolic discounting depend on whether the consumers are naive or sophisticated.[6] At any time, a naive consumer erroneously expects the self-control problem to vanish in the future, meaning that the naive consumer has an incentive to revise the consumption plan as time passes, while a sophisticated consumer recognizes that the self-control problem will be present also in future periods and implements a plan that his/her future selves will follow (as in, e.g., Laibson, 1996, 1997). We consider both naivety and sophistication in what follows. To our knowledge, it is not clear whether agents in real world economies behave more in accordance with naivety than sophistication or vice versa.[7] As the incentives to save differ between naive and sophisticated

[6]See, e.g., O'Donoghue and Rabin (1999, 2001) and Diamond and Köszegi (2003).

[7]Hey and Lotito (2009) address dynamically inconsistent decisions by using experimental data, and find behavioral patterns consistent with both naivety and sophistication, even if naivety seems to be a more common type of behavior than sophistication. The review by DellaVigna (2009) also shows behavioral patterns consistent with both naivety and sophistication.

consumers, this distinction is also important from a policy perspective.

Our benchmark model presented in Section 2 refers to a large open economy, in which the consumers may either invest at home or abroad, and where the government can use any desired combination of source based and residence based capital income taxes as well as the labor income tax for purposes of correction, revenue collection and redistribution. Furthermore, the government recognizes (and incorporates into its decision problem) that it may affect the world-market interest rate through public policy. As this model is formulated, it also nests two interesting special cases: a closed economy (where the net export of capital is always equal to zero) and a small open economy (where the government treats the world-market interest rate as exogenous). Section 3 focuses on public policy and, in particular, how the government may use tax policy to influence the private incentives to save and, therefore, correct for the self-control problem. We show how the government may implement the first best by a combination of residence based and source based capital taxes: access to both instruments enables the government to target the incentives to save *and* the world-market interest rate. We also explain how the tax policy simplifies in the special cases of a small open economy and closed economy, respectively.

In reality, it may be difficult for national governments to implement resource allocations based on either principle for capital taxation. For instance, restrictions on the use of source based taxation may arise from attempts to limit the scope for tax competition; an issue discussed at present in the European Union. Furthermore, the residence-principle relies on an information sharing system where source-countries assist in the collection of revenue.[8] This suggests to us that it is useful to relax the assumption that the government can freely use both pricinples for capital taxation. In Section 4, therefore, we consider situations where the government uses the

[8]This information exchange problem has been addressed by Baccetta and Espinosa (1995) and Eggert and Kolmar (2002).

labor income tax in combination with either the residence based or source based capital income tax. We show, among other things, how the government may use the labor income tax as an indirect instrument to influence the savings behavior. Section 5 provides a summary and discussion, while proofs are presented in the Appendix.

2 The Model

In this section, we present an OLG-economy in which each consumer lives for three periods and is subject to a self-control problem generated by quasi-hyperbolic discounting. We also present the production sector of the economy as well as the decision problem faced by the government.

2.1 Consumers

We assume that each consumer works in the first and second period of life and becomes a pensioner in the third. As the number of consumers is not important for the qualitative results to be derived below, the size of each cohort will be normalized to one. This means that one new consumer enters the economic system in each time period and that the population is constant. The consumers have identical preferences for consumption, c, and leisure, z. The instantaneous utility faced by a consumer of age $i = 0, 1, 2$ (0 = young, 1 = middle-aged and 2 = old) in period t can be written as $u_{i,t} = u\left(c_{i,t}, z_{i,t}\right)$, which is increasing in each argument and strictly concave. We also add the (conventional) assumptions that c and z are normal goods, and that consumption and leisure are complements in the utility function in the sense that $\partial^2 u_{i,t}/\partial c_{i,t}\partial z_{i,t} \geq 0$. Since the available time in each period can be used either for work or leisure, or a combination of them, the consumer also faces a time constraint, $H = l_{i,t} + z_{i,t}$, where H is a fixed time endowment and $l_{i,t}$ is the hours of work.

Following the approach developed by Phelps and Pollak (1968), and later used by, e.g., Laibson (1997) and O'Donoghue and Rabin (2003, 2006), the intertemporal objective in period t faced by generation t (i.e. individuals born in period t) can be written as follows:

$$U_{0,t} = u_{0,t} + \beta \sum_{i=1}^{2} \Theta^i u_{i,t+i} \tag{1}$$

where Θ is a conventional (exponential) utility discount factor while the parameter $\beta \in (0, 1)$ reflects the preference for immediate gratification.

As we mentioned above, the consumer has the option to invest his/her savings at home or abroad. At any time t, r_t denotes the domestic before-tax interest rate, while R_t^* denotes the rate of return before residence based taxation that domestic consumers may attain by investing abroad.[9] Capital income is taxed according to a mixed system, which contains a source based and a residence based part with marginal tax rates θ_t^s and $\theta_{i,t}^r$, respectively. Note that the source based tax rate is common to all consumers. It would be very difficult (if not impossible) to differentiate the source based tax rate among consumers, since those faced by the higher rate would invest their savings abroad instead of at home. The net (after-tax) interest rate faced by domestic consumers, if investing one dollar at home at time t, can then be written as

$$r_{i,t}^n = \underbrace{\left(1 - \theta_{i,t}^r\right)(1 - \theta_t^s)}_{=(1-\theta_{i,t})} r_t \tag{2}$$

where $\theta_{i,t} = \theta_{i,t}^r + \theta_t^s - \theta_{i,t}^r \theta_t^s$ will be referred to the *total marginal capital income tax rate* faced by age-group i in period t. If investing abroad, on the other hand, the consumer obtains the net return $\left(1 - \theta_{i,t}^r\right) R_t^*$. We assume that capital is perfectly mobile between countries, in which case the capital

[9]To be more specific, if other countries only use residence based capital income taxes at time t, R_t^* may be thought of as the foreign gross rate of return; if other countries use source based capital income taxes, we can interpret R_t^* as the foreign rate of return net of the source based capital income tax.

market equilibrium must obey the following condition:[10]

$$(1-\theta_t^s)\,r_t = R_t^*. \tag{3}$$

To simplify the notation, we abstract from bequests and assume that the initial wealth faced by each consumer is zero. The consumer earns labor income when young and middle-aged, and capital income when middle-aged and old. The gross wage rate is allowed to correlate with the age of the worker, meaning that the young and middle-aged worker may face different gross wage rates. Let $w_{i,t}$ denote the gross wage rate facing age-group i in period t, and $s_{i,t}$ denote savings. The marginal net (after-tax) wage rate can then be written as $w_{i,t}^n = w_{i,t}(1-\tau_{i,t})$, where $\tau_{i,t}$ is the marginal labor income tax rate. Note that the marginal income tax rates (attached to both labor and capital) are allowed to vary over time and across age-groups. The tax system also contains lump-sum components, $T_{i,t}$ $(i = 0, 1, 2)$, which may also vary over time and across age-groups. This flexible tax system provides a simple framework for studying the corrective and strategic use of taxation, as it implies that non-zero marginal income tax rates attached to labor and/or capital follow from optimization by the government and are not due to arbitrary restrictions on the tax instruments or the necessity to raise revenue per se.[11] The intertemporal budget constraint faced by an

[10]The capital market equilibrium condition can also be written as

$$(1-\theta_{i,t}^r)(1-\theta_t^s)r_t = R_t^*(1-\theta_{i,t}^r)$$

for $i = 1, 2$. Equation (3) then follows by eliminating $(1-\theta_{i,t}^r)$ on both sides. An alternative specification would be

$$(1-\theta_{i,t}^r-\theta_t^s)r_t = R_t^*(1-\theta_{i,t}^s)$$

for $i = 1, 2$. This formulation is clearly more restrictive than equation (3), as it would imply $\theta_{1,t}^r = \theta_{2,t}^r$.

[11]Similar tax systems have also been examined in other literature on optimal income taxation in dynamic economies; see, e.g., Brett (1997), Pirtillä and Tuomala (2001) and Aronsson and Johansson-Stenman (2010).

individual of generation t can then be written as

$$c_{0,t} = w_{0,t}^n l_{0,t} - T_{0,t} - s_{0,t} \tag{4}$$

$$c_{1,t+1} = w_{1,t+1}^n l_{1,t+1} + \left(1 + r_{1,t+1}^n\right) s_{0,t} - T_{1,t+1} - s_{1,t+1} \tag{5}$$

$$c_{2,t+2} = \left(1 + r_{2,t+2}^n\right) s_{1,t+1} - T_{2,t+2}. \tag{6}$$

Now, recall from the discussion in the introduction that although the consumers may suffer from self-control problems, it is not clear whether we should expect them to act in accordance with naivety or sophistication. We will, therefore, consider both these possibilities in the analysis below. Since the individual first order conditions as well as the policy rules for optimal taxation under naivety are interpretable as technical special cases of the corresponding first order conditions and policy rules, respectively, that follow under sophistication, we derive the results under the assumption that agents are sophisticated and then comment upon how the results are modified if agents instead were naive.

To arrive at a time-consistent solution for the sophisticated agents, their decision problems will be solved sequentially. We begin by briefly examining the labor supply and savings behavior of the middle-aged consumer, and then analyze the labor supply and savings behavior of the young sophisticated consumer who acts as a strategic leader vis-a-vis his/her middle-aged self. This strategic leadership motive is absent for naive consumers, who (erroneously) expect not to be facing the self-control problem in the future.

When middle-aged, the consumer chooses $l_{1,t+1}$ and $s_{1,t+1}$ to maximize $U_{1,t+1} = u_{1,t+1} + \beta\Theta u_{2,t+2}$ subject to equations (5) and (6), where the level of savings chosen when young, $s_{0,t}$, is treated as fixed. The first order conditions are

$$w_{1,t+1}^n \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \frac{\partial u_{1,t+1}}{\partial z_{1,t+1}} = 0 \tag{7}$$

$$\beta\Theta \left(1 + r_{2,t+2}^n\right) \frac{\partial u_{2,t+2}}{\partial c_{2,t+2}} - \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} = 0. \tag{8}$$

Note that quasi-hyperbolic discounting does not modify the atemporal trade-off between consumption and leisure, which means that the labor supply condition in equation (7) takes the same form as in a standard model, whereas $\beta < 1$ in equation (8) means that the consumer saves less than he/she would have done without any self-control problem, ceteris paribus. Since there is no incentive for the sophisticated middle-aged consumer to constrain his/her old self in our model (the old consumer makes no intertemporal choice), equations (7) and (8) take the same form independently of whether the consumer is naive or sophisticated. We can use equations (7) and (8) to derive the labor supply and savings functions

$$l_{1,t+1} = l_1\left(w^n_{1,t+1}, r^n_{1,t+1}, r^n_{2,t+2}, T_{1,t+1}, T_{2,t+2}, s_{0,t}\right) \quad (9)$$

$$s_{1,t+1} = s_1\left(w^n_{1,t+1}, r^n_{1,t+1}, r^n_{2,t+2}, T_{1,t+1}, T_{2,t+2}, s_{0,t}\right). \quad (10)$$

The sequential decision process means that $l_{1,t+1}$ and $s_{1,t+1}$ will be functions of $s_{0,t}$. As a consequence, equations (9) and (10) can be viewed as reaction functions via which the young consumer may influence the behavior of his/her middle-aged self. As an increase in $s_{0,t}$ typically means that more resources become available for consumption and saving when middle-aged, all results below will be interpreted under the assumption that $\partial s_{1,t+1}/\partial s_{0,t} > 0$.[12]

The young sophisticated consumer maximizes the objective defined in equation (1) subject to the life-time budget constraint presented in equations (4) - (6) as well as subject to the reaction functions defined by equations (9) - (10). The first order conditions for $l_{0,t}$ and $s_{0,t}$ can be written as

$$0 = w^n_{0,t}\frac{\partial u_{0,t}}{\partial c_{0,t}} - \frac{\partial u_{0,t}}{\partial z_{0,t}} \quad (11)$$

$$0 = \beta\Theta\left(1 + r^n_{1,t+1}\right)\frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \frac{\partial u_{0,t}}{\partial c_{0,t}} + \frac{\partial U_{0,t}}{\partial s_{1,t+1}}\frac{\partial s_{1,t+1}}{\partial s_{0,t}} \quad (12)$$

[12] This condition always applies except in the somewhat unlikely situation where an increase in $s_{0,t}$ leads to such a large reduction in $l_{1,t+1}$ that the resources available for consumption and savings when middle-aged actually decrease (recall that leisure is assumed to be a normal good).

in which

$$\frac{\partial U_{0,t}}{\partial s_{1,t+1}} = \frac{(1-\beta)}{\beta}\Theta\frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} > 0.$$

The final term on the right hand side of equation (12) captures how the savings by the young consumer, $s_{0,t}$, affects the savings by his/her middle-aged self. With $\partial s_{1,t+1}/\partial s_{0,t} > 0$, this effect constitutes an incentive for the young sophisticated consumer to save more than he/she would otherwise have done to counteract the tendency to undersave when middle-aged (which the sophisticated young consumer is fully aware of).[13] This effect would be absent for a naive consumer, who erroneously expects to have time-consistent preferences in the future, meaning that the first order condition for savings faced by the young naive consumer takes the same general form as equation (8) above. Equations (11) and (12) implicitly define the following labor supply and savings functions:

$$l_{0,t} = l_0\left(w^n_{0,t}, w^n_{1,t+1}, r^n_{1,t+1}, r^n_{2,t+2}, T_{0,t}, T_{1,t+1}, T_{2,t+2}\right) \quad (13)$$

$$s_{0,t} = s_0\left(w^n_{0,t}, w^n_{1,t+1}, r^n_{1,t+1}, r^n_{2,t+2}, T_{0,t}, T_{1,t+1}, T_{2,t+2}\right). \quad (14)$$

2.2 Production

Output is produced by identical competitive firms, the number of which is normalized to one. We assume that young and middle-aged workers are imperfect substitutes in the production and define the "effective labor" supplied in period t as follows; $L_t = l_{0,t} + al_{1,t}$, where a is a positive constant. If middle-aged workers are more (less) productive than young workers, then $a > 1$ (< 1). The production function is given by $F(L_t, K_t)$, which is increasing and strictly concave in its respective argument as well

[13] If we were to allow agents to be partially naive in the sense of O'Donoghue and Rabin (2001), the first order condition for savings would still take the form of equation (12); although based on an underestimation of the $(1-\beta)$ component of the final term on the right hand side (since the partially naive consumer understimates the magnitude of the future self-control problem).

as characterized by constant returns to scale. By using the normalizations $f(k_t) = F(L_t, K_t)/L_t$ and $k_t = K_t/L_t$, we obtain the standard first order conditions

$$r_t = f_k(k_t) \quad (15)$$

$$w_{0,t} = f(k_t) - r_t k_t \quad (16)$$

together with $w_{1,t} = a\ w_{0,t}$.

2.3 Equilibrium

The aggregate savings in period $t-1$, $s_{0,t-1} + s_{1,t-1}$, earns interest in period t. Each consumer may either invest his/her savings at home (in the form of domestic capital) or abroad (in the form of foreign capital), or may use a combination of these two options. Let Q_t denote the net export of capital in period t. It will then follow from the national accounts that[14]

$$s_{0,t-1} + s_{1,t-1} = K_t + Q_t. \quad (17)$$

If our home country is a large open economy, its net export of capital will influence the foreign rate of return on capital. Therefore, we can write the foreign rate of return as a function of the net export of capital from "our" country, $R_t^* = R(Q_t)$, and we assume that $dR_t^*/dQ_t < 0$. The latter is interpretable to mean that an increase in Q will increase the foreign capital stock and, therefore, reduce the foreign interest rate, ceteris paribus. The capital market equilibrium condition given by equation (3) can then be specified as follows:

$$(1 - \theta_t^s) r_t = R(Q_t). \quad (18)$$

Now, by using the identities $k_t = K_t/L_t$ and $L_t = l_{0,t} + a\ l_{1,t}$ in combination with equations (15)-(18), we can derive the following equations for the

[14]Recall that the size of each cohort has been normalized to one.

domestic factor prices and net export of capital:

$$r_t = r\left(\theta_t^s, l_{0,t}, l_{1,t}, s_{0,t-1}, s_{1,t-1}\right) \tag{19}$$

$$w_{0,t} = w_0\left(\theta_t^s, l_{0,t}, l_{1,t}, s_{0,t-1}, s_{1,t-1}\right) \tag{20}$$

$$Q_t = Q\left(\theta_t^s, l_{0,t}, l_{1,t}, s_{0,t-1}, s_{1,t-1}\right). \tag{21}$$

In equations (19)-(21), the variables $l_{0,t}$, $l_{1,t}$, $s_{0,t-1}$ and $s_{1,t-1}$ are, in turn, determined by equations (9), (10), (13) and (14). Therefore, equations (19)-(21) provide the channels through which public policy affects the domestic factor prices and net export of capital.

For further use, note also that our model nests two interesting special cases. First, if R_t^* is treated as exogenous for all t by the national government (instead of as a function of the net export of capital), our model describes a small open economy. Second, if $Q_t \equiv 0$ for all t, we have a closed economy. Both these special cases will be analyzed below along with the results of the more general model.

2.4 The Government

The government acts as first mover vis-a-vis the private sector (by recognizing how private agents respond to policy) and aims to correct for the self-control problem described above as well as raise revenue and achieve redistribution. Following earlier literature on optimal paternalism,[15] we assume that $\beta = 1$ from the point of view of the (paternalistic) government, meaning that the government wants to impose the following present value utility function on generation t: $\tilde{U}_{0,t} = u_{0,t} + \sum_{i=1}^{2} \Theta^i u_{i,t+i}$. Note that this function differs from the actual utility function faced by generation t in equation (1) above due to the consumer preference for immediate gratification. The social welfare function can then be written as

$$W = \sum_t \Theta^t \tilde{U}_{0,t}. \tag{22}$$

[15] See, e.g., O'Donoghue and Rabin (2003, 2006), Aronsson and Thunström (2008) and Aronsson and Granlund (2011).

To simplify the analysis, we ignore public debt.[16] By using equations (2) and (17), and that the marginal unit taxes of labor and capital facing the consumer of age i are given by $w_{i,t} - w_{i,t}^n = \tau_{i,t} w_{i,t}$ and $r_t - r_{i,t}^n = \theta_{i,t} r_t$, the budget constraint facing the government can be written as

$$\bar{g}_t = \sum_{i=0}^{2} T_{i,t} + \sum_{i=0}^{1} \left(w_{i,t} - w_{i,t}^n\right) l_{i,t} + \sum_{i=0}^{1} \left(r_t - r_{i,t}^n\right) s_{i,t-1} - \theta_t^s r_t Q_t \qquad (23)$$

for all t, where r_t, $w_{0,t}$ and Q_t are given by equations (19), (20) and (21) above, and $w_{1,t} = a w_{0,t}$. The variable $\bar{g}_t \geq 0$ represents a fixed revenue requirement. The final term on the right hand side of equation (23) follows because the government can only levy source based taxes on the domestic capital stock, i.e. $r_t K_t = r_t \sum_i s_{t-1}^i - r_t Q_t$ is the tax base for the source based tax.

The public decision problem is to choose $w_{0,t}^n$, $w_{1,t}^n$, $r_{1,t}^n$, $r_{2,t}^n$, $T_{0,t}$, $T_{1,t}$, $T_{2,t}$ and θ_t^s for all t to maximize the social welfare function in equation (22) subject to the budget constraint presented in equation (23).[17] The whole time sequence of each policy instrument is decided upon, and announced, at time zero.[18] The Lagrangean associated with this policy problem and the corresponding first order conditions are presented in the Appendix. Here, we concentrate on the implications of these first order conditions for public policy.

[16]Although this assumption limits the scope for redistribution over time, it is not important for the qualitative results derived below with respect to how the government uses marginal income taxation to correct for the self-control problem.

[17]Note that marginal tax rates and marginal factor prices (net of tax) are equivalent policy instruments.

[18]Note that the problem of time-inconsistent public policy does not arise here. In a more general model with information asymmetries between the government and the private sector, on the other hand, a policy based on commitment may no longer be time-consistent; see Aronsson and Sjögren (2011) for a study of time-consistent optimal taxation without commitment under quasi-hyperbolic discounting in a closed economy.

3 Tax Policy in the Benchmark Model

In this section, we examine the optimal tax policy in our benchmark model for a large open economy, whose government recognizes that it may influence the world-market interest rate. We will then turn to the special cases mentioned above, i.e. the small open economy and the closed economy, respectively.

The tax policy used by the paternalistic government in a large open economy with the full set of instruments described above is summarized as follows:

Proposition 1. *In the benchmark model for a large open economy, the optimal tax policy can be characterized as follows for all t:*

(i) *Marginal labor income tax rates:*

$\tau_{0,t} = \tau_{1,t} = 0$;

(ii) *Source based capital income tax rate:*

$\theta_t^s < 0$ *if* $Q_t > 0$, $\theta_t^s > 0$ *if* $Q_t < 0$, *and* $\theta_t^s = 0$ *if* $Q_t = 0$;

(iii) *Total marginal capital income tax rates:*

$\theta_{1,t} < 0$ *and* $\theta_{2,t} < 0$, *where* $|\theta_{1,t}| < |\theta_{2,t}|$ *if the consumers are sophisticated, while* $\theta_{1,t} = \theta_{2,t}$ *if they are naive;*

(iv) *Residence based capital income tax rates:*

$\theta_{1,t}^r < 0$ *and* $\theta_{2,t}^r < 0$ *if* $Q_t \leq 0$, *while* $\theta_{1,t}^r \gtreqless 0$ *and* $\theta_{2,t}^r \gtreqless 0$ *if* $Q_t > 0$.

Proof: see the Appendix.

Part (i) of Proposition 1 reflects the principle of targeting. The intuition is that for the large open economy examined here, the two capital tax instruments can be used simultaneously to excercise control over the private incentives to save *and* excercise market power to influence the world-market interest rate. As a consequence, there is no reason to use the labor income tax as a supplemental instrument for correction.

Part (ii) describes how the government uses the source based tax to affect the foreign rate of return. If the country is a net exporter of capital at time t, so $Q_t > 0$, an increase in the foreign rate of return will lead to a higher domestic national income. This can be accomplished via a source based capital subsidy implemented at home: this reduces the net export of capital which, in turn, contributes to increase the foreign rate of return. If the country is a net importer of capital, the argument for a positive source based tax is analogous. These results follow immediately from the tax formula for θ_t^s, which is given by

$$\frac{\theta_t^s}{1-\theta_t^s} = \frac{Q_t}{R_t^*}\frac{dR_t^*}{dQ_t}. \tag{24}$$

Equation (24) is a variant of the standard inverse elasticity rule for optimal taxation.

The total marginal capital income tax rates described in part (iii) reflect correction for the self-control problem (that would otherwise manifest itself in terms of too little saving). Note that the distinction between naivety and sophistication is important here. If consumers are naive, $\theta_{1,t}$ and $\theta_{2,t}$ satisfy

$$\theta_{1,t} = \theta_{2,t} = \left(\frac{\beta-1}{\beta}\right)\left(\frac{1+r_t}{r_t}\right) < 0, \tag{25}$$

whereas the corresponding policy with sophisticated consumers can be summarized as

$$\theta_{1,t} = \left(\frac{\beta-1}{\beta}\right)\left(\frac{1+r_t}{r_t}\right) - \left(\frac{\beta-1}{\beta}\right)\frac{1}{r_t}\frac{\partial s_{1,t}}{\partial s_{0,t-1}} < 0 \tag{26}$$

$$\theta_{2,t} = \left(\frac{\beta-1}{\beta}\right)\left(\frac{1+r_t}{r_t}\right) < 0. \tag{27}$$

The difference between equations (25) and (26) follows because the young sophisticated consumer acts strategically, i.e. chooses a higher level of $s_{0,t}$ to stimulate increased savings by his/her middle-aged self. Therefore, the second term on the right hand side of equation (26) will counteract - although it does not fully offset - the tendency to undersave caused by the preference for immediate gratification.[19] This "strategic leadership effect" is absent for naive consumers, which explains why the government needs to subsidize the savings by young consumers at a higher rate under naivety than under sophistication, ceteris paribus. The subsidy rate attached to the savings by the middle-aged does not depend on whether the consumers are naive or sophisticated in our model, as the middle-aged consumer has no incentive to act strategically vis-a-vis his/her old self.

Finally, since the two capital tax instruments are linked via the total marginal capital income tax rate, part (iv) is interpretable to mean that the residence based tax serves are a "residual" such as to make the source based tax compatible with the total marginal capital income tax rate (which is the tax measure of relevance for private saving). By using the expression for the total marginal capital income tax rate presented in Section 2, we can solve for the residence based tax, i.e. $\theta^r_{i,t} = (\theta_{i,t} - \theta^s_t)/(1 - \theta^s_t)$ for $i = 1, 2$. Since $\theta_{i,t} < 0$, we have $\theta^r_{i,t} < 0$ if $0 < \theta^s_t < 1$ (applicable to a net capital importer), while $\theta^r_{i,t}$ can be either positive or negative if $\theta^s_t < 0$ (applicable to a net capital exporter). If the self-control problem were absent, such that $\beta = 1$, we can immediately see that $\theta_{i,t}$ should be equal to zero, in which case the residence based tax would be used to fully offset the savings tax-wedge created by the source based tax.

3.1 Two Useful Special Cases of the Benchmark Model

The benchmark model set out in Section 2, and examined above in this section, refers to a large open economy. As we mentioned before, this benchmark model nests two interesting special cases: first, if the foreign rate of return is treated as fixed by the national government (instead of as a function of the net export of capital), the model reduces to that of a small open economy and, second, if the net export of capital is equal to zero in each period, our model corresponds to a closed economy. These two special cases

[19] Comparative statics based on equations (9) and (10) imply $\partial s_{1,t+1}/\partial s_{0,t} < (1 + r_t)$.

share common policy-elements, which are summarized in Proposition 2:

Proposition 2. *In a small open economy where R_t^* is treated at exogenous by the domestic government for all t, and in a closed economy where $Q_t \equiv 0$ for all t, the optimal tax policy satisfies conditions (i) and (iii) in Proposition 1.*

The proof of Proposition 2 follows immediately from the proof of Proposition 1 and is, therefore, omitted. The intuition is straight forward. If applied to a small open economy, the inverse elasticity rule in equation (24) means that the government will not use the source based tax instrument ($\theta_t^s = 0$, $\theta_{1,t}^r = \theta_{1,t}$ and $\theta_{2,t}^r = \theta_{2,t}$ for all t). By analogy, the source based tax is a redundant instrument in the closed economy, simply because the world-market interest rate is no longer an interesting target variable for the domestic government. In either case, therefore, the marginal capital income tax structure is characterized solely by equation (25) if the consumers are naive, and solely by equations (26) and (27) if they are sophisticated, which constitute perfect instruments for internalizing the internal externalities generated by quasi-hyperbolic discounting. In other words, a simple savings policy enables the government to implement the first best resource allocation, which also explains why the marginal labor income tax rates remain equal to zero.

4 Restricted Capital Taxation

The above analysis of tax policy in (large and small) open economies was carried out under the assumption that the government may implement both source based and residence based capital taxes. As we mentioned in the introduction, it may in reality be difficult for national governments to fully implement any of these two instruments, due to international agreements that serve to limit the scope for tax competition, and/or costs and uncertainty associated with information sharing. It is, therefore, useful to consider each principle for capital taxation separately, and analyze how the government in each such case uses the mix of labor and capital taxation to correct for the self-control problem generated by quasi-hyperbolic discounting.

4.1 Residence Based Taxation

If the residence based tax constitutes the only available instrument for capital taxation, it follows that all capital income faced by domestic residents will be taxed at home independently of source. In this case, where $\theta_t^s \equiv 0$ for all t by assumption, the capital market equilibrium condition given by equation (3) simplifies to read $r_t = R_t^*$, which means that the gross domestic interest rate is equal to the rate of return that consumers attain if investing abroad.

In a small open economy, whose government treats R_t^* for all t as exogenous, it follows immediately from Proposition 2 that the optimal mix of labor income taxation and residence based capital income taxation satisfies conditions (i) and (iii) of Proposition 1. The intuition is, of course, that even if it were optional to use source based taxation (in addition to the other tax instruments), the small open economy would not implement such a tax. Therefore, the additional restriction that $\theta_t^s \equiv 0$ for all t introduced here is of no practical relevance.

Instead, let us focus attention on the large open economy. As we mentioned above, the government of such an economy treats R_t^* as a function of the domestic net export of capital, Q_t, which is, in turn, determined by domestic tax policy according to equation (21). A restriction on the use of source based capital taxation may in this case have important implications for how the government uses its other instruments. By solving the public decision-problem set out in subsection 2.4 under the additional restriction that $\theta_t^s \equiv 0$ for all t, we can derive the following result:

Proposition 3. *In a large open economy, whose government does not have access to the source based capital tax, i.e.* $\theta_t^s \equiv 0$ *for all* t*, the optimal tax policy can be characterized as follows for all* t*:*

(i) *Marginal labor income tax rates:*
$\tau_{0,t} < 0$ *and* $\tau_{1,t} < 0$ *if* $Q_t > 0$*,* $\tau_{0,t} > 0$ *and* $\tau_{1,t} > 0$ *if* $Q_t < 0$*, and* $\tau_{0,t} = \tau_{1,t} = 0$ *if* $Q_t = 0$*;*

(ii) *Residence based marginal capital income tax rates:*
$\theta_{1,t}^r < 0$ *and* $\theta_{2,t}^r < 0$ *if* $Q_t \leq 0$*, and* $\theta_{1,t}^r \lesseqgtr 0$ *and* $\theta_{2,t}^r \gtreqless 0$ *if* $Q_t > 0$*.*

Proof: see the Appendix.

The policy presented in Proposition 3 differs from that of Proposition 1, due to that the source based tax instrument is no longer available for influencing the foreign rate of return. Instead, both the labor income tax and the residence based capital income tax will in this case partly serve as (imperfect) instruments for exercising this market power. In the Appendix, we derive the following expression for the marginal labor income tax rate faced by age-group i in period t:

$$\tau_{i,t} = -\frac{Q_t}{w_{i,t}} \frac{dR_t^*}{dQ_t} \frac{\partial Q_t}{\partial l_{i,t}}. \tag{28}$$

Note that $\partial Q_t / \partial l_{i,t} < 0$ due to complementarity between labor and domestic capital: increased use of labor in the domestic production will, therefore, contribute to reduce the net export of capital. A decrease in the net capital export leads to an increase in the rate of return that domestic consumers obtain if investing abroad, which is desirable if the country is a next exporter of capital. This decrease in the net capital export can be accomplished by subsidizing labor. The argument for a positive marginal labor income tax rate in case the country is a net importer of capital is analogous.

Turning to capital income taxation, note that a change in the level of savings by any age-group affects the net export of capital and, therefore, the foreign interest rate. This effect is captured by introducing the following variable:

$$\zeta_{i,t} = -\frac{Q_t}{\beta r_t}\frac{dR_t^*}{dQ_t}\frac{\partial Q_t}{\partial s_{i,t-1}} \text{ for } i = 0, 1.$$

One can show that equation (21) implies $\partial Q_t/\partial s_{i,t-1} > 0$, while $dR_t^*/dQ_t < 0$ by the assumptions made earlier. The residence based marginal capital income tax rates (which are equal to the total marginal capital income tax rates) can then be characterized as follows if the consumers are sophisticated:

$$\theta_{1,t}^r = \theta_{1,t} = \left(\frac{\beta-1}{\beta}\right)\left(\frac{1+r_t}{r_t}\right) - \left(\frac{\beta-1}{\beta}\right)\frac{1}{r_t}\frac{\partial s_{1,t}}{\partial s_{0,t-1}} + \zeta_{0,t} \quad (29)$$

$$\theta_{2,t}^r = \theta_{2,t} = \left(\frac{\beta-1}{\beta}\right)\left(\frac{1+r_t}{r_t}\right) + \zeta_{1,t}. \quad (30)$$

With naive consumers, the only modification would be that the second term on the right hand side of equation (29) vanishes. Therefore, and by analogy to the results derived in Section 3, pure correction for the self-control problem would necessitate that the savings by young consumers be subsidized at a higher rate under naivety than under sophistication.

The new aspect in equations (29) and (30) is that the total marginal capital income tax rates (the rates of relevance when deciding upon savings) no longer only reflect policies to correct for the self-control problem, as they did in Section 3; instead, the final term on the right hand side is due to that the government also uses the residence based tax instrument to exercise market power in the international capital market (due to that the source based tax instrument is absent here). Since part of a given increase in the level of savings may be invested abroad, it follows that increased savings by domestic consumers contributes to reduce the foreign rate of return, ceteris paribus. This is desirable for a net importer of capital and undesirable for a net exporter. As a consequence, $\zeta_{i,t} < 0$ (> 0) for a net capital importer (expoter), which explains why $\theta_{1,t}$ and $\theta_{2,t}$ are both negative if the country is a net capital importer, and ambiguous in sign if it is a net capital exporter.

4.2 Source Based Taxation

The residence based capital income tax constitutes a direct instrument for influencing the savings by domestic residents. Access to residence based taxation, therefore, means that the government uses this (and no other) instrument to correct for the self-control problem, which would otherwise manifest itself in terms of too little saving, although it may also use the residence based tax for other purposes (as we saw in equations (29) and (30) in the previous subsection). However, without the residence based tax, there is no longer a direct instrument by which the government can target the incentives to save, in which case the labor income tax and - if the economy is large in the sense described above - the source based capital income tax might be used as indirect instruments to correct for the self-control problem.[20]

Small Open Economy

Consider first the tax policy of a small open economy, whose government treats the foreign rate of return, R_t^*, in equation (3) as exogenous for all t. If the government does not have access to the residence based capital income tax, how should the labor income tax be used in response to the preference for immediate gratification? Basic intuition suggests that a marginal labor subsidy might accomplish this task; such a subsidy typically leads to increased hours of work and income which, in turn, leads to increased savings. We start by showing that this argument is correct in a simplified version of the model, irrespective of whether the consumers are naive or sophisticated. The simplified version of the model means that the labor supply is fixed for

[20] A possible argument against this approach is that other means of influencing the the incentives to save, such as information campaigns, may be more useful that the blunt instruments considered here. However, information campaigns are less likely to be successful in reaching their intended effects if the intertemporal choices are governed by a preference for immediate gratification. It is, therefore, of clear value to understand how the tax system ought to be modified in response self-control problems, even if direct instruments for targeting savings behavior are absent.

the middle-aged generation although flexible for the young, allowing us to avoid intertemporal labor supply responses to changes in $\tau_{0,t}$. We will then return to the general model where also $l_{1,t+1}$ is flexible.

Therefore, by solving the public decision problem in subsection 2.4 subject to $\theta^r_{i,t} \equiv 0$ for $i = 1, 2$ and all t, as well as subject to the additional restrictions that R^*_t and $l_{1,t+1}$ are exogenous for all t, we have derived the following result:

Proposition 4. *Suppose that the residence based tax instrument is not available, so $\theta^r_{i,t} \equiv 0$ for $i = 1, 2$ and all t. If the labor supply is fixed for the middle-aged, then the optimal tax policy in a small open economy satisfies (i) $\tau_{0,t} < 0$ for all t, and (ii) $\theta^s_t = 0$ for all t.*

Proof: see the Appendix.

The intuition behind part (ii) is the same as before. To interpret part (i), let us introduce the following compensated labor supply and savings responses to an increase in the marginal wage rate:

$$\frac{\partial \tilde{l}_{0,t}}{\partial w^n_{0,t}} = \frac{\partial l_{0,t}}{\partial w^n_{0,t}} + l_{0,t}\frac{\partial l_{0,t}}{\partial T_{0,t}} > 0 \tag{31}$$

$$\frac{\partial \tilde{s}_{0,t}}{\partial w^n_{0,t}} = \frac{\partial s_{0,t}}{\partial w^n_{0,t}} + l_{0,t}\frac{\partial s_{0,t}}{\partial T_{0,t}} > 0, \tag{32}$$

where the right hand side of equation (32) is positive due to complementarity between consumption and leisure in the utility function. Then, by using $\gamma_t > 0$ to denote the Lagrange multiplier associated with the government's budget constraint (i.e. the marginal cost of public funds measured in terms of utility) as well as the short notation

$$\alpha_{0,t} = \frac{\partial \tilde{s}_{0,t}/\partial w^n_{0,t}}{\partial \tilde{l}_{0,t}/\partial w^n_{0,t}} > 0$$

one can show that the marginal labor income tax rate implemented for the young naive consumer in period t is given by

$$\tau_{0,t} = \frac{\alpha_{0,t}}{\gamma_t w_{0,t}} \left(\frac{\beta - 1}{\beta} \right) \left(\frac{\partial u_{0,t}}{\partial c_{0,t}} + \Theta \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \right) < 0. \tag{33}$$

With sophisticated consumers, the corresponding formula becomes

$$\tau_{0,t} = \frac{\alpha_{0,t}}{\gamma_t w_{0,t}} \left(\frac{\beta - 1}{\beta} \right) \frac{\partial u_{0,t}}{\partial c_{0,t}} < 0. \tag{34}$$

Equation (33) contains an additional negative term by comparison with equation (34), which suggest that the subsidy may be larger with naive than sophisticated consumers. The intuition is, of course, that sophisticated consumers partly internalize the behavioral failure themselves.[21] However, notice that the magnitudes of the labor supply and savings responses following this subsidy, as reflected in the variable $\alpha_{0,t}$, may also depend on whether the consumers are naive or sophisticated, which renders the comparison inconclusive without further assumptions.

The reason as to why the subsidy in equation (33) and (34), respectively, reflects compensated - instead of uncompensated - labor supply and savings responses to an increase in the marginal wage rate is that this subsidy follows from a simultaneous, and optimal, choice of both $\tau_{0,t}$ and $T_{0,t}$. For given utility levels faced by the middle-aged and old in period t, the only variables through which the government may influence the welfare faced by the young consumer, and at the same time retain public sector budget balance, are $\tau_{0,t}$ and $T_{0,t}$. As a consequence, the policy response to quasi-hyperbolic discounting in Proposition 4 is interpretable as a self-financed reallocation of the tax schedule, whereby labor is subsidized and the subsidy financed through a higher lump-sum component (or intercept) of the tax function. Taking this interpretation beyond the present model, the reader may think of a policy where decreased marginal labor income taxation is combined with a less generous limit for tax deductions, such that the net tax revenue paid by the young consumers is held constant.

[21]This is analogous to the result derived in Section 3 that the government ought to subsidize savings at a higher rate if the consumers are naive than if they are sophisticated.

Returning to the general model in which both $l_{0,t}$ and $l_{1,t+1}$ are flexible, the subsidy result presented in Proposition 4 does no longer necessarily apply. The reason is that public policy has intertemporal consequences, and a labor tax/subsidy when middle-aged, if expected when young, may influence the hours of work and savings behavior both when young and when middle-aged. We exemplify by considering the marginal labor income tax rates implemented for the sophisticated generation t, although the qualitative results also apply to naive consumers. We show in the Appendix that the labor income tax structure can be characterized as

$$\tau_{0,t} = \frac{\beta-1}{\beta}\left[a_{0,t}\frac{\partial u_{0,t}}{\partial c_{0,t}}\frac{\partial \tilde{s}_{0,t}}{\partial w^n_{0,t}} - b_{0,t}\frac{\partial u_{1,t+1}\Theta}{\partial c_{1,t+1}}\frac{\partial l_{1,t+1}}{\partial s_{0,t}}\frac{\partial \tilde{s}_{0,t}}{\partial w^n_{0,t}}\right] \quad (35)$$

$$\tau_{1,t+1} = \frac{\beta-1}{\beta}a_{1,t+1}\left\{\frac{\partial u_{1,t+1}\Theta}{\partial c_{1,t+1}}\frac{\partial \tilde{s}_{1,t+1}}{\partial w^n_{1,t+1}} + \frac{\partial u_{0,t}}{\partial c_{0,t}}\frac{\partial \tilde{s}_{0,t}}{\partial w^n_{1,t+1}}\right\} - \frac{\beta-1}{\beta}b_{1,t+1}\frac{\partial u_{0,t}}{\partial c_{0,t}}\frac{\partial \tilde{l}_{0,t}}{\partial w^n_{1,t+1}}, \quad (36)$$

in which $a_{0,t} = [\partial \tilde{l}_{1,t+1}/\partial w^n_{1,t+1}]/\varphi_{0,t}$, $b_{0,t} = [\partial \tilde{s}_{1,t+1}/\partial w^n_{1,t+1}]/\varphi_{0,t}$, $a_{1,t+1} = [\partial \tilde{l}_{0,t}/\partial w^n_{0,t}]/\varphi_{1,t+1}]$ and $b_{1,t+1} = [\partial \tilde{s}_{0,t}/\partial w^n_{0,t}]/\varphi_{1,t+1}$, while (for $i = 0, 1$)

$$\varphi_{i,t+i} = \gamma_{t+i}w^n_{i,t+i}\left[\frac{\partial \tilde{l}_{0,t}}{\partial w^n_{0,t}}\left(\frac{\partial \tilde{l}_{1,t+1}}{\partial w^n_{1,t+1}} + \frac{\partial l_{1,t+1}}{\partial s_{0,t}}\frac{\partial \tilde{s}_{0,t}}{\partial w^n_{1,t+1}}\right) - \frac{\partial \tilde{l}_{0,t}}{\partial w^n_{1,t+1}}\frac{\partial l_{1,t+1}}{\partial s_{0,t}}\frac{\partial \tilde{s}_{0,t}}{\partial w^n_{0,t}}\right].$$

In general, neither equation (35) nor equation (36) can be signed unambiguously, since the intertemporal, compensated changes in $l_{0,t}$ and $s_{0,t}$ following a labor subsidy to the consumer's middle-aged self, i.e.

$$\frac{\partial \tilde{l}_{0,t}}{\partial w^n_{1,t+1}} = \frac{\partial l_{0,t}}{\partial w^n_{1,t+1}} + l_{1,t+1}\frac{\partial l_{0,t}}{\partial T_{1,t+1}}$$

$$\frac{\partial \tilde{s}_{0,t}}{\partial w^n_{1,t+1}} = \frac{\partial s_{0,t}}{\partial w^n_{1,t+1}} + l_{1,t+1}\frac{\partial s_{0,t}}{\partial T_{1,t+1}}$$

can be either positive or negative. The changes in $\tilde{l}_{0,t}$ and $\tilde{s}_{0,t}$ captured by these compensated derivatives have direct effects on $\tau_{1,t+1}$ in equation (36), as well as indirect effects on both $\tau_{0,t}$ and $\tau_{1,t+1}$ through the variable $\varphi_{i,t+i}$. However, if $\partial \tilde{l}_{0,t}/\partial w^n_{1,t+1}$ and $\partial \tilde{s}_{0,t}/\partial w^n_{1,t+1}$ are small in absolute value by comparison with their atemporal counterparts, i.e. $\partial \tilde{l}_{0,t}/\partial w_{0,t}^{n}$ and $\partial \tilde{s}_{0,t}/\partial w_{0,t}^{n}$, then Proposition 4 continues to apply with the qualification that also $\tau_{1,t+1} < 0$.[22] In that case, we can derive the following generalization of Proposition 4:

Proposition 5. *If* $\partial \tilde{l}_{0,t}/\partial w_{1,t+1}^{n}$ and $\partial \tilde{s}_{0,t}/\partial w_{1,t+1}^{n}$ *are sufficiently small in absolute value for all* t*, and if the residence based tax instrument is not available, so* $\theta_{i,t}^{r} \equiv 0$ *for* $i = 1, 2$ *and all* t*, the optimal policy mix in the small open economy satisfies* $\tau_{0,t} < 0$ *and* $\tau_{1,t+1} < 0$ *for all* t*.*

Proposition 5 follows directly from inspection of equations (35) and (36).

Large Open Economy

By comparison with the small open economy examined above, the large open economy constitutes a much more complex framework, as the government in such an economy typically implements a source based capital income tax in addition to the labor income tax. Therefore, to be able to concentrate on basic intuition, and avoid unnecessarily complicated policy rules, we will again consider a simplified version of the model where the hours of work are held constant for the middle-aged.

Let us once again focus on sophisticated consumers, although the qualitative results derived in Proposition 6 below also apply under naivety. To simplify the comparison with the small open economy, we use $\tau_{0,t}^{small}$ as a short notation for the marginal labor income tax formula derived for the small open economy in eqution equation (34), i.e.

[22] Contrary to the atemporal measures of how the compensated labor supply and savings respond to a change in the wage rate (which can be estimated by using existing empirical evidence), it is not straight forward to assess the magnitudes of the derivatives $\partial \tilde{l}_{0,t}/\partial w_{1,t+1}^{n}$ and $\partial \tilde{s}_{0,t}/\partial w_{1,t+1}^{n}$. However, note that quasi-hyperbolic discountring may, in itself, contribute to reduce absolute value of both these derivatives; in fact, if $\beta = 0$, both of them are equal to zero.

$$\tau_{0,t}^{small} = \frac{\alpha_{0,t}}{\gamma_t w_{0,t}} \left(\frac{\beta - 1}{\beta}\right) \frac{\partial u_{0,t}}{\partial c_{0,t}} < 0,$$

although it is, in this case, evaulated in the large open economy being examined here. We show in the Appendix that the marginal labor income tax facing the young consumer in period t and the source based tax implemented in time period $t+1$, respectively, can be written as

$$\tau_{0,t} = \tau_{0,t}^{small} - \frac{\alpha_{0,t}}{\gamma_t w_{0,t}} \left(\theta_{t+1}^s \gamma_{t+1} r_{t+1} + \theta_{t+2}^s \gamma_{t+2} r_{t+2} \frac{\partial s_{1,t+1}}{\partial s_{0,t}}\right) \quad (37)$$

$$\frac{\theta_t^s}{(1-\theta_t^s)} = \frac{Q_t}{R_t^*} \frac{dR_t^*}{dQ_t} + \frac{\Gamma_t}{\gamma_t R_t^*} \frac{dR_t^*}{dQ_t}. \quad (38)$$

Let us begin by interpreting the formula for the source based capital income tax, θ_{t+1}^s, given in equation (38). Since $dR_t^*/dQ_t < 0$, the first term on the right hand side contributes to decrease the source based tax if the country is a net exporter of capital ($Q > 0$) and increase the source based tax if it is a net importer of capital ($Q < 0$). This mechanism corresponds to the inverse elasticity rule presented in equation (24): as such, it reflects an incentive facing the domestic government to influence the foreign interest rate.

The second term arises because the government lacks a direct instrument for influencing the savings behavior. In the absence of a residence based tax, the source based tax (as well as the labor income tax discussed below) will, therefore, also serve as an indirect instrument for correction of the internal externalities that quasi-hyperbolic discounting give rise to. The variable

$$\begin{aligned}
\Gamma_t = & \left(\frac{\partial u_{1,t}}{\partial c_{1,t}} - \gamma_t\right) s_{0,t-1} + \left(\frac{\partial u_{2,t}}{\partial c_{2,t}} - \gamma_t\right) s_{1,t-1} \\
& + \mu_{0,t-2} \frac{\partial s_{0,t-2}}{\partial R_t^*} + \mu_{0,t-1} \frac{\partial s_{0,t-1}}{\partial R_t^*} + \mu_{1,t-1} \frac{\partial s_{1,t-1}}{\partial R_t^*} + \mu_{1,t} \frac{\partial s_{1,t}}{\partial R_t^*} \\
& + \eta_{t-1} \frac{dR_{t-1}^*}{dQ_{t-1}} \frac{\partial Q_{t-1}}{\partial l_{0,t-1}} \frac{\partial l_{0,t-1}}{\partial R_t^*} + \gamma_{t-1} \left(w_{0,t-1} - w_{0,t-1}^n\right) \frac{\partial l_{0,t-1}}{\partial R_t^*} \\
& - \gamma_{t-1} \left[\left(1 - \theta_{t-1}^s\right) K_{t-1} \frac{\partial r_{t-1}}{\partial l_{0,t-1}} + \theta_{t-1}^s r_{t-1} \frac{\partial Q_{t-1}}{\partial l_{0,t-1}}\right] \frac{\partial l_{0,t-1}}{\partial R_t^*}
\end{aligned}$$

reflects this additional incentive, and the sign of Γ_t is, in general, ambiguous (see the Appendix).

Equation (37) shows how the optimal marginal labor income tax (or subsidy) rate implemented in the large open economy differs from that of a small open economy (presented in equation (34) and discussed in Proposition 4). We have derived the following result:

Proposition 6. *In a large open economy, whose government does not have access to the residence based capital income tax, i.e.* $\theta_t^r \equiv 0$ *for all* t*, the marginal labor income tax rate satisfies:*

(i) $\tau_{0,t} < \tau_{0,t}^{small}$ *if* $\theta_{t+1}^s > 0$ *and* $\theta_{t+2}^s > 0$*, which means that* $\tau_{0,t} < 0$*,*

(ii) $\tau_{0,t} > \tau_{0,t}^{small}$ *if* $\theta_{t+1}^s < 0$ *and* $\theta_{t+2}^s < 0$*, which means that* $\tau_{0,t}$ *can be either positive or negative.*

To facilitate the interpretation of Proposition 6, suppose that the sign of the source based tax is driven by the first term on the right hand side of equation (38), meaning that its sign depends on whether the country is a net importer or net exporter of capital (as in the absence of any resitriction on the residence based tax described in Proposition 1). The first part of Proposition 6 is then interpretable in terms of a net importer of capital, where the government implements a positive source based capital tax to push down the foreign rate of return. This will exacerberate the undersavings problem due to quasi-hyperbolic discounting and provides, therefore, an additional incentive for the government to subsidize labor at the margin (in addition to the incentive to subsidize labor in a small open economy), which explains why $\tau_{0,t} < 0$, and $\tau_{0,t} < \tau_{0,t}^{small}$. By analogy, a net exporter of capital implements a source based capital subsidy ($\theta < 0$), which pushes up the foreign rate of return. As such, this counteracts the undersavings problem caused by quasi-hyperbolic discounting, which explains why the marginal labor subsidy rate (that serves to increase savings) needs not be as large here as for the capital importing country; in fact, $\tau_{0,t}$ may be either positive or negative here depending on the relative size of the terms in equation (37).

5 Concluding Remarks

To our knowledge, this is the first paper dealing with the optimal mix of labor and capital income taxation in an OLG model of an open economy with mobile capital, where the consumer preferences are characterized by a self-control problem caused by quasi-hyperbolic discounting. As such, we make a distinction between (i) small and large open economies, (ii) naivety and sophistication from the perspective of consumer behavior, and (iii) the instruments available for taxing or subsidizing savings, i.e. residence based and source based capital income taxes/subsidies; all of which are motivated based on earlier literature on savings, taxation and self-control problems.

We would like to emphasize three broad conclusions. First, and given a full set of tax instruments (that contains both the residence based and source based taxes), the residence based capital income tax plays a residual role: it is set such that the total marginal savings subsidy - the effective subsidy rate attached to savings - corrects for the self-control problem (that would otherwise manifest itself in terms of too little savings). The source based tax will only be used to affect the foreign interest rate (large open economy) or not used at all (small open economy). Furthermore, note that the labor income tax serves no corrective purpose if the government can tax capital income both according to the residence-principle and source-principle. Second, the total marginal capital subsidy faced by a young consumer is larger under naivety than under sophistication. The intuition is that the young sophisticated consumer internalizes part of the internal externality himself/herself, through strategic interaction vis-a-vis his/her middle-aged self, while the young naive consumer (who erroneously expects not to be time-inconsistent in the future) does not. Third, in the absence of the residence based instrument (or if this instrument is subject to a binding restriction), the labor income tax plays a distinct role as an (imperfect) instrument to correct for the self-control problem. In that case, our results show that labor ought to be subsidized at the margin, which leads to higher income and, therefore, increased savings, ceteris paribus, and the marginal labor subsidy is likely to be larger under naivety than sophistication.

Future research may take several directions and we briefly discuss two of them. First, our analysis assumes nonlinear taxation, where the tax schedule contains slope as well as intercept components that are subject to choice by the government, and we also abstract from other policy instruments than taxation. With a more restrictive tax system, such as a system with linear taxation, the results may differ from those derived above. In addition, if public provision schemes can be designed to influence the savings behavior, such instruments are clearly interesting to consider as a supplement to taxation in the regimes where the residence based tax is not available (or restricted). Second, we have neglected other forms of capital accumulation than physical capital. For instance, quasi-hyperbolic discounting is also likely to affect the incentives underlying human capital accumulation faced by the consumers. In that case, the (corrective) role of the labor income tax will also differ from that described here. We leave these and other extensions for future research.

6 Appendix

Proof of Proposition 1

The Lagrangean of the policy problem in the Benchmark Model can be written as

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty}\Theta^t\tilde{U}_{0,t}+\sum_{t=0}^{\infty}\sum_{i=1}^{2}\alpha_{i,t+i}\Theta^{t+i}\left[\left(1-\theta_{i,t+i}^{r}\right)R_{t+i}^{*}-r_{i,t+i}^{n}\right]\\&\quad+\sum_{t=0}^{\infty}\gamma_t\Theta^t\left[\sum_{i=0}^{1}\left(w_{i,t}-w_{i,t}^{n}\right)l_{i,t}+\sum_{i=1}^{2}\left(r_t-r_{i,t}^{n}\right)s_{i-1,t-1}+\sum_{i=0}^{2}T_{i,t}-\theta_t^s r_t Q_t\right]\\&\quad+\sum_{t=0}^{\infty}\sum_{i=0}^{1}\mu_{i,t}\Theta^t\left[s_{i,t}\left(\cdot\right)-s_{i,t}\right]+\sum_{t=0}^{\infty}\eta_{t+1}\Theta^{t+1}\left[R_{t+1}^{*}\left(Q_{t+1}\right)-R_{t+1}^{*}\right] \qquad \text{(A.1)}\end{aligned}$$

where a, γ, μ and η are current value Lagrange multipliers. The functions $s_{i,t}\left(\cdot\right)$ are given in equations (10) and (14). Let $S_t = s_{0,t-1}+s_{1,t-1}$ and define the short notation

$$\begin{aligned}
A_{t+1} &= \left(1-\theta^s_{t+1}\right) Q_{t+1}\left(\frac{\partial r_{t+1}}{\partial s_{0,t}} + \frac{\partial r_{t+1}}{\partial l_{1,t+1}}\frac{\partial l_{1,t+1}}{\partial s_{0,t}}\right) \\
&\quad -\theta^s_{t+1} r_{t+1}\left(\frac{\partial Q_{t+1}}{\partial s_{0,t}} + \frac{\partial Q_{t+1}}{\partial l_{1,t+1}}\frac{\partial l_{1,t+1}}{\partial s_{0,t}}\right) \\
A_{t+2} &= \left(1-\theta^s_{t+2}\right) Q_{t+2}\frac{\partial r_{t+2}}{\partial s_{1,t+1}} - \theta^s_{t+2} r_{t+2}\frac{\partial Q_{t+2}}{\partial s_{1,t+1}} \\
B_{t+j} &= \left(1-\theta^s_{t+j}\right) Q_{t+j}\frac{\partial r_t}{\partial l_{j,t+j}} - \theta^s_{t+j} r_{t+j}\frac{\partial Q_{t+j}}{\partial l_{j,t+j}} \text{ for } j=0,1 \\
\frac{\partial \Psi_t}{\partial s_{0,t}} &= \Theta\left(1+r^n_{1,t+1}\right)\frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \frac{\partial u_{0,t}}{\partial c_{0,t}} \\
\frac{\partial \Psi_t}{\partial s_{1,t+1}} &= \Theta^2\left(1+r^n_{2,t+2}\right)\frac{\partial u_{2,t+2}}{\partial c_{2,t+2}} - \Theta\frac{\partial u_{1,t+1}}{\partial c_{1,t+1}}.
\end{aligned}$$

The first order conditions can then be written as

$$\begin{aligned}
w^n_{0,t} \quad : \quad 0 &= l_{0,t}\left(\frac{\partial u_{0,t}}{\partial c_{0,t}} - \gamma_t\right) + \gamma_t\left(w_{0,t} - w^n_{0,t}\right)\frac{\partial l_{0,t}}{\partial w^n_{0,t}} \\
&\quad +\gamma_t B_t \frac{\partial l_{0,t}}{\partial w^n_{0,t}} + \mu_{0,t}\frac{\partial s_{0,t}}{\partial w^n_{0,t}} + \eta_t\frac{\partial R^*_t}{\partial Q_t}\frac{\partial Q_t}{\partial l_{0,t}}\frac{\partial l_{0,t}}{\partial w^n_{0,t}}
\end{aligned} \tag{A.2a}$$

$$\begin{aligned}
T_{0,t} \quad : \quad 0 &= \left(\gamma_t - \frac{\partial u_{0,t}}{\partial c_{0,t}}\right) + \gamma_t\left(w_{0,t} - w^n_{0,t}\right)\frac{\partial l_{0,t}}{\partial T_{0,t}} \\
&\quad +\gamma_t B_t \frac{\partial l_{0,t}}{\partial T_{0,t}} + \mu_{0,t}\frac{\partial s_{0,t}}{\partial T_{0,t}} + \eta_t\frac{\partial R^*_t}{\partial Q_t}\frac{\partial Q_t}{\partial l_{0,t}}\frac{\partial l_{0,t}}{\partial T_{0,t}}
\end{aligned} \tag{A.2b}$$

$$\begin{aligned}
w^n_{1,t+1} \quad : \quad 0 &= l_{1,t+1}\left(\Theta\frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \gamma_{t+1}\right) + \mu_{0,t}\frac{\partial s_{0,t}}{\partial w^n_{1,t+1}} + \mu_{1,t+1}\frac{\partial s_{1,t+1}}{\partial w^n_{1,t+1}} \\
&\quad +\gamma_t\left(w_{0,t} - w^n_{0,t}\right)\frac{\partial l_{0,t}}{\partial w^n_{1,t+1}} + \gamma_t B_t\frac{\partial l_{0,t}}{\partial w^n_{1,t+1}} \\
&\quad +\gamma_{t+1}\left(w_{1,t+1} - w^n_{1,t+1}\right)\frac{\partial l_{1,t+1}}{\partial w^n_{1,t+1}} + \gamma_{t+1} B_{t+1}\frac{\partial l_{1,t+1}}{\partial w^n_{1,t+1}} \\
&\quad +\eta_t\frac{\partial R^*_t}{\partial Q_t}\frac{\partial Q_t}{\partial l_{0,t}}\frac{\partial l_{0,t}}{\partial w^n_{1,t+1}} + \eta_{t+1}\frac{\partial R^*_{t+1}}{\partial Q_{t+1}}\frac{\partial Q_{t+1}}{\partial l_{1,t+1}}\frac{\partial l_{1,t+1}}{\partial w^n_{1,t+1}}
\end{aligned} \tag{A.2c}$$

$$T_{j,t+j} \quad : \qquad 0 = \left(\gamma_{t+j} - \Theta^j \frac{\partial u_{j,t+j}}{\partial c_{j,t+j}}\right) + \mu_{0,t}\frac{\partial s_{0,t}}{\partial T_{j,t+j}} + \mu_{1,t+1}\frac{\partial s_{1,t+1}}{\partial T_{j,t+j}}$$
$$+\gamma_t\left(w_{0,t} - w^n_{0,t}\right)\frac{\partial l_{0,t}}{\partial T_{j,t+j}} + \gamma_t B_t \frac{\partial l_{0,t}}{\partial T_{j,t+j}}$$
$$+\gamma_{t+1}\left(w_{1,t+1} - w^n_{1,t+1}\right)\frac{\partial l_{1,t+1}}{\partial T_{j,t+j}} + \gamma_{t+1} B_{t+1}\frac{\partial l_{1,t+1}}{\partial T_{j,t+j}}$$
$$+\eta_t \frac{\partial R^*_t}{\partial Q_t}\frac{\partial Q_t}{\partial l_{0,t}}\frac{\partial l_{0,t}}{\partial T_{j,t+j}} + \eta_{t+1}\frac{\partial R^*_{t+1}}{\partial Q_{t+1}}\frac{\partial Q_{t+1}}{\partial l_{1,t+1}}\frac{\partial l_{1,t+1}}{\partial T_{j,t+j}} \tag{A.2d}$$

$$r^n_{j,t+j} \quad : \qquad 0 = \left(\Theta^j \frac{\partial u_{j,t+j}}{\partial c_{j,t+j}} - \gamma_{t+j}\right) s_{j-1,t+j-1} + \mu_{0,t}\frac{\partial s_{0,t}}{\partial r^n_{j,t+j}} + \mu_{1,t+1}\frac{\partial s_{1,t+1}}{\partial r^n_{j,t+j}}$$
$$+\gamma_t\left(w_{0,t} - w^n_{0,t}\right)\frac{\partial l_{0,t}}{\partial r^n_{j,t+j}} + \gamma_t B_t \frac{\partial l_{0,t}}{\partial r^n_{j,t+j}} - \alpha_{1,t+1}$$
$$+\gamma_{t+1}\left(w_{1,t+1} - w^n_{1,t+1}\right)\frac{\partial l_{1,t+1}}{\partial r^n_{j,t+j}} + \gamma_{t+1} B_{t+1}\frac{\partial l_{1,t+1}}{\partial r^n_{j,t+j}}$$
$$+\eta_t \frac{\partial R^*_t}{\partial Q_t}\frac{\partial Q_t}{\partial l_{0,t}}\frac{\partial l_{0,t}}{\partial r^n_{j,t+j}} + \eta_{t+1}\frac{\partial R^*_{t+1}}{\partial Q_{t+1}}\frac{\partial Q_{t+1}}{\partial l_{1,t+1}}\frac{\partial l_{1,t+1}}{\partial r^n_{j,t+j}} \tag{A.2e}$$

$$\theta^r_{j,t+j} \quad : \qquad 0 = -\alpha_{j,t+j} R^*_{t+1} \tag{A.2f}$$

$$\theta^s_t \quad : \qquad 0 = \eta_t \frac{\partial R^*_t}{\partial Q_t}\frac{\partial Q_t}{\partial \theta^s_t} + \gamma_t\left(s_{0,t-1} + s_{1,t-1}\right)\frac{\partial r_t}{\partial \theta^s_t}$$
$$+\gamma_t\left[L_t \frac{\partial w_{0,t}}{\partial \theta^s_t} - r_t Q_t - \theta^s_t r_t \frac{\partial Q_t}{\partial \theta^s_t} - \theta^s_t Q_t \frac{\partial r_t}{\partial \theta^s_t}\right] \tag{A.2g}$$

$$R^*_{t+1}\!: \quad 0 = \alpha_{1,t+1}\left(1 - \theta^r_{1,t+1}\right) + \alpha_{2,t+1}\left(1 - \theta^r_{2,t+1}\right) - \eta_{t+1} \tag{A.2h}$$

$$s_{0,t} \quad : \qquad 0 = \mu_{0,t} - \left[\frac{\partial \Psi_t}{\partial s_{0,t}} + \gamma_{t+1}\left(r_{t+1} - r^n_{1,t+1}\right)\right] - \gamma_{t+1} A_{t+1} - \mu_{1,t+1}\frac{\partial s_{1,t+1}}{\partial s_{0,t}}$$
$$-\gamma_{t+1}\left(w_{1,t+1} - w^n_{1,t+1}\right)\frac{\partial l_{1,t+1}}{\partial s_{0,t}} - \eta_{t+1}\frac{\partial R^*_{t+1}}{\partial Q_{t+1}}\frac{\partial Q_{t+1}}{\partial s_{0,t}} \tag{A.2i}$$

$$s_{1,t+1} \quad : \qquad 0 = \mu_{1,t+1} - \left[\frac{\partial \Psi_t}{\partial s_{1,t+1}} + \gamma_{t+2} \left(r_{t+2} - r^n_{1,t+2} \right) \right] - \gamma_{t+2} A_{t+2}$$
$$-\eta_{t+2} \frac{\partial R^*_{t+2}}{\partial Q_{t+2}} \frac{\partial Q_{t+2}}{\partial s_{1,t+1}} \tag{A.2j}$$

for $j = 1, 2$.

To derive the formula for the source based capital income tax, use the first order conditions for $\theta^r_{1,t+1}$, $\theta^r_{2,t+1}$ and R^*_{t+1} together with the identity $s_{0,t-1} + s_{1,t-1} = K_t + Q_t$, and the zero profit condition. Note that the zero profit condition means

$$0 = K_t \frac{dr_t}{dx} + (l_{0,t} + a l_{1,t}) \frac{dw_{0,t}}{dx} \tag{A.3}$$

for any policy variable, x. The first order condition for θ^s_t in equation (A.2g) can then be rewritten as

$$0 = (1 - \theta^s_t) Q_t \frac{\partial r_t}{\partial \theta^s_t} - r_t Q_t - \theta^s_t r_t \frac{\partial Q_t}{\partial \theta^s_t}. \tag{A.4}$$

Differentiating the capital market equilibrium condition presented in equation (3) with respect to θ^s_t and substituting the resulting expression into equation (A.4) gives

$$0 = \frac{\partial R^*_t}{\partial Q_t} Q_t - \theta^s_t r_t. \tag{A.5}$$

Using equation (3) to substitute for r_t and rearranging gives in equation (24), which implies part (ii) of Proposition 1.

To derive the expressions for the marginal income tax rates, combine the first order conditions for $w^n_{0,t}$ and $T_{0,t}$; $w^n_{1,t+1}$ and $T_{1,t+1}$; $r^n_{1,t+1}$ and $T_{1,t+1}$; and $r^n_{2,t+2}$ and $T_{2,t+2}$, respectively. Using the first order conditions for $s_{0,t}$ and $s_{1,t+1}$ to eliminate $\mu_{0,t}$ and $\mu_{1,t+1}$ and rearranging gives the following equation system:

$$
\begin{aligned}
0 &= \chi_1 \frac{\partial \tilde{s}_{0,t}}{\partial w^n_{0,t}} + \chi_2 \frac{\partial \tilde{l}_{0,t}}{\partial w^n_{0,t}} + \chi_3 \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w^n_{0,t}} + \chi_4 \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w^n_{0,t}} && \text{(A.6)} \\
0 &= \chi_1 \frac{\partial \tilde{s}_{0,t}}{\partial w^n_{1,t+1}} + \chi_2 \frac{\partial \tilde{l}_{0,t}}{\partial w^n_{1,t+1}} + \chi_3 \left[\frac{\partial \tilde{s}_{1,t+1}}{\partial w^n_{1,t+1}} + \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w^n_{1,t+1}} \right] \\
&\quad + \chi_4 \left(\frac{\partial \tilde{l}_{1,t+1}}{\partial w^n_{1,t+1}} + \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w^n_{1,t+1}} \right) && \text{(A.7)} \\
0 &= \chi_1 \frac{\partial \tilde{s}_{0,t}}{\partial r^n_{1,t+1}} + \chi_2 \frac{\partial \tilde{l}_{0,t}}{\partial r^n_{1,t+1}} + \chi_3 \left[\frac{\partial \tilde{s}_{1,t+1}}{\partial r^n_{1,t+1}} + \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial r^n_{1,t+1}} \right] \\
&\quad + \chi_4 \left(\frac{\partial \tilde{l}_{1,t+1}}{\partial r^n_{1,t+1}} + \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial r^n_{1,t+1}} \right) && \text{(A.8)} \\
0 &= \chi_1 \frac{\partial \tilde{s}_{0,t}}{\partial r^n_{2,t+2}} + \chi_2 \frac{\partial \tilde{l}_{0,t}}{\partial r^n_{2,t+2}} + \chi_3 \left[\frac{\partial \tilde{s}_{1,t+1}}{\partial r^n_{2,t+2}} + \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial r^n_{2,t+2}} \right] \\
&\quad + \chi_4 \left(\frac{\partial \tilde{l}_{1,t+1}}{\partial r^n_{2,t+2}} + \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial r^n_{2,t+2}} \right) && \text{(A.9)}
\end{aligned}
$$

where $\tilde{l}$ and $\tilde{s}$ denote compensated labor supply and savings function, respectively, and

$$
\begin{aligned}
\chi_1 &= \Theta \left(1 + r^n_{1,t+1}\right) \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \frac{\partial u_{0,t}}{\partial c_{0,t}} + \gamma_{t+1} \left(r_{t+1} - r^n_{1,t+1}\right) \\
\chi_2 &= \gamma_t \left(w_{0,t} - w^n_{0,t}\right) \\
\chi_3 &= \Theta^2 \left(1 + r^n_{2,t+2}\right) \frac{\partial u_{2,t+2}}{\partial c_{2,t+2}} - \Theta \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} + \gamma_{t+2} \left(r_{t+2} - r^n_{1,t+2}\right) \\
\chi_4 &= \gamma_{t+1} \left(w_{1,t+1} - w^n_{1,t+1}\right).
\end{aligned}
$$

Note that equations (A.6) - (A.9) are satisfied if

$$\chi_1 = \chi_2 = \chi_3 = \chi_4 = 0. \tag{A.10}$$

Clearly, $\chi_2 = 0$ and $\chi_4 = 0$ imply $\tau_{0,t} = 0$ and $\tau_{1,t+1} = 0$, respectively. Finally, note that equations (A.6) - (A.9) imply $\mu_{0,t} = \mu_{1,t+1} = 0$ and

$$\gamma_t = \frac{\partial u_{0,t}}{\partial c_{0,t}}, \quad \gamma_{t+1} = \Theta \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}}, \quad \gamma_{t+2} = \Theta^2 \frac{\partial u_{2,t+2}}{\partial c_{2,t+2}}. \tag{A.11}$$

Using the private first order condition for $s_{0,t}$ together with equations (A.11) and $\theta_{1,t+1} r_{t+1} = r_{t+1} - r^n_{1,t+1}$ give equation (26). Similarly, the private first order condition for $s_{1,t+1}$ together with equations (A.11) and $\theta_{2,t+2}r_{t+2} = r_{t+2} - r^n_{2,t+2}$ give equation (27). Parts (i), (iii) and (iv) of Proposition 1 immedately follow.■

Proof of Proposition 3

The Lagrangean associated with the government's decision-problem is identical to equation (A1) with the exception that $\theta^s_t = 0$ for all t. As a consequence, the first order conditions characterizing the public decision problem take the same form as in the Benchmark model with the exception that $\theta^s_t = 0$ for all t here.

By combining the first order conditions for $w^n_{0,t}$ and $T_{0,t}$; $w^n_{1,t+1}$ and $T_{1,t+1}$; $r^n_{1,t+1}$ and $T_{1,t+1}$; and $r^n_{2,t+2}$ and $T_{2,t+2}$, respectively, we can derive the following equation system:

$$0 = \mu_{0,t}\frac{\partial \tilde{s}_{0,t}}{\partial w^n_{0,t}} + \gamma_t\left[\left(w_{0,t} - w^n_{0,t}\right) + Q_t\frac{\partial r_t}{\partial l_{0,t}}\right]\frac{\partial \tilde{l}_{0,t}}{\partial w^n_{0,t}} \quad \text{(A.12)}$$

$$\begin{aligned} 0 = {} & \mu_{0,t}\frac{\partial \tilde{s}_{0,t}}{\partial w^n_{1,t+1}} + \gamma_t\left[\left(w_{0,t} - w^n_{0,t}\right) + Q_t\frac{\partial r_t}{\partial l_{0,t}}\right]\frac{\partial \tilde{l}_{0,t}}{\partial w^n_{1,t+1}} + \mu_{1,t+1}\frac{\partial \tilde{s}_{1,t+1}}{\partial w^n_{1,t+1}} \\ & + \gamma_{t+1}\left[\left(w_{1,t+1} - w^n_{1,t+1}\right) + Q_{t+1}\frac{\partial r_{t+1}}{\partial l_{1,t+1}}\right]\frac{\partial \tilde{l}_{1,t+1}}{\partial w^n_{1,t+1}} \end{aligned} \quad \text{(A.13)}$$

$$\begin{aligned} 0 = {} & \mu_{0,t}\frac{\partial \tilde{s}_{0,t}}{\partial r^n_{1,t+1}} + \gamma_t\left[\left(w_{0,t} - w^n_{0,t}\right) + Q_t\frac{\partial r_t}{\partial l_{0,t}}\right]\frac{\partial \tilde{l}_{0,t}}{\partial r^n_{1,t+1}} + \mu_{1,t+1}\frac{\partial \tilde{s}_{1,t+1}}{\partial r^n_{1,t+1}} \\ & + \gamma_{t+1}\left[\left(w_{1,t+1} - w^n_{1,t+1}\right) + Q_{t+1}\frac{\partial r_{t+1}}{\partial l_{1,t+1}}\right]\frac{\partial \tilde{l}_{1,t+1}}{\partial r^n_{1,t+1}} \end{aligned} \quad \text{(A.14)}$$

$$\begin{aligned} 0 = {} & \mu_{0,t}\frac{\partial \tilde{s}_{0,t}}{\partial r^n_{2,t+2}} + \gamma_t\left[\left(w_{0,t} - w^n_{0,t}\right) + Q_t\frac{\partial r_t}{\partial l_{0,t}}\right]\frac{\partial \tilde{l}_{0,t}}{\partial r^n_{2,t+2}} + \mu_{1,t+1}\frac{\partial \tilde{s}_{1,t+1}}{\partial r^n_{2,t+2}} \\ & + \gamma_{t+1}\left[\left(w_{1,t+1} - w^n_{1,t+1}\right) + Q_{t+1}\frac{\partial r_{t+1}}{\partial l_{1,t+1}}\right]\frac{\partial \tilde{l}_{1,t+1}}{\partial r^n_{2,t+2}} \end{aligned} \quad \text{(A.15)}$$

Note that equations (A.12) - (A.15) are satisfied if the following conditions hold:

$$
\begin{aligned}
0 &= \left(w_{0,t} - w_{0,t}^{n}\right) + Q_t \frac{\partial r_t}{\partial l_{0,t}} && \text{(A.16)} \\
0 &= \left(w_{1,t+1} - w_{1,t+1}^{n}\right) + Q_{t+1} \frac{\partial r_{t+1}}{\partial l_{1,t+1}} && \text{(A.17)} \\
0 &= \mu_{0,t} && \text{(A.18)} \\
0 &= \mu_{1,t+1}. && \text{(A.19)}
\end{aligned}
$$

Differentiating the capital market equilibrium condition $R_t^* \left(Q_t\left(\cdot\right)\right) = r_t\left(\cdot\right)$ w.r.t. $l_{0,t}$ and $l_{1,t+1}$ and substituting the resulting expressions into equation (A.16) and (A.17), respectively, produces the marginal labor income tax formula in equation (28).

Finally, if equations (A.16) - (A.19) are satisfied, the first order conditions for $T_{0,t}$, $T_{1,t+1}$ and $T_{2,t+2}$ can be written as in equations (A.11). By using equations (A.11) together with (a) the private first order conditions for $s_{0,t}$ and $s_{1,t+1}$, (b) the expressions for unit savings taxes, $\theta_{1,t+1} r_{t+1} = r_{t+1} - r_{1,t+1}^{n}$ and $\theta_{2,t+2} r_{t+2} = r_{t+2} - r_{2,t+2}^{n}$, and (c) the capital market equilibrium condition, it is straight forward to derive equations (29) and (30).■

Proof of Proposition 4 and Derivation of Equations (35) and (36)

The small open economy with source based capital income taxation means that the capital market equilibrium condition is given by $\left(1-\theta_t^s\right) r_t = R_t^*$, with R_t^* treated as fixed by the domestic government. Since we assume away residence based capital income taxes, i.e. $\theta_{i,t}^{r} \equiv 0$, the Lagrangean associated with the public decision-problem can be written as

$$
\begin{aligned}
\mathcal{L} &= \sum_{t=0}^{\infty} \Theta^t \tilde{U}_{0,t} + \sum_{t=0}^{\infty} \sum_{i=0}^{1} \mu_{i,t} \Theta^t \left[s_{i,t}\left(\cdot\right) - s_{i,t}\right] \\
&\quad + \sum_{t=0}^{\infty} \gamma_t \Theta^t \left[\sum_{i=0}^{1} \left(w_{i,t} - w_{i,t}^{n}\right) l_{i,t} + \theta_t^s r_t K_t + \sum_{i=0}^{2} T_{i,t}\right]. \qquad \text{(A.20)}
\end{aligned}
$$

Starting with the source based capital income tax, the first order condition for θ_t^s is written as

$$
0 = \gamma_t \left[\frac{dw_{0,t}}{d\theta_t^s} l_{0,t} + a \frac{dw_{0,t}}{d\theta_t^s} l_{1,t} + r_t K_t + \theta_t^s r_t \frac{\partial K_t}{\partial \theta_t^s} + \theta_t^s K_t \frac{dr_t}{d\theta_t^s}\right]. \qquad \text{(A.21)}
$$

Now, by using equation (A.3), and then differentiating the capital market equuilibrium condition with respect to θ_t^s and substituting into equation (A.21), we obtain

$$0 = \theta_t^s r_t \frac{\partial K_t}{\partial \theta_t^s}. \tag{A.22}$$

Equation (A.21) implies $\theta_t^s = 0$.

To derive the expressions for the marginal labor income tax rates, we use the following first order conditions:

$$\begin{aligned} w_{0,t}^n \quad : \quad 0 &= l_{0,t}\left(\frac{\partial u_{0,t}}{\partial c_{0,t}} - \gamma_t\right) + \mu_{0,t}\frac{\partial s_{0,t}}{\partial w_{0,t}^n} \\ &+\gamma_t\left[\left(w_{0,t} - w_{0,t}^n\right) + \theta_t^s r_t \frac{\partial K_t}{\partial l_{0,t}}\right]\frac{\partial l_{0,t}}{\partial w_{0,t}^n} \end{aligned} \tag{A.23a}$$

$$\begin{aligned} T_{0,t} \quad : \quad 0 &= \left(\gamma_t - \frac{\partial u_{0,t}}{\partial c_{0,t}}\right) + \mu_{0,t}\frac{\partial s_{0,t}}{\partial T_{0,t}} \\ &+\gamma_t\left[\left(w_{0,t} - w_{0,t}^n\right) + \theta_t^s r_t \frac{\partial K_t}{\partial l_{0,t}}\right]\frac{\partial l_{0,t}}{\partial T_{0,t}} \end{aligned} \tag{A.23b}$$

$$\begin{aligned} w_{1,t+1}^n \quad : \quad 0 &= l_{1,t+1}\left(\Theta\frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \gamma_{t+1}\right) + \mu_{0,t}\frac{\partial s_{0,t}}{\partial w_{1,t+1}^n} + \mu_{1,t+1}\frac{\partial s_{1,t+1}}{\partial w_{1,t+1}^n} \\ &+\gamma_t\left[\left(w_{0,t} - w_{0,t}^n\right) + \theta_t^s r_t \frac{\partial K_t}{\partial l_{0,t}}\right]\frac{\partial l_{0,t}}{\partial w_{1,t+1}^n} \\ &+\gamma_{t+1}\left[\left(w_{1,t+1} - w_{1,t+1}^n\right) + \theta_{t+1}^s r_{t+1} \frac{\partial K_{t+1}}{\partial l_{1,t+1}}\right]\frac{\partial l_{1,t+1}}{\partial w_{1,t+1}^n} \end{aligned} \tag{A.23c}$$

$$\begin{aligned} T_{1,t+1} \quad : \quad 0 &= \left(\gamma_{t+1} - \Theta\frac{\partial u_{1,t+1}}{\partial c_{1,t+1}}\right) + \mu_{0,t}\frac{\partial s_{0,t}}{\partial T_{1,t+1}} + \mu_{1,t+1}\frac{\partial s_{1,t+1}}{\partial T_{1,t+1}} \\ &+\gamma_t\left[\left(w_{0,t} - w_{0,t}^n\right) + \theta_t^s r_t \frac{\partial K_t}{\partial l_{0,t}}\right]\frac{\partial l_{0,t}}{\partial T_{1,t+1}} \\ &+\gamma_{t+1}\left[\left(w_{1,t+1} - w_{1,t+1}^n\right) + \theta_{t+1}^s r_{t+1} \frac{\partial K_{t+1}}{\partial l_{1,t+1}}\right]\frac{\partial l_{1,t+1}}{\partial T_{1,t+1}} \end{aligned} \tag{A.23d}$$

$$
\begin{aligned}
s_{0,t} \quad : \qquad & 0=\mu_{0,t}-\frac{\partial\Psi_t}{\partial s_{0,t}}-\gamma_{t+1}\left(w_{1,t+1}-w^n_{1,t+1}\right)\frac{\partial l_{1,t+1}}{\partial s_{0,t}} \\
& -\gamma_{t+1}\theta^s_{t+1}r_{t+1}\frac{\partial K_{t+1}}{\partial l_{1,t+1}}\frac{\partial l_{1,t+1}}{\partial s_{0,t}}-\mu_{1,t+1}\frac{\partial s_{1,t+1}}{\partial s_{0,t}} \qquad \text{(A.23e)}
\end{aligned}
$$

$$
s_{1,t+1}\colon \quad 0=\mu_{1,t+1}-\frac{\partial\Psi_t}{\partial s_{1,t+1}} \qquad \text{(A.23f)}
$$

where $\partial\Psi_t/\partial s_{0,t}$ and $\partial\Psi_t/\partial s_{1,t+1}$ take the same form as in the proof of Proposition 1 above; yet with the modification that $r^n_{1,t+1}=R^*_{t+1}$ and $r^n_{2,t+2}=R^*_{t+2}$.

By combining equations (A.23a) and (A.23b), and equations (A.23c) and (A.23d), respectively, using equations (A.23e) and (A.23f) to eliminate Lagrange multipliers, as well as using that $\theta^s_t=0$, we can derive

$$
\begin{aligned}
0 \;=\; & \gamma_t\tau_{0,t}w_{0,t}\frac{\partial\tilde{l}_{0,t}}{\partial w^n_{0,t}}+\gamma_{t+1}\tau_{1,t+1}w_{1,t+1}\frac{\partial l_{1,t+1}}{\partial s_{0,t}}\frac{\partial\tilde{s}_{0,t}}{\partial w^n_{0,t}} \\
& +\left(\frac{\partial\Psi_t}{\partial s_{0,t}}+\frac{\partial\Psi_t}{\partial s_{1,t+1}}\frac{\partial s_{1,t+1}}{\partial s_{0,t}}\right)\frac{\partial\tilde{s}_{0,t}}{\partial w^n_{0,t}} \qquad \text{(A.24a)}
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\; & \gamma_t\tau_{0,t}w_{0,t}\frac{\partial\tilde{l}_{0,t}}{\partial w^n_{1,t+1}}+\gamma_{t+1}\tau_{1,t+1}w_{1,t+1}\left(\frac{\partial\tilde{l}_{1,t+1}}{\partial w^n_{1,t+1}}+\frac{\partial l_{1,t+1}}{\partial s_{0,t}}\frac{\partial\tilde{s}_{0,t}}{\partial w^n_{1,t+1}}\right) \\
& +\left(\frac{\partial\Psi_t}{\partial s_{0,t}}+\frac{\partial\Psi_t}{\partial s_{1,t+1}}\frac{\partial s_{1,t+1}}{\partial s_{0,t}}\right)\frac{\partial\tilde{s}_{0,t}}{\partial w^n_{1,t+1}}+\frac{\partial\Psi_t}{\partial s_{1,t+1}}\frac{\partial\tilde{s}_{1,t+1}}{\partial w^n_{1,t+1}}. \qquad \text{(A.24b)}
\end{aligned}
$$

Solving equation system (A.24) gives the expressions for $\tau_{0,t}$ and $\tau_{1,t+1}$ in equations (35) and (36). In the special case where $l_{1,t+1}$ is fixed, we can use equation (A.24a) to derive equation (33), which proves part (i) of Proposition 4.■

Derivation of Equations (37) and (38)

This is a restricted version of the Benchmark model where $\theta^r_{i,t}\equiv 0$. Therefore, since $R^*_{t+1}\left(Q_{t+1}\right)=R^*_{t+i}=r^n_{i,t+i}$, the restriction $R^*_{t+i}(1-\theta^r_{i,t+1})=r^n_{i,t+i}$

in the Benchmark model will be redundant here, and the Lagrangean can be written as

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty} \Theta^t \tilde{U}_{0,t} + \sum_{t=0}^{\infty}\sum_{i=0}^{1} \mu_{i,t}\left[s_{i,t}(\cdot) - s_{i,t}\right] + \sum_{t=0}^{\infty} \eta_{t+1}\Theta^{t+1}\left[R^*_{t+1}(Q_{t+1}) - R^*_{t+1}\right] \\ &+ \sum_{t=0}^{\infty} \gamma_t \Theta^t \, [\sum_{i=0}^{1}\left(w_{i,t} - w^n_{i,t}\right) l_{i,t} + \sum_{i=1}^{2}\left(r_t - r^n_{i,t}\right) s_{i-1,t-1} \\ &+ \sum_{i=0}^{2} T_{i,t} - \theta^s_t r_t Q_t] \qquad (A.25)\end{aligned}$$

The first order conditions for θ^s_t and R^*_t are given by

$$\theta^s_t \;:\; 0 = \eta_t \frac{dR^*_t}{dQ_t}\frac{\partial Q_t}{\partial \theta^s_t} + \gamma_t\left[r_t K_t - (1-\theta^s_t) K_t \frac{\partial r_t}{\partial \theta^s_t} - \theta^s_t r_t \frac{\partial Q_t}{\partial \theta^s_t}\right] \qquad (A.26a)$$

$$\begin{aligned} R^*_t \;:\; 0 &= \frac{\partial u_{1,t}}{\partial c_{1,t}} s_{0,t-1} + \frac{\partial u_{2,t}}{\partial c_{2,t}} s_{1,t-1} - \eta_t \qquad (A.26b) \\ &+\mu_{0,t-2}\frac{\partial s_{0,t-2}}{\partial R^*_t} + \mu_{0,t-1}\frac{\partial s_{0,t-1}}{\partial R^*_t} + \mu_{1,t-1}\frac{\partial s_{1,t-1}}{\partial R^*_t} + \mu_{1,t}\frac{\partial s_{1,t}}{\partial R^*_t} \\ &+\eta_{t-1}\frac{\partial R^*_{t-1}}{dQ_{t-1}}\frac{\partial Q_{t-1}}{\partial l_{0,t-1}}\frac{\partial l_{0,t-1}}{\partial R^*_t} + \gamma_{t-1}\left(w_{0,t-1} - w^n_{0,t-1}\right)\frac{\partial l_{0,t-1}}{\partial R^*_t} \\ &-\gamma_{t-1}\left[\left(1-\theta^s_{t-1}\right) K_{t-1}\frac{\partial r_{t-1}}{\partial l_{0,t-1}} + \theta^s_{t-1} r_{t-1}\frac{\partial Q_{t-1}}{\partial l_{0,t-1}}\right]\frac{\partial l_{0,t-1}}{\partial R^*_t}. \end{aligned}$$

Note that the capital market equilibrium condition $R^*_t(Q_t) = r_t(Q_t)[1-\theta^s_t]$ implies

$$\frac{dR^*_t}{dQ_t}\frac{\partial Q_t}{\partial \theta^s_t} = (1-\theta^s_t)\frac{\partial r_t}{\partial \theta^s_t} - r_t. \qquad (A.27)$$

Substituting equation (A.27) into equation (A.24a), while using equation (A.24b) to solve for η_t, gives equation (38).

To derive equation (37), we use the first order conditions for $w^n_{0,t}$, $T_{0,t}$, $s_{0,t}$ and $s_{1,t+1}$

$$
\begin{aligned}
w^n_{0,t} \quad : \quad 0 = {} & l_{0,t}\left(\frac{\partial u_{0,t}}{\partial c_{0,t}} - \gamma_t\right) + \mu_{0,t}\frac{\partial s_{0,t}}{\partial w^n_{0,t}} + \gamma_t\left(w_{0,t} - w^n_{0,t}\right)\frac{\partial l_{0,t}}{\partial w^n_{0,t}} \\
& -\gamma_t\left[\left(1-\theta^s_t\right)K_t\frac{\partial r_t}{\partial l_{0,t}} + \theta^s_t r_t\frac{\partial Q_t}{\partial l_{0,t}}\right]\frac{\partial l_{0,t}}{\partial w^n_{0,t}} \\
& +\eta_t\frac{dR^*_t}{dQ_t}\frac{\partial Q_t}{\partial l_{0,t}}\frac{\partial l_{0,t}}{\partial w^n_{0,t}} \qquad \text{(A.28a)}
\end{aligned}
$$

$$
\begin{aligned}
T^n_{0,t} \quad : \quad 0 = {} & \left(\gamma_t - \frac{\partial u_{0,t}}{\partial c_{0,t}}\right) + \mu_{0,t}\frac{\partial s_{0,t}}{\partial T^n_{0,t}} + \gamma_t\left(w_{0,t} - w^n_{0,t}\right)\frac{\partial l_{0,t}}{\partial T^n_{0,t}} \\
& -\gamma_t\left[\left(1-\theta^s_t\right)K_t\frac{\partial r_t}{\partial l_{0,t}} + \theta^s_t r_t\frac{\partial Q_t}{\partial l_{0,t}}\right]\frac{\partial l_{0,t}}{\partial T^n_{0,t}} \\
& +\eta_t\frac{dR^*_t}{dQ_t}\frac{\partial Q_t}{\partial l_{0,t}}\frac{\partial l_{0,t}}{\partial T^n_{0,t}} \qquad \text{(A.28b)}
\end{aligned}
$$

$$
\begin{aligned}
s_{0,t} \quad : \quad 0 = {} & \frac{\partial \Psi_t}{\partial s_{0,t}} - \mu_{0,t} + \eta_{t+1}\frac{dR^*_{t+1}}{\partial Q_{t+1}}\frac{\partial Q_{t+1}}{\partial s_{0,t}} + \mu_{1,t+1}\frac{\partial s_{1,t+1}}{\partial s_{0,t}} \qquad \text{(A.28c)} \\
& +\gamma_{t+1}\left[\theta^s_{t+1}r_{t+1} - \theta^s_{t+1}r_{t+1}\frac{dQ_{t+1}}{\partial s_{0,t}} - \left(1-\theta^s_{t+1}\right)K_{t+1}\frac{\partial r_{t+1}}{\partial s_{0,t}}\right]
\end{aligned}
$$

$$
\begin{aligned}
s_{1,t+1} \quad : \quad 0 = {} & \frac{\partial \Psi_t}{\partial_{1,t+1}} - \mu_{1,t+1} + \eta_{t+2}\frac{dR^*_{t+2}}{dQ_{t+2}}\frac{\partial Q_{t+2}}{\partial s_{1,t+1}} \qquad \text{(A.28d)} \\
& +\gamma_{t+2}\left[\theta^s_{t+2}r_{t+2} - \theta^s_{t+2}r_{t+2}\frac{\partial Q_{t+2}}{\partial s_{1,t+1}} - \left(1-\theta^s_{t+2}\right)K_{t+2}\frac{\partial r_{t+2}}{\partial s_{1,t+1}}\right].
\end{aligned}
$$

Combine equations (A.28a) and (A.28b) to derive

$$
\begin{aligned}
0 = {} & \mu_{0,t}\frac{\partial \tilde{s}_{0,t}}{\partial w^n_{0,t}} + \gamma_t\left(w_{0,t} - w^n_{0,t}\right)\frac{\partial \tilde{l}_{0,t}}{\partial w^n_{0,t}} + \eta_t\frac{dR^*_t}{dQ_t}\frac{\partial Q_t}{\partial l_{0,t}}\frac{\partial \tilde{l}_{0,t}}{\partial w^n_{0,t}} \\
& -\gamma_t\left[\left(1-\theta^s_t\right)K_t\frac{\partial r_t}{\partial l_{0,t}} + \theta^s_t r_t\frac{\partial Q_t}{\partial l_{0,t}}\right]\frac{\partial \tilde{l}_{0,t}}{\partial w^n_{0,t}} \qquad \text{(A.29)}
\end{aligned}
$$

Define $K_t = s_{0,t-1} + s_{1,t-1} - Q_t$, and observe that $\Gamma_t = \eta_t - \gamma_t\left(s_{0,t-1} + s_{1,t-1}\right)$ and $\tau_{0,t}w_{0,t} = w_{0,t} - w^n_{0,t}$. Then, by using the capital market equilibrium condition $R^*_t(Q_t) = r_t(Q_t)[1-\theta^s_t]$, and that θ^s_t can be written as

$$\theta_t^s = \frac{Q_t}{r_t}\frac{dR_t^*}{dQ_t} + \frac{\Gamma_t}{\gamma_t r_t}\frac{dR_t^*}{dQ_t},$$

equation (A.29) can be rewritten to read

$$\tau_{0,t} = -\frac{\mu_{0,t}}{\gamma_t w_{0,t}}\frac{\partial \tilde{s}_{0,t}/\partial w_{0,t}^n}{\partial \tilde{l}_{0,t}/\partial w_{0,t}^n}. \tag{A.30}$$

Finally, use equations (A.28c) and (A.28d) together with the capital market equilibrium condition to substitute for $\mu_{0,t}$ and $\mu_{1,t+1}$, and define $\tau_{0,t}^{small}$ as in the text. This gives equation (37).

References

[1] Aronsson, T. and Granlund, D. (2011) Public Goods and Optimal Paternalism under Present-Biased Preferences. *Economics Letters* **113**, 54-57.

[2] Aronsson, T. and Johansson-Stenman, O. (2010) Positional Concerns in an OLG Model: Optimal Labor and Capital Income Taxation. *International Economic Review* **51**, 1071-1095.

[3] Aronsson, T. and Sjögren, T. (2011) Quasi-Hyperbolic Discounting, Paternalism and Optimal Mixed Taxation. Revised manuscript (working paper version: Umeå Economic Studies no 778, 2009).

[4] Aronsson, T. and Thunström, L. (2008) A Note on Optimal Paternalism and Health Capital Subsidies. *Economic Letters* **101**, 241-242.

[5] Bacchetta, P. and Espinosa, M. (1995) Information Sharing and Tax Competition among Governments. *Journal of International Economics* **39**, 103-121.

[6] Bernheim, D. (2002) Taxation and Saving. In Auerbach, A. and Feldstein, M. (eds) *Handbook of Public Economics*. Elsevier Science B.V.

[7] Bernheim, D., Skinner, J. and Weinberg S. (2001) What Accounts for the Variation in Retirement Wealth Among U.S. Households? *American Economic Review* **91**, 832-857.

[8]Brett, C. (1997) A Note on Nonlinear Taxation in an Overlapping Generations Model. Mimeo.

[9]Brown, A., Chua, Z., and Camerer, C. (2009) Learning and Visceral Temptation in Dynamic Saving Experiments. *Quarterly Journal of Economics* **124**, 197-231.

[10]DellaVigna, S. (2009) Psychology and Economics: Evidence from the Field. *Journal of Economic Literature* **47**, 315–372.

[11]Diamond, P. and Köszegi, B. (2003) Quasi-Hyperbolic Discounting and Retirement. *Journal of Public Economics* **87**, 1839-1872.

[12]Eggert, W. and Kolmar, M. (2002) residence-Based Capital Taxation in a Small Open Economy: Why Information is Voluntarily Exchanged and Why it is not. *International Tax and Public Finance* **9**, 465-482.

[13]Feldstein, M. (2008) Resolving the Global Imbalance: The U.S. Dollar and the U.S. Saving Rate. *Journal of Economic Perspectives* **22**, 113-125.

[14]Fredreick, S., Loewenstein, G. and O'Donoghue, T. (2002) Time Discounting and Time Preference: A Critical Review. *Journal of Economic Literature* **11**, 351-401.

[15]Guidolin, M. and La Jeunesse, E. (2007) The Decline in the U.S. Personal Saving Rate: Is it real and is it a Puzzle? *The Federal Reserve bank of St. Louis Review*, November/December, 491-514.

[16]Hey, J. and Lotito, G. (2009) Naive, resolute or Sophisticated? A Study of Dynamic Decision Making. *Journal of Risk and Uncertainty* **38**, 1-25.

[17]Laibson, D. (1996) Hyperbolic Discount Functions, Undersaving and Savings Policy. NBER Working Paper 5635.

[18]Laibson, D. (1997) Golden Eggs and Hyperbolic Discounting. *Quarterly Journal of Economics* **112**, 443-477.

[19]Mastrobuoni, G. and Weinberg, S.(2009) Heterogeneity in Intra-Monthly Consumption Patterns, Self-Control, and Savings at Retirement. *American Economic Journal: Economic Policy* **1**, 163-189.

[20]O'Donoghue, T. and Rabin, M. (1999) Doing it Now or Later. *American Economic Review*, March, 103-124.

[21]O'Donoghue, T. and Rabin, M. (2001) Choice and Procrastination. *Quarterly Journal of Economics* **116**, 121-160.

[22]O'Donoghue, T. and Rabin, M. (2003) Studying Optimal Paternalism, Illustrated by a Model of Sin Taxes. *American Economic Review*, May, 186-191.

[23]O'Donoghue, T. and Rabin, M. (2006) Optimal Sin Taxes. *Journal of Public Economics* **90**, 1825-1849.

[24]Phelps, E. and Pollak, R. (1968) On Second Best National Saving and Game Equilibrium Growth. *Review of Economic Studies* **35**, 185-199.

[25]Pirttilä, J. and Tuomala, M. (2001) On Optimal Non-Linear Taxation and Public Good Provision in an Overlapping Generations Economy. *Journal of Public Economics* **79**, 485-501.

[26]Rubenstein, A. (2003) "Economics and Psychology"? The Case of Hyperbolic Discounting. *International Economic Review* **44**, 1207-1216.

[27]Thaler, R. (1981) Some Empirical Evidence on Dynamic Inconsistency. *Economic Letters* **8**, 201-207.

[28]Viscusi, K., Huber, J. and Bell, J. (2008) Estimating Discount Rates for Environmental Quality from Utility-Based Choice Experiments. *Journal of Risk and Uncertainty* **37**, 199-220.