State-Variable Public Goods When Relative Consumption Matters: A Dynamic Optimal Taxation Approach^{**}

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Abstract

This paper concerns the optimal provision of a state-variable public good, where the global climate is the prime example. The analysis is based on a two-type optimal income tax model with overlapping generations, where people care about their relative consumption. We consider both keeping-up-with-the-Joneses preferences (where people compare their own current consumption with others' current consumption) and catching-up-with-the-Joneses preferences (where people compare their own current consumption). The extent to which the rule for public provision ought to be modified is shown to depend crucially on the preference elicitation format.

Keywords: State variable public goods, asymmetric information, relative consumption, status, positional preferences, climate policy.

JEL Classification: D62, H21, H23, H41

^{**} The authors are grateful for constructive comments from seminars participants at Lund University, Stockholm School of Economics, Uppsala University and University of Arizona. They would also like to thank FORMAS, FAS and the Swedish Research Council for generous research grants.

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1. Introduction

How much should we invest in a state-variable public good? Applied to the global climate, how much should we invest to combat climate change? This is one of the most important and discussed questions of our time. Naturally, the answer depends on many factors such as discounting issues, how the funds for the investment are raised and on related second-best problems of the economy, including how such investments interact with the use of other policy instruments, and on the nature of our preferences. In particular, while the dominating bulk of the economics literature considers the case where the utility of private consumption is driven solely by the absolute consumption, there is now a substantial body of empirical evidence suggesting that people are also concerned with their own consumption relative to that by others.¹ The present paper analyzes the optimal provision of a state-variable public good - which may be interpreted as global climate quality - in an economy where people have positional preferences in the sense of deriving utility from their own private consumption relative to that of other people. To deal with the dynamic character of the problem of providing a state-variable public good, while recognizing that informational limitations may necessitate distortive means of raising revenue, our analysis is based on an overlapping generations (OLG) model with asymmetric information between the government and the private sector, where the government raises revenue through nonlinear taxation of labor income and capital income. As far as we know, there are only two earlier studies dealing with public good provision under relative consumption concerns in a second-best economy: Wendner and Goulder (2008) and Aronsson and Johansson-Stenman (2008).² Both are based on static models, and the former also assumes linear tax instruments.

¹ This includes happiness research (e.g. Easterlin 2001; Blanchflower and Oswald 2004; Ferrer-i-Carbonell 2005; Luttmer 2005; Clark and Senik 2010), questionnaire-based experiments (e.g. Johansson-Stenman et al. 2002; Solnick and Hemenway 2005; Carlsson et al. 2007; Corazzini, Esposito and Majorano, forthcoming) and, more recently, brain science (Fliessbach et al. 2007; Dohmen et al. forthcoming). There are also recent evolutionary models consistent with relative consumption concerns (Samuelson 2004; Rayo and Becker 2007). Stevenson and Wolfers (2008) constitute a recent exception in the happiness literature, claiming that the role of relative income is overstated.

 $^{^{2}}$ Ng (1987) and Brekke and Howarth (2002) have analyzed public good provision and relative concerns based on different first-best models.

This paper makes at least two distinct contributions to the literature, both of which are related to the intertemporal aspects of the analysis. First, since we use a dynamic model, we are able to consider a state-variable public good, i.e. a stock that accumulates over time both due to the instantaneous contributions and depreciation. This is both a theoretically relevant and practically important extension, in particular because the global climate has this character. The quality, and characteristics more generally, of the atmosphere are clearly not only affected by actions taken today (such as current public abatement activities); they are also strongly affected by actions taken in previous periods (cf. Stern 2007). This argument applies to many other public goods as well, including infrastructural investments such as roads, schools and hospitals. Thus, while the reader may interpret the state-variable public good in the subsequent analysis as a quality measure of the climate, the analysis is more general and applies to virtually all (pure or impure) public goods.

Second, the dynamic framework also allows us to simultaneously consider *keeping*-up-with-the-Joneses preferences, i.e. relative consumption comparisons between people within the same period, and *catching*-up-with-the-Joneses preferences, i.e. relative consumption comparisons over time.³ The extension to intertemporal social comparisons is important for several reasons: (*i*) There is empirical evidence suggesting that people make comparisons with their own past consumption (e.g., Loewenstein and Sicherman, 1991; Frank and Hutchens, 1993). (*ii*) It makes intuitive sense that old people compare their own consumption with several different reference levels, including what they recall about their own and others' consumption when they were young, and most people probably receive information from parents and grandparents about the living conditions of the previous generation; see Senik (2009) for recent estimates regarding the importance of different kinds of comparisons over time. (*iii*) The appearance of intertemporal social comparisons is consistent with the empirical pattern of some financial puzzles such as the equity premium puzzle (e.g. Abel,

³ The notion "keeping up with the Joneses" is unfortunately used with different meanings in the literature. Either it is used, as here, to indicate social comparisons in the sense that my utility depends on your consumption, or it is used with more specific meanings, e.g. such that if you consume more now I will also consume more now. Similarly, the notion "catching up with the Joneses", may either, as here, simply mean that my utility today depend on your previous consumption, or it may reflect something more specific such that my consumption today increases with your previous consumption. No results in the present paper depends directly on the direction of people's consumption and leisure adjustment to changed reference consumption. The crucial element is instead the externalities that relative consumption imply.

1990; Campbell and Cochrane, 1999). (*iv*) Such comparisons are finally also in line with recent research based on evolutionary models, such as Rayo and Becker (2007), in which there are evolutionary reasons for why people should compare their current consumption with (*a*) others' current consumption, (*b*) their own past consumption and (*c*) others' past consumption. In the present paper, we consider all three kinds of comparisons. Given the dynamic nature of the policy problem analyzed below, the paper will also, implicitly, touch on the discounting problem; yet, this is not the main task, and we will not discuss any intergenerational equity issues.⁴

Policy rules for public goods typically depend on the set of tax instruments that the government has at its disposal. Since we derive the optimal provision rules conditional on the existence of fully nonlinear taxes on labor and capital income, the tax instruments considered here are based on informational limitations and not on any other a priori restrictions such as linearity. A nonlinear tax system gives a reasonably realistic description of the options for taxation available to real world governments; it also allows us to capture redistributive and corrective aspects of public policy (as well as interaction effects between them) in a relatively simple way. While most earlier studies dealing with public policy issues under relative consumption concerns are based on linear tax instruments,⁵ there are a few exceptions including Oswald (1983), Tuomala (1990) and Aronsson and Johansson-Stenman (2008, 2010a, b). The model in the present paper builds on the model in Aronsson and Johansson-Stenman (2010b), which address the optimal use of nonlinear income taxes in an OLG model but do not consider public goods, the main concern in the present paper.⁶

⁴ The choice of discount rate is perhaps the most discussed issue in the economics of global warming; see e.g. Nordhaus (2007) and Stern (2007).

⁵ See e.g. Boskin and Sheshinski (1978), Layard (1980), Ng (1987), Blomquist (1993), Corneo and Jeanne (1997, 2001), Ireland (2001), Brekke and Howarth (2002), Abel (2005), Wendner and Goulder (2008), Kanbur and Tuomala (2010) and Wendner (2010a, b, forthcoming). Clark et al. (2008) provide a good overview of both the empirical evidence and economic implications of relative consumption concerns.

⁶ Of course, it builds on many other important papers as well: The seminal papers on optimal public good provision under optimal nonlinear income taxation are due to Hylland and Zeckhauser (1979) and Christiansen (1981), whereas Boadway and Keen (1993) was the first paper dealing with this problem based on the self-selection approach independently developed by Stiglitz (1982) and Stern (1982). Pirttilä and Tuomala (2001) was the first paper to consider the optimal provision of a state variable public good under optimal nonlinear income taxation, whereas Ng (1987) was the first paper dealing with public good provision when relative consumption matters (in a static first-best model).

Section 2 presents the OLG framework and the individual preference structure, while Section 3 defines the corresponding measures of the extent to which different kinds of relative consumption matters, denoted "degrees of positionality". Section 4 considers the optimization problems of individuals and firms, while Section 5 describes the corresponding optimization problem facing the government. Section 6 presents rather general expressions for the optimal provision rule, which are valid for all kinds of social comparisons. Yet, while these results provide general insights on the incentives for public provision under relative consumption concerns, they are not directly interpretable in terms of the strength of such concerns. Therefore, the provision rules derived in the following sections 7 and 8 are expressed directly in terms of the degrees of positionality.

Section 7 concerns the somewhat simplified case where the individual only compares his/her own current consumption with other people's current consumption (Keeping-up-with-the-Joneses preferences); as such, it builds on the model by Aronsson and Johansson-Stenman (2010a) and extends it to encompass public goods. The results here are shown to depend crucially on the preference elicitation format. If people's marginal willingness to pay for the public good is measured independently, i.e. without considering that other people also have to pay for increased public provision, then relative consumption concerns typically (for reasonable parameter values) work in the direction of increasing the optimal provision of the public good. However, this is not the case when a referendum format is used, so that people are asked for their marginal willingness to pay conditional on that all people will have to pay for increased public provision. Conditions are also presented for when a dynamic analogue of the conventional Samuelson (1954) rule applies.

Section 8 considers the more general case with both keeping-up and catching-up-with-the-Joneses preferences simultaneously, i.e. where people derive utility from their own consumption relative to the current *and* past consumption of others, as well as relative to their own past consumption; as such, the model extends the one in Aronsson and Johansson-Stenman (2010b) to encompass public goods. Under some further simplifying assumptions (e.g., about the population size and how the concerns for relative consumption change over time), it is shown that the policy rule for public provision can be written as a straightforward extension of the corresponding policy rule derived solely on the basis of keeping-up-with-theJoneses preferences in Section 7. Section 9 provides some concluding remarks. Proofs of all propositions (along with some other calculations) are provided in the Appendix.

2. The OLG Framework and Individual Preferences

Consider an OLG economy where individuals live for two periods. Following the convention in earlier literature, we assume that each individual works during the first period of life and does not work during the second. There are two types of individuals, where the low-ability type (type 1) is less productive than the high-ability type (type 2). The number of individuals of ability-type *i* who were born at the beginning of period *t* is denoted n_t^i . Each such individual cares about his/her consumption when young and when old, c_t^i and x_{t+1}^i ; his/her leisure when young, z_t^i , given by a time endowment, *H*, less the hours of work, l_t^i (when old, all available time is leisure); and the amount of the public good available when young and when old, G_t and G_{t+1} .

In addition, the individuals also compare their own consumption with one or several reference points, to be called "reference consumption" in what follows; we follow earlier comparable literature in assuming that the private consumption good (the consumption of which is denoted c when young and x when old) is, in part, a positional good, whereas leisure and the publicly provided good are completely non-positional.⁷ Also in accordance with the bulk of earlier comparable literature, we focus on *difference* comparisons, where relative consumption is defined by the difference between the individual's own consumption and a corresponding measure of reference consumption.⁸ The appropriate measure of reference

⁷ See Aronsson and Johansson-Stenman (2009) for an analysis of the case where also relative leisure matters. It is, of course, also possible to extend the analysis by allowing people to care about their relative benefit from a publicly provided good. We leave this to future research. Our conjecture is that the qualitative insights will still hold as long as private consumption is more positional than the publicly provided good, which is consistent with the limited empirical evidence (Solnick and Hemenway, 2005).

⁸ See, e.g., Akerlof (1997), Corneo and Jeanne (1997), Ljungqvist and Uhlig (2000), Bowles and Park (2005), and Carlsson et al. (2007). Alternative approaches include ratio comparisons (Boskin and Sheshinski, 1978; Layard, 1980; Abel, 2005; Wendner and Goulder, 2008) and comparisons of ordinal rank (Frank, 1985; Hopkins and Kornienko, 2004, 2009). Dupor and Liu (2003) consider a specific flexible functional form that includes the difference comparison and ratio comparison approaches as special cases. A recent paper by Mujcic and Frijters

consumption at the individual level is, of course, an empirical question; yet, as we indicated above, there is not much information available. Our approach is to follow Rayo and Becker (2007), who argue in the context of an evolutionary model of happiness that the reference point of an individual might be determined by three components: (i) other people's current consumption, (ii) his/her own past consumption, and (iii) other people's past consumption. In the context of our model, we interpret these three components such that people care about three different kinds of relative consumption: their own current consumption relative to (*i*) the current average consumption when young and when old, i.e., $c_i^i - \overline{c_i}$ and $x_{t+1}^i - \overline{c_{t+1}}$; (*ii*) their own consumption one period earlier, i.e., $c_i^i - \overline{c_{t-1}}$ and $x_{t+1}^i - \overline{c_t}$.⁹ For further use, note that the average consumption in period *t* is defined as

$$\overline{c}_{t} = \frac{\sum_{i} \left[n_{i}^{i} c_{t}^{i} + n_{t-1}^{i} x_{t}^{i} \right]}{\sum_{i} \left[n_{t}^{i} + n_{t-1}^{i} \right]}$$

The utility function of ability-type *i* born in the beginning of period *t* can then be written as

$$U_{t}^{i} = V_{t}^{i}(c_{t}^{i}, z_{t}^{i}, x_{t+1}^{i}, c_{t}^{i} - \overline{c}_{t}, x_{t+1}^{i} - \overline{c}_{t+1}, x_{t+1}^{i} - c_{t}^{i}, c_{t}^{i} - \overline{c}_{t-1}, x_{t+1}^{i} - \overline{c}_{t}, G_{t}, G_{t+1})$$

$$= v_{t}^{i}(c_{t}^{i}, z_{t}^{i}, x_{t+1}^{i}, c_{t}^{i} - \overline{c}_{t}, x_{t+1}^{i} - \overline{c}_{t+1}, c_{t}^{i} - \overline{c}_{t-1}, x_{t+1}^{i} - \overline{c}_{t}, G_{t}, G_{t+1})$$

$$= u_{t}^{i}(c_{t}^{i}, z_{t}^{i}, x_{t+1}^{i}, \overline{c}_{t-1}, \overline{c}_{t}, \overline{c}_{t+1}, G_{t}, G_{t+1})$$

$$(1)$$

The public good is a state variable governed by the difference equation

$$G_t = g_t + (1 - \xi)G_{t-1},$$
(2)

⁹ Although one can easily imagine that each individual compares himself/herself more with some people than with others, we follow the bulk of earlier comparable literature by using the average consumption as a basis for the reference points. Aronsson and Johansson-Stenman (2010a) also consider alternative reference measures based on within-generation and upward comparisons, respectively, in a study of optimal taxation and find policy responses that are qualitatively similar to those that follow if the reference point is based solely on the average consumption.

⁽²⁰¹¹⁾ compare models based on difference comparisons, ratio comparisons and rank-based comparisons, applied on a questionnaire-experimental data-set, without being able to discriminate between them.

where g_t is the addition to the public good in period *t*, provided by the government, and ξ is the rate of depreciation. Therefore, the traditional flow-variable public good appears as the special case where $\xi = 1$.

We assume that the functions $V_t^i(\cdot)$ and $v_t^i(\cdot)$ are increasing in each argument and strictly concave, meaning that $u_t^i(\cdot)$ is decreasing in \overline{c}_{t-1} , \overline{c}_t and \overline{c}_{t+1} , while increasing in the other arguments and strictly concave. The first line of equation (1) is expressed in terms of the five consumption differences described above, as well as in terms of leisure, private consumption when young and when old, and public consumption when young and when old, respectively. Note also that, since c_t^i and x_{t+1}^i are decision variables of the individual, we can without loss of generality rewrite this utility formulation as the "reduced form" function on the second line, where the effect of $x_{t+1}^i - c_t^i$ on utility is embedded in the effects of c_t^i and x_{t+1}^i . Therefore, the only difference between the first and second lines of equation (1) is that the partial derivatives with respect to c_t^i and x_{t+1}^i will have a more complex interpretation on the second line; for instance, the partial derivative of $v_t^i(\cdot)$ with respect to c_t^i reflects both the direct utility effect of increased absolute consumption when young and the (presumably negative) utility effect due to lower relative consumption when old compared to when young. This means that all analytical results derived in a model where individuals do not compare their own current and past consumption will continue to hold also in the case where people make such comparisons. The intuition is that people internalize such comparisons perfectly.

The third line contains the most general utility formulation and resembles a classical externality problem. Here, we do not specify anything regarding the structure of the social comparisons, beyond that others' consumption gives rise to negative externalities. As will be demonstrated, for some results we do not need any stronger assumptions regarding the preference structure. However, we need the more restrictive utility formulation based on the function $v_t^i(\cdot)$, where we specify that people care about additive comparisons, to establish a relationship between, on the one hand, the optimal provision of the public good and, on the other, the degree to which the utility gain from higher consumption is associated with increased relative consumption. The definition of such measures is the issue to which we turn next.

3. Positionality Degrees

Since much of the subsequent analysis is focused on the relative consumption concerns, it is useful to introduce measures of the degree to which such concerns matter for each individual. By using $\Delta_t^{i,c} \equiv c_t^i - \overline{c}_t$, $\Delta_{t+1}^{i,x} \equiv x_{t+1}^i - \overline{c}_{t+1}$, $\delta_t^{i,c} \equiv c_t^i - \overline{c}_{t-1}$, and $\delta_{t+1}^{i,x} \equiv x_{t+1}^i - \overline{c}_t$ as short notations for the four differences in the function $v_t^i(\cdot)$ in equation (1), we follow Aronsson and Johansson-Stenman (2010b) and define the *degree of current consumption positionality* when young and when old, respectively, as

$$\alpha_{t}^{i,c} \equiv \frac{v_{t,\Delta^{c}}^{i}}{v_{t,\Delta^{c}}^{i} + v_{t,\delta^{c}}^{i} + v_{t,c}^{i}} , \qquad (3a)$$

$$\alpha_{t+1}^{i,x} \equiv \frac{v_{t,\Delta^x}^i}{v_{t,\Delta^x}^i + v_{t,\delta^x}^i + v_{t,x}^i},$$
(3b)

where sub-indexes indicate partial derivative, i.e. $v_{t,c}^{i} \equiv \partial v_{t}^{i}(\cdot) / \partial c_{t}^{i}$ and similarly for the other terms. The variable $\alpha_{t}^{i,c}$ can be interpreted as the fraction of the overall utility increase from an additional dollar spent when young in period *t* that is due to the increased consumption relative to the average consumption in period *t*, whereas $\alpha_{t+1}^{i,x}$ has a corresponding interpretation when old in period *t*+1. For example, if $\alpha_{t}^{i,c} = 0.3$ then 30% of the utility increase from the last dollar spent by an individual of ability-type *i* when young in period *t* is due to the increased relative consumption compared to other people's current consumption in the same period; hence, 70% is due to a combination of increased absolute consumption and increased relative consumption compared to other people's past consumption.By analogy, we can define the *degree of intertemporal consumption positionality* when young and when old, respectively, as

$$\beta_{t}^{i,c} \equiv \frac{v_{t,\delta^{c}}^{i}}{v_{t,\Delta^{c}}^{i} + v_{t,\delta^{c}}^{i} + v_{t,c}^{i}} , \qquad (4a)$$

$$\beta_{t+1}^{i,x} = \frac{v_{t,\delta^x}^i}{v_{t,\Delta^x}^i + v_{t,\delta^x}^i + v_{t,x}^i}.$$
(4b)

The variables $\beta_t^{i,c}$ and $\beta_{t+1}^{i,x}$ reflect the fraction of the overall utility increase from an additional dollar spent in period *t* and *t*+1 (i.e., when young and when old), respectively, that is due to the increased consumption relative to other people's past consumption. The assumptions made above imply that $0 < \alpha_t^{i,c}, \alpha_{t+1}^{i,x}, \beta_t^{i,c}, \beta_{t+1}^{i,x} < 1$ for all *t*. We can interpret the current degrees of positionality (measured by the two α -variables) as capturing the extent to which the keeping-up-with-the-Joneses motive for relative consumption comparisons is important for the individual, while the intertemporal degrees of positionality (measured by the β -variables) capture the corresponding catching-up-with-the-Joneses motive.

Let us also define the notions of *the average degree of current consumption positionality* and *the average degree of intertemporal consumption positionality*, which are given by

$$\overline{\alpha}_{t} = \sum_{i} \alpha_{t}^{i,x} \frac{n_{t-1}^{i}}{N_{t}} + \sum_{i} \alpha_{t}^{i,c} \frac{n_{t}^{i}}{N_{t}} \in (0,1), \qquad (5a)$$

$$\overline{\beta}_{t} = \sum_{i} \beta_{t}^{i,x} \frac{n_{t-1}^{i}}{N_{t}} + \sum_{i} \beta_{t}^{i,c} \frac{n_{t}^{i}}{N_{t}} \in (0,1) , \qquad (5b)$$

respectively, where $N_t \equiv \sum_i [n_{t-1}^i + n_t^i]$ is the total population in period *t*. Note that both $\overline{\alpha}_t$ and $\overline{\beta}_t$ are measured among those alive in period *t*.

4. Individual and Firm Behavior

Each individual of any generation *t* treats the measures of reference consumption, i.e. \overline{c}_{t-1} , \overline{c}_t and \overline{c}_{t+1} , as exogenous during optimization (while these measures are of course endogenous to the government, as will be explained below). The individual budget constraint is given by

$$w_{t}^{i}l_{t}^{i} - T_{t}(w_{t}^{i}l_{t}^{i}) - s_{t}^{i} = c_{t}^{i},$$
(6)

$$s_t^i(1+r_{t+1}) - \Phi_{t+1}(s_t^i r_{t+1}) = x_{t+1}^i,$$
(7)

where w_t^i is the before-tax wage rate, s_t^i is savings, r_{t+1} is the market interest rate, and $T_t(\cdot)$ and $\Phi_{t+1}(\cdot)$ denote the payments of labor income and capital income taxes, respectively. The first order conditions for the hours of work and savings can be written as

$$u_{t,c}^{i}w_{t}^{i}\left[1-T_{t}^{'}(w_{t}^{i}l_{t}^{i})\right]-u_{t,z}^{i}=0,$$
(8)

$$-u_{t,c}^{i} + u_{t,x}^{i} \left[1 + r_{t+1} \left(1 - \Phi_{t+1}^{i} (s_{t}^{i} r_{t+1}) \right) \right] = 0, \qquad (9)$$

in which $u_{t,c}^i \equiv \partial u_t^i / \partial c_t^i$, $u_{t,z}^i \equiv \partial u_t^i / \partial z_t^i$ and $u_{t,x}^i \equiv \partial u_t^i / \partial x_{t+1}^i$, and $T_t^i(w_t^i l_t^i)$ and $\Phi_{t+1}^i(s_t^i r_{t+1})$ are the marginal labor income tax rate and the marginal capital income tax rate, respectively.

The production sector consists of identical competitive firms, which number is normalized to one for notational convenience, producing a homogenous good under constant returns to scale. Following Aronsson and Johansson-Stenman (2010a), tThe production function is given by

$$F(L_{t}^{1}, L_{t}^{2}, K_{t}; t) = f(\theta^{1}L_{t}^{1} + \theta^{2}L_{t}^{2}, K_{t}; t),$$
(10)

where $L_t^i \equiv n_t^i l_t^i$ is the total number of hours of work supplied by ability-type *i* in period *t*, and K_t is the capital stock in period *t*; θ^1 and θ^2 are positive constants such that $\theta^2 > \theta^1$. The direct time-dependency implies that we allow for exogenous technological change. The firm obeys the necessary optimality conditions

$$F_{L^i}(L^1_t, L^2_t, K_t; t) = \frac{\partial f}{\partial(\theta^1 L^1_t + \theta^2 L^2_t)} \theta^i = w^i_t \quad \text{for } i=1, 2,$$
(11)

$$F_{K}(L_{t}^{1}, L_{t}^{2}, K_{t}; t) = \frac{\partial f}{\partial K_{t}} = r_{t}.$$
(12)

Note that equation (11) implies that the relative wage rate between the two ability-types is constant both within each period and over time, i.e. $w_t^1 / w_t^2 = \theta^1 / \theta^2 = \phi$, where ϕ is a constant. This property simplifies the analysis; it is not important for the qualitative results,

since endogenous relative wage rates would not affect the policy rules for public provision derived below.

5. The Government's Optimization Problem

We assume that the government faces a general social welfare function as follows:

$$W = W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \dots),$$
(13)

which is increasing in each argument.¹⁰

The informational assumptions are conventional. The government is able to observe income, although ability is private information. As in most of the earlier literature on redistribution under asymmetric information, we assume that the government wants to redistribute from the high-ability to the low-ability type. This means that the most interesting self-selection constraint is to prevent the high-ability type from pretending to be a low-ability type. The self-selection constraint that may bind then becomes

$$U_{t}^{2} = u_{t}^{2}(c_{t}^{2}, z_{t}^{2}, x_{t+1}^{2}, \overline{c}_{t-1}, \overline{c}_{t}, \overline{c}_{t+1}, G_{t}, G_{t+1})$$

$$\geq u_{t}^{2}(c_{t}^{1}, H - \phi l_{t}^{1}, x_{t+1}^{1}, \overline{c}_{t-1}, \overline{c}_{t}, \overline{c}_{t+1}, G_{t}, G_{t+1}) = \hat{U}_{t}^{2} , \qquad (14)$$

where $\phi \equiv w_t^1 / w_t^2 < 1$ is the (constant) wage ratio, i.e. relative wage rate. The expression on the right-hand side of the weak inequality in (14) is the utility of the mimicker. Although the mimicker enjoys the same consumption as the low-ability type in each period, he/she enjoys more leisure (as the mimicker is more productive than the low-ability type). Note also that, given the set of available tax instruments, it is possible for the government to control the

¹⁰ A similar formulation is used by Pirttilä and Tuomala (2001), although they in addition assume that the social welfare function is utilitarian within each generation. All results derived below could also have been obtained by explicitly solving for the Pareto efficient allocation by maximizing the utility of one ability-type born in a certain period, while holding the utility constant for all other agents (the other ability-type born in the same period and both ability-types born in all other periods). The chosen strategy is motivated by convenience, as it simplifies the presentation.

present and future consumption as well as the hours of work of each ability-type (this is discussed more thoroughly below). As a consequence, in order to be a mimicker, the high-ability type must mimic the point chosen by the low-ability type on each tax function (both the labor income tax and the capital income tax) and, therefore, consume the same amount as the low-ability type in both periods.

Note that $T_t(\cdot)$ is a general labor income tax, which can be used to implement any desired combination of l_t^1 , c_t^1 , l_t^2 , and c_t^2 , given the savings chosen by each ability-type. Similarly, the general capital income tax, $\Phi_{t+1}(\cdot)$, can be used to implement any desired combination of c_t^1 , x_{t+1}^1 , c_t^2 , x_{t+1}^2 , and K_{t+1} , given the labor income of each individual. Therefore, instead of using the parameters of the labor income and capital income tax functions as decision-variables (via which the government may control the hours of work, private consumption and capital formation), we follow convention in earlier literature on redistribution under asymmetric information in OLG economies in writing the second best problem as a direct decision-problem, where l_t^1 , c_t^1 , x_t^1 , l_t^2 , c_t^2 , x_t^2 , K_t , g_t and G_t for all t constitute decision-variables. The resource constraint is given by

$$F(L_t^1, L_t^2, K_t; t) + K_t - \sum_{i=1}^2 \left[n_t^i c_t^i + n_{t-1}^i x_t^i \right] - K_{t+1} - g_t = 0,$$
(15)

which means that output is used for private consumption as well as private and public investment.

Formally, the decision-problem faced by the government is to choose l_t^1 , c_t^1 , x_t^1 , l_t^2 , c_t^2 , x_t^2 , K_t , g_t and G_t for all *t* to maximize the social welfare function presented in equation (13) subject to equations (2), (14) and (15). The government also recognizes that the measures of reference consumption are endogenous as defined by the mean-value formula presented in Section 2. The Lagrangean can be written as

$$\mathcal{L} = W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \dots) + \sum_t \lambda_t \left[U_t^2 - \hat{U}_t^2 \right] + \sum_t \gamma_t \left[F(L_t^1, L_t^2, K_t; t) + K_t - \sum_{i=1}^2 [n_i^i c_i^i + n_{t-1}^i x_t^i] - K_{t+1} - g_t \right].$$
(16)
$$+ \sum_t \mu_t \left[g_t + (1 - \xi) G_{t-1} - G_t \right]$$

To facilitate comparison with earlier studies on optimal income taxation and/or public provision in dynamic models with self-selection constraints (e.g., Brett, 1997; Pirttilä and Tuomala, 2001; Aronsson, Sjögren and Dalin (2009); Aronsson and Johansson-Stenman, 2010a), we assume that the government commits to its tax and expenditure policies in what follows. Although we realize that our study shares a potential time-inconsistency problem with this earlier literature, it is beyond the scope of the present paper to develop a framework for public good provision without commitment.¹¹ The first-order conditions are presented in the Appendix.

6. A General Rule for Public Good Provision when Relative Consumption Matters

In this section we present general optimality conditions for the public good provision in a format that facilitates straightforward economic interpretations and comparisons with the benchmark case with no relative consumption concerns.

Let $\hat{u}_t^2 = u_t^2(c_t^1, H - \phi l_t^1, x_{t+1}^1, \overline{c}_{t-1}, \overline{c}_t, \overline{c}_{t+1}, G_t, G_{t+1})$ denote the utility faced by the mimicker of generation *t* based on the function $u_t^2(\cdot)$ in equation (1). We can then define the marginal rate of substitution between the public good and private consumption for the young and old ability-type *i*, and for the young and old mimicker, in period *t* as follows:

$$MRS_{G,c}^{i,t} \equiv \frac{u_{t,G_t}^i}{u_{t,c}^i}, \ MRS_{G,x}^{i,t} \equiv \frac{u_{t-1,G_t}^i}{u_{t-1,x}^i}, \ M\hat{R}S_{G,c}^{2,t} \equiv \frac{\hat{u}_{t,G_t}^2}{\hat{u}_{t,c}^2} \ \text{and} \ M\hat{R}S_{G,x}^{2,t} \equiv \frac{\hat{u}_{t-1,G_t}^2}{u_{t-1,x}^2}.$$

To shorten the formulas to be derived, we shall also use the short notations

¹¹ See Brett and Weymark (2008) for a recent study of (time-consistent) optimal nonlinear income taxation without commitment. See also Klein, Krusell and Ríos-Rull (2008) for an analysis of time-consistent policy which includes public expenditures, in a rather different theoretical set-up.

$$MB_{t,G} \equiv \sum_{i} n_{t}^{i} MRS_{G,c}^{i,t} + \sum_{i} n_{t-1}^{i} MRS_{G,x}^{i,t}$$
(17)

$$\Omega_{t} \equiv \lambda_{t} \hat{u}_{t,c}^{2} \left[MRS_{G,c}^{1,t} - M\hat{R}S_{G,c}^{2,t} \right] + \lambda_{t-1} \hat{u}_{t-1,x}^{2} \left[MRS_{G,x}^{1,t} - M\hat{R}S_{G,x}^{2,t} \right]$$
(18)

for the sum of the marginal willingness to pay for the public good (measured as the marginal rate of substitution between the public good and private consumption) among those alive in period *t* and the difference in the marginal value attached to the public good between low-ability-type and the mimicker (measured both for the young and old) in period *t*, respectively. To facilitate later interpretations, we assume that $MB_{t,G}$ is decreasing in G_t .¹² We are nowable to derive the following result:

Proposition 1. The optimal provision of the public good is characterized by

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} + \Omega_{t+\tau} - \frac{MB_{t+\tau,G}}{N_{t+\tau}\gamma_{t+\tau}} \frac{\partial \mathcal{L}}{\partial \overline{c}_{t+\tau}} \right] \left[1 - \xi \right]^{\tau} = 1.$$
⁽¹⁹⁾

Following Aronsson and Johansson-Stenman (2010a), the partial derivative of the Lagrangean with respect to the reference consumption in period *t*, i.e. $\partial \mathcal{L} / \partial \overline{c}_t$, will be called the *positionality effect* in period *t*, and reflects the overall welfare consequences of an increase in \overline{c}_t , holding each individual's own consumption constant. As such, it is a measure of the "positional externality" of private consumption. While it is reasonable to expect $\partial \mathcal{L} / \partial \overline{c}_t$ to be negative, since for each individual $u_{t,\overline{c}_t}^i < 0$ and $u_{t,\overline{c}_{t-1}}^i < 0$, it is theoretically possible that it is positive due to effects through the self-selection constraint to be discussed more thoroughly in the following sections.

Before interpreting Proposition 1 further, let us first consider the special case where $\xi = 1$, in which the state-variable public good is equivalent to an atemporal control (or flow) variable, i.e. $G_t = g_t$, so that we can simplify equation (19) and obtain:¹³

¹² A sufficient – yet not necessary – condition for this property to hold is that private and public consumption, if measured in the same period, are weak complements in the utility function.

¹³ For thorough discussions of public good provision in static economies with asymmetric information, although without relative consumption concerns, see Christiansen (1981) and Boadway and Keen (1993).

Corollary 1. *If the public good is a flow variable, so that* $\xi = 1$ *, then the optimal provision of the public good satisfies*

$$MB_{t,G} + \Omega_t - \frac{MB_{t,G}}{N_t \gamma_t} \frac{\partial \mathcal{L}}{\partial \overline{c}_t} = 1$$
(20)

Equation (20) is analogous to the formula for public provision derived in a static model by Aronsson and Johansson-Stenman (2008). The right-hand side is the direct marginal cost of providing the public good, which is measured as the marginal rate of transformation between the public good and the private consumption good and is normalized to one, whereas the lefthand side is interpretable as the marginal benefit of the public good adjusted for the influences of the self-selection constraint and positional preferences, respectively. With a flow-variable public good, the main differences between a static model and the intertemporal model analyzed here are that the self-selection effect and positionality effect relevant for public provision in period t reflect the incentives facing generations t and t-1, as the highability type in each of these generations may act as a mimicker in period t.

Let us now return to the case with a state variable public good, i.e. where $\xi < 1$. Equation (19) essentially combines the policy rule for a state-variable public good in an OLG model without positional preferences derived by Pirttilä and Tuomala (2001), with the policy rule for a flow-variable public good summarized by equation (20). Again, the right-hand side is the direct marginal cost of a small increase in the contribution to the public good and the private consumption good, whereas the left-hand side measures the marginal benefit of an increase in the contribution to the public good and the self-selection constraint and positional preferences, respectively. Note that this measure of adjusted marginal benefit is intertemporal as an increase in g_t , *ceteris paribus*, affects the utility of each ability-type, as well as the self-selection constraint and the welfare the government attaches to increased reference consumption, in all future periods.

7. Optimal Provision Rules with Keeping-up-with-the-Joneses Preferences

The positionality effect included in equation (19) is crucial for our understanding of how the incentives underlying public provision depend on the relative consumption concerns. In this

section, we assume that the positional preferences are of the keeping-up-with-the-Joneses type, meaning that each individual derives utility from his/her own current consumption relative to the current average consumption in the in the economy as a whole, and that each individual makes this comparison both when young and when old. As indicated above, we abstract from catching-up-with-the-Joneses comparisons here; such comparisons are addressed in Section 8 below. This simplification means that the variables $\delta_t^{i,c} \equiv c_t^i - \overline{c_{t-1}}$, and $\delta_{t+1}^{i,x} \equiv x_{t+1}^i - \overline{c_t}$ vanish from equation (1) and, as a consequence, that the intertemporal degrees of positionality are equal to zero.

When the preferences are of the keeping-up-with-the-Joneses type, the positionality effect only reflects current degrees of positionality. Let α_t^d be a summary measure of differences in the current degree of positionality between the mimicker and the low-ability type in period *t* such that

$$\alpha_t^d \equiv \frac{\lambda_{t-1}\hat{u}_{t-t,x}^2}{\gamma_t N_t} \Big[\hat{\alpha}_t^{2,x} - \alpha_t^{1,x} \Big] + \frac{\lambda_t \hat{u}_{t,c}^2}{\gamma_t N_t} \Big[\hat{\alpha}_t^{2,c} - \alpha_t^{1,c} \Big],$$

where the symbol "^" denotes "mimicker" (as before), while the superindex "d" stands for "difference." Thus, α_t^d reflects an aggregate measure of the positionality differences between the young mimicker and the young low-ability type and between the old mimicker and the old low-ability type, respectively. Consequently, $\alpha_t^d > 0$ (<0) if the mimicker is always, i.e. both when young and old, more (less) positional than the low-ability type. Following Aronsson and Johansson-Stenman (2010a), the positionality effect associated with the keeping-up-with-the-Joneses type of positional preferences can be written as follows;

$$\frac{\partial \mathcal{L}}{\partial \overline{c}_t} = -N_t \gamma_t \frac{\overline{\alpha}_t - \alpha_t^d}{1 - \overline{\alpha}_t}.$$
(21)

Therefore, the overall welfare effects of an increase in the level of reference consumption in period *t*, *ceteris paribus*, contains two components. The first is the average degree of current positionality, $\overline{\alpha}_t$, which contributes to decrease the right hand side of equation (21). This negative effect arises because the utility facing each individual of generation *t* depends negatively on \overline{c}_t via the argument $c_t^i - \overline{c}_t$ in the utility function, and the utility facing each

individual of generation *t*-1 depends negatively on \overline{c}_t via the argument $x_t^i - \overline{c}_t$. As such, the average degree of current positionality reflects the importance of the positional externality. The second component in equation (21), α_t^d , appears because the mimicker and the (mimicked) low-ability type typically differs with respect to the degree of positionality, which the government may exploit to relax the self-selection constraint. If $\alpha_t^d > 0$ (<0), increased reference consumption in period *t* leads to a relaxation (tightening) of the self-selection constraint, as it means that the mimicker is hurt more (less) than the low-ability type. As a consequence, this effect may either counteract ($\alpha_t^d > 0$) or reinforce ($\alpha_t^d < 0$) the negative positional consumption externality.

By substituting equation (21) into equation (19), we can derive the following result:

Proposition 2. The optimal provision of the public good based on keeping-up-with-the-Joneses preferences is characterized by

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} \frac{1-\alpha_{t+\tau}^d}{1-\overline{\alpha}_{t+\tau}} + \Omega_{t+\tau} \right] \left[1-\xi \right]^{\tau} = 1.$$
(22)

The interesting aspect of Proposition 2 is that the effects of positional concerns are captured by a single multiplier, $(1-\alpha_{t+\tau}^d)/(1-\overline{\alpha}_{t+\tau})$, which is interpretable as the "positionalityweight" in period $t + \tau$. The average degree of positionality, $\overline{\alpha}_{t+\tau}$, contributes to scale up the aggregate instantaneous marginal benefit and, therefore, increases the provision of the public good. As explained above, the effect of $\alpha_{t+\tau}^d$ (the measure of differences in the degree of positionality between the mimicker and the low-ability type) can be either positive or negative. If $\alpha_{t+\tau}^d > 0$, this mechanism contributes to scale down the marginal benefit of public consumption in period $t + \tau$. The intuition is, of course, that additional resources spent on private consumption leads to a relaxation of the self-selection constraint in this case (as the mimicker is more positional than the low-ability type). If $\alpha_{t+\tau}^d < 0$, on the other hand, this mechanism works in this opposite direction., Therefore, a sufficient (not necessary) condition for the positionality weight in period $t + \tau$ to scale up the aggregate instantaneous marginal benefit of the public good in that period is that $\alpha_{t+\tau}^d \leq 0$, meaning that the low-ability type is at least as positional as the mimicker. In the Appendix, we derive the following result more generally:

Proposition 3. A neccessary and sufficient condition for the joint impact of present and future positionality effects to increase the contribution to the public good in period t is that

$$\sum_{\tau=0}^{\infty} MB_{t+\tau,G} \frac{\overline{\alpha}_{t+\tau} - \alpha_{t+\tau}^{d}}{1 - \overline{\alpha}_{t+\tau}} [1 - \xi]^{\tau} > 0.$$

Hence, a sufficient condition is that the low-ability type is predominantly at least as positional as the mimicker in the sense that

$$\sum_{\tau=0}^{\infty} MB_{t+\tau,G} \frac{\alpha_{t+\tau}^d}{1-\overline{\alpha}_{t+\tau}} \left[1-\xi\right]^{\tau} < 0.$$

Note that even though the second condition in Proposition 3 is much stronger than the first, it still does not require the low-ability types to be at least as positional as the mimickers in all periods. Instead, as long as the low-ability type is predominantly as least as positional as the mimicker – which imposes a condition on a weighted average of future differences in the degree of positionality – this is perfectly consistent with the possibility that the mimicker is more positional than the low-ability type during certain periods or intervals of time.

Let us next consider conditions for when the second-best adjustments through the impacts on the self-selection constraints, i.e. the effects of the variables Ω_t and α_t^d for all *t*, vanish from the policy rule for public provision. We have derived the following result;

Proposition 4. If leisure is weakly separable from private and public consumption in the sense that the utility function can be written as $U_t^i = q_t^i(h_t(c_t^i, x_{t+1}^i, \Delta_t^{i,c}, \Delta_{t+1}^{i,x}, G_t, G_{t+1}), z_t^i)$ for all *t*, then the optimal policy rule for the public good is given by

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \frac{MB_{t+\tau,G}}{1-\overline{\alpha}_{t+\tau}} \left[1-\xi\right]^{\tau} = 1.$$
(23)

Note that while we still allow for type-specific preferences, here the function $h_t(\cdot)$ is common to all consumers of generation t. Equation (23) resembles a result derived by Pirttilä and Tuomala (2001) in a model without relative consumption concerns, in which they were able to show that leisure separability implies that the provision of the public good is governed by what resembles an intertemporal analogue to the Samuelson condition. Their result is modified here because the policy rule still reflects the desire to correct for positional externalities, which works to increase the marginal benefit of the public good (since $1/(1-\overline{\alpha}_t) > 1$ for all t by assumption). The intuition behind Proposition 4 is that if leisure is weakly separable from the other goods in the utility function – and with the additional restriction that the function $h_t(\cdot)$ is common for the two ability-types – it follows that $\Omega_t = \alpha_t^d = 0$ for all t, i.e. neither the marginal willingness to pay for the public good nor the

degree of positionality differs between the mimicker and the low-ability type. As a consequence, there is no longer an incentive for the government to modify the provision of the public good to relax the self-selection constraint.

Following Aronsson and Johansson-Stenman (2008), it is interesting to consider the role of preference elicitation for the public good. Note first that individual benefits of the public good are so far measured by each individual's marginal willingness to pay for a small increment, ceteris paribus, i.e. while holding everything else, including others' private consumption, fixed. At the same time, increased public provision typically comes together with other changes, notably that one's own as well as other people's taxes or charges are increased. In one frequently used method, the contingent valuation method, it is typically recommended (see Arrow et al. 1993) that a realistic payment vehicle is used when asking people about their maximum willingness to pay. One commonly used payment vehicle is to ask subjects how they would vote in a referendum where everybody would have to pay a certain amount, the same for all, through increased taxes (or charges) for the improvement. In the standard case where people do not care about relative consumption, this formulation has no important theoretical implication. Here, however, it does. To see this, let us define the marginal rate of substitution between the public good and private consumption at any time, t, conditional on the requirement that $c_t^i - \overline{c_t}$ and $x_{t+1}^i - \overline{c_{t+1}}$ remain constant, which would follow if the willingness to pay question were supplemented by the information that everybody has to pay the same amount for an incremental public good.

With reference to equation (1), this measure of marginal willingness to pay, conditional on that others would have to pay equally much on the margin, can then be defined as follows:

Definition. An individual's conditional marginal willingness to pay for the public good when young and old, respectively, is defined by:

$$CMRS_{G,c}^{i,t} \equiv \frac{v_{t,G_t}^i}{v_{t,c}^i},$$
(24a)

$$CMRS_{G,x}^{i,t} = \frac{v_{t-1,G_t}^i}{v_{t-1,x}^i}.$$
(24b)

In a way similar to equation (17), we may construct an aggregate marginal benefit measure consisting of the sum of all people's (alive in period t) marginal willingness to pay for the public good, conditional on that others will also have to pay equally much on the margin, as follows:

$$CMB_{t,G} \equiv \sum_{i} n_{t}^{i} CMRS_{G,c}^{i,t} + \sum_{i} n_{t-1}^{i} CMRS_{G,x}^{i,t} .$$
(25)

The question is then how the optimal provision rule will change if expressed as a function of $CMB_{t,G}$ instead of $MB_{t,G}$? By using

$$\Psi_{t} \equiv \operatorname{cov}\left(\frac{1-\alpha_{t}}{1-\overline{\alpha}_{t}}, \frac{CMRS_{G,c}^{t}}{CMRS_{G,c}^{t}}\right)$$

as a short notation for the (normalized) covariance between the degree of non-positionality, measured by $1-\alpha_t^i$, and the marginal willingness to pay for the public good, we have:

Proposition 5. *Based on keeping-up-with-the-Joneses preferences, and expressed in terms of conditional marginal WTPs, the optimal policy rule for public good provision is given by*

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \Big[CMB_{t+\tau,G} \Big[1 + \Psi_t \Big] \Big[1 - \alpha_{t+\tau}^d \Big] + \Omega_{t+\tau} \Big] \Big[1 - \xi \Big]^{\tau} = 1.$$
(26)

Compared to equation (22), we can observe two differences (in addition to the replacement of $MB_{t+r,G}$ by $CMB_{t+r,G}$): First and foremost, the benefit-amplifying factor $1/(1-\overline{\alpha}_{t+r})$ is not part of equation (26). The intuition is straightforward. If others' consumption is held constant, each individual's willingness to pay for increased public good provision would be reduced by the fact that his/her relative consumption decreases. However, if each individual's relative consumption is held constant, as in Proposition 5, there is obviously no such effect. Second, equation (26) include a factor $[1+\Psi_t]$, for which the intuition can be given as follows. If the conditional marginal WTP differs between the types, and all people will have to pay the same amount on the margin, then those with a higher conditional marginal WTP will obtain a utility increase, while the others will face a utility loss. The utility increase, in monetary terms, will more than outweigh the utility loss if and only if the covariance between the degree of non-positionality and the conditional marginal WTP is positive.

It is finally interesting to analyze whether there is some special case in which the second-best policy rule for the public good reduces to a first-best policy rule. It turns out that there is, and the following result gives sufficient conditions under which an intertemporal analogue to the Samuelson rule applies:

Proposition 6. If - in addition to the conditions underlying Proposition 4 - we assume that (i) the degree of positionality is the same for both ability-types in all periods, both when young and when old, and (ii) the market interest rate is constant over time, then the optimal provision of the public good, expressed in terms of conditional marginal WTPs, is given by

$$\sum_{\tau=0}^{\infty} \frac{CMB_{t,G}}{(1+r)^{\tau}} \left[1-\xi\right]^{\tau} = 1.$$
(27)

Given that the degree of current consumption positionality is the same for everybody (in which case there is no correlation between the marginal willingness to pay and the degree of current consumption positionality at the individual level), then a weighted sum over time of instantaneous marginal benefits should equal the marginal cost of an incremental public good. In other words, an intertemporal analogue to the traditional Samuelson rule applies. What is less clear, perhaps, is how we should apply this or any other policy rule in practice,

as we would need information about the willingness to pay for the public good by future generations. However, before taking the discussion about implementation to any greater detail, it is important to know the point of departure.

Note also that in the special case where $\xi = 1$, i.e. the case where the public good is a flow variable, Proposition 6 implies:

Corollary 2. In addition to the conditions governing Proposition 6 (except for a constant interest rate which is not needed here), suppose that the public good is a flow variable, so that $\xi = 1$. The policy rule for the public good then simplifies to read

$$CMB_{t,G} = 1. \tag{28}$$

Corollary 1 thus implies that the conventional Samuelson (1954) rule, expressed in marginal willingness to pay conditional on the fact that also others have to pay on the margin, holds for each moment in time.

8. Optimal Provision of the Public Good under both keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences

The analysis carried out in earlier sections is based on the assumption that the only measure of reference consumption at the individual level, in any period, is based on the average consumption in that particular period. Although this idea accords well with earlier literature on public policy and positional preferences, it neglects the possibility that agents also compare their own current consumption with both their own past consumption and that of other people. In this section, we will present and analyze the more general model that takes all these comparisons into account.

Note once again that equation (19) holds generally, i.e. irrespective of which form the relative consumption concerns take. To be able to consider keeping-up-with-the-Joneses preferences simultaneously with catching-up-with-the-Joneses preferences (remember that comparisons with one's own previous consumption do not affect the provision rules), we must explore the positionality effect, $\partial \mathcal{L} / \partial \overline{c_t}$, for this more general case. By analogy to the variable α_t^d

defined in Section 7, which is a summary measure of differences in the degree of current positionality between the mimicker and the low-ability type in period t, we also define a corresponding measure of differences in the degree of intertermporal positionality between the mimicker and the low-ability type,

$$\beta_t^d = \frac{\lambda_{t-1}\hat{u}_{t-t,x}^2}{\gamma_t N_t} \Big[\hat{\beta}_t^{2,x} - \beta_t^{1,x} \Big] + \frac{\lambda_t \hat{u}_{t,c}^2}{\gamma_t N_t} \Big[\hat{\beta}_t^{2,c} - \beta_t^{1,c} \Big]$$

with the same general interpretation as α_t^d . In other words, $\beta_t^d > 0$ (<0) if the young and old mimicker in period *t* are more (less) positional than the corresponding low-ability type, where positionality is measured in the intertemporal dimension, i.e. relative to other people's past consumption. To simplify the notation and facilitate comparison with equation (21), we use the following short notation:

$$B_{t} = \frac{N_{t}\gamma_{t}[\alpha_{t}^{d} - \overline{\alpha}_{t}]}{1 - \overline{\alpha}_{t}} + \frac{N_{t+1}\gamma_{t+1}[\beta_{t+1}^{d} - \overline{\beta}_{t+1}]}{1 - \overline{\alpha}_{t}}.$$

We show in the Appendix (along with the proof of Proposition 7 below) that the positionality effect associated with this more general model can be written as

$$\frac{\partial \mathcal{L}}{\partial \overline{c}_{t}} = B_{t} + \sum_{i=1}^{\infty} B_{t+i} \prod_{j=1}^{i} \frac{\overline{\beta}_{t+j}}{1 - \overline{\alpha}_{t+j-1}} \,.$$
⁽²⁹⁾

The variable B_t in equation (29) is analogous to the right hand side of equation (21) with the modification that it also reflects the intertemporal (not just the current) degrees of positionality. As such, it contains two additional components. First, $-N_{t+1}\gamma_{t+1}\overline{\beta}_{t+1}/(1-\overline{\alpha}_t) < 0$ is interpretable as the value of the positional externality associated with the catching-up-with-the-Joneses motive for relative consumption comparisons. The underlying mechanism is, of course, that \overline{c}_t directly affects individual utility negatively via the argument $x_{t+1}^i - \overline{c}_t$ in the utility function. Second, the component $N_{t+1}\gamma_{t+1}\beta_{t+1}^d/(1-\overline{\alpha}_t)$ reflects the corresponding welfare effects through the self-selection mechanism in period t+1. In a way similar to the analogous measure of differences in the current degree of positionality between the mimicker and the low-ability type, this effect means that increased reference consumption in period t may either contribute to relax ($\beta_{t+1}^d > 0$) or tighten ($\beta_{t+1}^d < 0$) of the self-selection constraint. The final component on the right hand side of equation (29) arises due to an intertemporal chain reaction: the intuition is that the catching-up-with-the-Joneses motive for consumption

comparisons, i.e., that other people's past consumption affects utility, means that the welfare effects of changes in the reference consumption are no longer time-separable (as they would be without intertemporal consumption comparisons).

By substituting equation (29) into equation (19), we can derive the following result:

Proposition 7. The optimal provision of the public good based on keeping.-up-with-the-Joneses and catching-up-with-the-Joneses preferences is characterized by

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} + \Omega_{t+\tau} - \frac{MB_{t+\tau,G}}{N_{t+\tau}\gamma_{t+\tau}} \left[B_t + \sum_{i=1}^{\infty} B_{t+i} \prod_{j=1}^{i} \frac{\overline{\beta}_{t+j}}{1 - \overline{\alpha}_{t+j-1}} \right] \right] \left[1 - \xi \right]^{\tau} = 1.$$
(30)

The basic intuition behind Proposition 7 is analogous to that of Proposition 2; yet with the modification that the catching-up-with-the-Joneses motive for consumption comparisons is present in equation (30). This means that (i) increases in the average degrees of positionality (both in the current and intertemporal dimensions) typically contribute to increased provision of the public good, and (ii) differences in the degrees of positionality between the mimicker and the low-ability type contribute to increase (decrease) the optimal provision of the public good if the low-ability type is predominantly more (less) positional than the mimicker is both dimensions.

At the same time, equation (30) is not very tractable, due to the intertemporal chain reaction caused by the catching-up-with-the-Joneses motive for relative consumption, and the interpretation of its different components is far from obvious. Yet, by making some additional simplifying assumptions, we are able to simplify the positionality effect given by equation (29) considerably, and also derive a policy rule for public provision that takes the same form as equation (22).

Assumptions A. $\overline{\alpha}_t = \overline{\alpha}$, $\overline{\beta}_t = \overline{\beta}$, $\alpha_t^d = \alpha^d$, $\beta_t^d = \beta^d$, $N_t = N$ and $r_t = r \quad \forall t$

In other words, our measures of degrees of average positionality and positionality differences between the types of people as well as the population size and the interest rate are assumed to be constant over time. While these are of course important restrictions, they are hardly very strong assumptions, and similar assumptions are frequently made in the comparable catchingup-with-the-Joneses literature.¹⁴ It should also be noted that the model is still general enough to reflect different preferences between ability-types types, including different degrees of positionality.

Let us now define the average degree of *total* consumption positionality and the difference in the degree of total consumption positionality between the mimicker and the low-ability type, respectively, in present value terms as

$$\overline{\rho} \equiv \overline{\alpha} + \frac{\overline{\beta}}{1+r},$$
$$\rho^d \equiv \alpha^d + \frac{\beta^d}{1+r}.$$

Therefore, the average degree of total consumption positionality is measured as the average degree of current consumption positionality plus the present value of the average degree of intertemporal consumption positionality; the reason for calculating the present value is, of course, that the intertemporal externality caused in period *t* gives rise to disutility in period *t*+1. In a similar way, the measure of differences in the degree of total consumption positionality between the mimicker and the low-ability type, ρ^d , reflects differences in the degree of current consumption positionality, α^d , and the (present value of) differences in intertemporal positionality, $\beta^d / (1+r)$. We can then, under Assumptions A, show (see the Appendix) that equation (29) reduces to read

$$\frac{\partial \mathcal{L}}{\partial \overline{c}_t} = N \gamma_t \frac{\rho^d - \overline{\rho}}{1 - \overline{\rho}},$$

which takes the same general form as in the absence of the catching-up-with-the-Joneses motive for relative consumption, i.e. as equation (21). This implies that we are able to present straightforward extensions of Propositions 2-6 and Corollary 2 to the more general case, where we also consider catching-up-with-the-Joneses preferences:

Proposition 2'. Under Assumptions A, and based on keeping.-up-with-the-Joneses and catching-up-with-the-Joneses preferences, the optimal provision of the public good is characterized by

¹⁴ See, e.g., Campbell and Cochrane (1999) and Díaz et al. (2003).

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} \frac{1-\rho^d}{1-\overline{\rho}} + \Omega_{t+\tau} \right] [1-\xi]^{\tau} = 1.$$
(31)

Technically, the only difference compared to equation (22) is that the positionality-weight based on current degrees, $(1-\alpha^d)/(1-\overline{\alpha})$, is here replaced by a corresponding positionality-weight based on total degrees, $(1-\rho^d)/(1-\overline{\rho})$. The intuition is that comparisons with other people's past consumption imply that the (young and old) individuals alive today impose a negative positional externality on the individuals alive in the next period, i.e. the higher the consumption in period *t*, ceteris paribus, the greater will be the utility loss due to lower relative consumption in period *t*+1. As such, this intertemporal externality must be considered simultaneously with the (atemporal) externality that affects others today. Due to Assumptions A, the striking implication of Proposition 2' is that the current and intertermporal aspects of consumption positionality affect the incentives for public good provision in *exactly* the same way. Therefore, the following results for when positional concerns lead to increased contributions to the public good are analogous to Proposition 3:

Proposition 3'. Under Assumptions A, and based on keeping.-up-with-the-Joneses and catching-up-with-the-Joneses preferences, a neccessary and sufficient condition for the joint impact of present and future positionality effects to increase the contribution to the public good in period t is that $\overline{\rho} - \rho^d > 0$. Hence, a sufficient condition is that the low-ability type is at least as positional as the mimicker in the sense that $\rho^d < 0$.

Again, the result from Section 7 carries over with the only modifications that $\overline{\alpha}$ and α^d are replaced by $\overline{\rho}$ and ρ^d , respectively. Let us then consider conditions for when the second-best adjustments through the impacts on the self-selection constraints vanish from the policy rule for public provision. We can derive the following analogue to Proposition 4:

Proposition 4'. Given the conditions underlying Proposition 3', and if leisure is weakly separable from private and public consumption in the sense that the utility function can be written as $U_t^i = q_t^i(h_t(c_t^i, x_{t+1}^i, \Delta_t^{i,c}, \Delta_{t+1}^{i,c}, \delta_t^{i,c}, \delta_{t+1}^{i,x}, G_t, G_{t+1}), z_t^i)$ for all t, then the optimal provision of the public good is characterized by

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \frac{MB_{t+\tau,G}}{1-\overline{\rho}} [1-\xi]^{\tau} = 1.$$
(32)

Note that the separability condition in this case also includes the variables reflecting consumption comparisons over time. Consequently, the conditions for when the second-best adjustments through the impacts on the self-selection constraints vanish also carry over to this more general case. Note also that $\bar{\rho}$ still remains in equation (32); the intuition is, of course, that the government has an incentive to correct for (current and intertemporal) positional externalities, even if it is unable to use the public good as an instrument the relax the self-selection constraint.

As a final concern, let us once again examine the payment vehicle, where we ask subjects how they would vote in a referendum where everybody have to pay the same amount for increased public provision. What are the implications of adding the catching-up-with-the-Joneses preferences to the model analyzed in Section 7? By using

$$\Lambda_{t} = \operatorname{cov}\left(\frac{1 - \alpha_{t} - \beta_{t}}{1 - \overline{\alpha}_{t} - \overline{\beta}_{t}}, \frac{CMRS_{G,c}^{t}}{\overline{CMRS}_{G,c}^{t}}\right)$$

to denote the (normalized) covariance between the total degree of non-positionality, measured by $1 - \alpha_t - \beta_t$, and the conditional marginal WTP for the public good, we have the following analogue to Proposition 5:

Proposition 5'. The optimal provision of the public good based on the keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences and Assumptions A, and expressed in terms of conditional marginal WTPs, can be characterized as

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[CMB_{t,G} \frac{1-\overline{\alpha}-\overline{\beta}}{1-\overline{\rho}} \left[1+\Lambda_t\right] \left[1-\rho^d\right] + \Omega_{t+\tau} \left[1-\xi\right]^{\tau} = 1.$$
(33)

There is one important difference between Propositions 5 and 5'. If all relative consumption concerns are governed by the keeping-up-with-the-Joneses motive, as in Proposition 5, then the average degree of consumption positionality vanishes from the policy rule for the contribution to the public good. In equation (26), therefore, there was no incentive to modify

the formula for public provision in order to correct for positional externalities. This result no longer applies in equation (33), since $(1 - \overline{\alpha} - \overline{\beta})/(1 - \overline{\rho}) < 1$. Although the marginal WTPs for each generation are measured with all aspects of relative consumption held constant at the individual level, discounting of intertemporal positionality degrees tends to reduce the social cost of increased reference consumption. As a consequence, if the relative consumption concerns (or parts thereof) are driven by a catching-up-with-the-Joneses motive, there is an incentive for the government to reduce the contribution to the public good, ceteris paribus, to reach the optimal level of correction for positional externalities. The intuition is that the catching-up-with-the-Joneses type of externality is characterized by a lime-lag between cause and effect and must, therefore, be internalized before the welfare loss actually surfaces; e.g., because each individual's consumption in period t leads to positional externalities in period t+1. The social cost of spending one additional dollar on public consumption in period t, relative to spending it in period t+1, is given by $\gamma_t / \gamma_{t+1} = 1 + r$. As such, if the government at any time t plans to internalize a positional externality in period t+1, it is more costly to do so if this externality is generated by a catching-up-with-the-Joneses comparison than a keepingup-with-the-Joneses comparison, which explains why externality-correction leads to a smaller contribution to the public good in equation (33) than in equation (26), ceteris paribus.

For the same reason, adding a catching-up-with-the-Joneses motive for relative consumption also implies that we have to modify the results presented in Proposition 6 and Corollary 2, the analogues of which are presented as follows:

Proposition 6'. If - in addition to the conditions in Proposition 4' - we assume that the degree of current and intertemporal positionality, respectively, does not differ among ability-types, neither for young nor for old individuals, then the optimal provision of the public good, expressed in terms of conditional marginal WTPs, is given by

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} CMB_{t,G} \frac{1-\overline{\alpha}-\overline{\beta}}{1-\overline{\rho}} [1-\xi]^{\tau} = 1.$$
(34)

Corollary 2'. *If* - *in addition to the conditions in Proposition* 6' - *the public good is a flow variable, so that* $\xi = 1$ *, then the optimal provision of the public good is given by*

$$CMB_{t,G} \frac{1 - \overline{\alpha} - \overline{\beta}}{1 - \overline{\rho}} = 1.$$
(35)

Equation (35) is interpretable as an analogue to the basic Samuelson rule, expressed in terms of marginal willingness to pay conditional on that others will also have to pay the same amount for the public good on the margin. Note also that if the interest rate is small enough, meaning that the scale factor on the left hand side of equation (35) is close to one, then the conventional Samuelson condition would still provide a reasonable rule-of-thumb for public provision. This insight is clearly remarkable, since we simultaneously consider (i) a second-best problem with asymmetric information between the government and the private sector, (ii) distributional concerns, (iii) keeping-up-with-the-Joneses preferences, (iv) catching-up-with-the-Joneses preferences and (v) internal habit formation. Yet, needless to say, the fact that we are not able to claim that any of the assumptions underlying Corollary 2' are biased in a certain direction does of course not mean that they constitute good approximations to the real world.

9. Conclusion

As far as we know, the present paper is the first to consider public good provision in a dynamic second-best economy with asymmetric information under optimal taxation, where people care about relative consumption. The model used is an extension of the standard optimal nonlinear income tax model with two ability-types. Our approach recognizes three mechanisms behind the positional concerns: each individual compares his/her current consumption with (i) his/her own past consumption, (ii) other people's current consumption (keeping-up-with-the-Joneses), and (iii) other people's past consumption (catching-up-with-the-Joneses).

We began by analyzing the simple case where the comparison with other people's consumption is limited to their current consumption. This situation enabled us to derive several distinct results with respect to the consequences of positional preferences for the optimal public provision rule.

As the public good in our model is a state variable, the effects of positional preferences are more complex than in the static model analyzed by Aronsson and Johansson-Stenman (2008).

The reason is that the marginal benefit of an incremental contribution to the public good in period t is intertemporal (it reflects the present value of all future instantaneous marginal benefits), meaning that it is governed by the preferences of the current and all future generations. If an individual's marginal willingness to pay for the public good is measured by holding the contributions made by others constant, it follows that the more positional people are on average now and in the future, *ceteris paribus*, the larger the optimal contributions to the public good compared to the case where relative consumption comparisons are absent. However, it also matters whether the low-ability type is more or less positional than the mimicker (both at present and in the future), as this determines whether an incremental contribution to the public good in period t relaxes or tightens the self-selection constraint.

We also show that the adjustment of the formula for public provision implied by relative consumption concerns depends on whether each individual's marginal willingness to pay is elicited by holding everything else constant, or by using a payment vehicle implying that each individual knows that other agents also have to pay. If people's marginal willingness to pay for the public good is measured independently, i.e. without considering that other people also have to pay for increased public provision, then relative consumption concerns typically work in the direction of increasing the optimal provision of the public good. Yet, this is not the case when a referendum format is used where people are asked for their marginal willingness to pay conditional on the fact that others will also have to pay for the increased public provision. In the latter case, additional conditions are presented for when a dynamic analogue of the conventional Samuelson (1954) rule applies.

Adding the intertemporal aspects of relative consumption comparisons, i.e. that each individual also compares his/her current consumption with his/her own and others' past consumption, gives a richer structure, as it enables us to distinguish between the current and intertemporal degrees of consumption positionality. Although a catching-up-with-the-Joneses motive for relative consumption gives rise to the same basic policy incentives as those caused by a keeping-up-with-the-Joneses motive, comparisons with others' past consumption makes the analysis more complex, as the welfare effects of a change in the reference consumption in period t effectively become dependent on the preferences of all future generations. Still, for the case where the degrees of (current and intertemporal) consumption positionality are constant over time, and with some additional assumptions, we derive a set of distinct results for public good provision when the relative consumption concerns are governed both by the

keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences. Here, the total degree of consumption positionality plays the same general role as the current degree of positionality does when all relative consumption concerns are driven by the keeping-up-with-the-Joneses type of preferences. As a consequence, the incentives created by relative consumption concerns depend on the average degree of total consumption positionality, differences between the mimicker and the (mimicked) low-ability type with respect to the degree of total consumption positionality, as well as on the method for eliciting the marginal willingness to pay for the public good.

Let us finally return to the problem of climate change. The results here show that relative consumption concerns, whether in comparison with others presently living or with others' previous consumption, do have important implications for the calculations of future costs and benefits of climate change (i.e. the change of the state-variable public good called the climate). However, it is also demonstrated that it is possible to use the conventional (i.e. without relative consumption concerns) dynamic (second-best) cost-benefit model provided that the individual future costs and benefits associated with climate change are calculated such that each individual's relative consumption is held fixed).¹⁵

The present paper has in several respects generalized the literature on optimal public expenditure when relative consumption matters. Yet, there are still many important aspects left to explore. Examples include public provision of private goods, heterogeneous relative consumption concerns (e.g., that people may compare themselves more with their own ability-type), a multi-country setting, and the case where agents also have positional preferences for public consumption. We hope to address these issues in future research.

Appendix

First-order conditions

The first-order conditions for l_t^1 , c_t^1 , x_{t+1}^1 , l_t^2 , c_t^2 , x_{t+1}^2 , K_{t+1} , G_t and g_t are given by

¹⁵ The practical problems associated with calculating such costs and benefits for future generations are of course immense, and beyond the scope of this paper.

$$\frac{\partial W}{\partial (n_t^1 U_t^1)} n_t^1 u_{t,z}^1 + \lambda_t \hat{u}_{t,z}^2 \phi_t + \gamma_t n_t^1 w_t^1 = 0, \qquad (A1)$$

$$\frac{\partial W}{\partial (n_t^1 U_t^1)} n_t^1 u_{t,c}^1 - \lambda_t \hat{u}_{t,c}^2 - \gamma_t n_t^1 + \frac{n_t^1}{N_t} \frac{\partial \mathcal{L}}{\partial \overline{c_t}} = 0, \qquad (A2)$$

$$\frac{\partial W}{\partial (n_t^1 U_t^1)} n_t^1 u_{t,x}^1 - \lambda_t \hat{u}_{t,x}^2 - \gamma_{t+1} n_t^1 + \frac{n_t^1}{N_{t+1}} \frac{\partial \mathcal{L}}{\partial \overline{c}_{t+1}} = 0, \qquad (A3)$$

$$-\left[\frac{\partial W}{\partial (n_t^2 U_t^2)}n_t^2 + \lambda_t\right]u_{t,z}^2 + \gamma_t n_t^2 w_t^2 = 0, \qquad (A4)$$

$$\left[\frac{\partial W}{\partial (n_t^2 U_t^2)} n_t^2 + \lambda_t\right] u_{t,c}^2 - \gamma_t n_t^2 + \frac{n_t^2}{N_t} \frac{\partial \mathcal{L}}{\partial \overline{c}_t} = 0, \qquad (A5)$$

$$\left[\frac{\partial W}{\partial (n_t^2 U_t^2)} n_t^1 + \lambda_t\right] u_{t,x}^2 - \gamma_{t+1} n_t^2 + \frac{n_t^2}{N_{t+1}} \frac{\partial \mathcal{L}}{\partial \overline{c}_{t+1}} = 0, \qquad (A6)$$

$$\gamma_{t+1}(1+r_{t+1}) - \gamma_t = 0, \qquad (A7)$$

$$\sum_{i=1}^{2} \left[\frac{\partial W}{\partial (n_{t}^{i} U_{t}^{i})} n_{t}^{i} u_{t,G_{t}}^{i} + \frac{\partial W}{\partial (n_{t-1}^{i} U_{t-1}^{i})} n_{t-1}^{i} u_{t-1,G_{t}}^{i} \right] + \lambda_{t} \left[u_{t,G_{t}}^{2} - \hat{u}_{t,G_{t}}^{2} \right] , \qquad (A8)$$
$$+ \lambda_{t-1} \left[u_{t-1,G_{t}}^{2} - \hat{u}_{t-1,G_{t}}^{2} \right] + \mu_{t+1} (1 - \xi) - \mu_{t} = 0$$
$$-\gamma_{t} + \mu_{t} = 0 , \qquad (A9)$$

where we have used that $w_t^i = F_{L^i}(L_t^1, L_t^2, K_t; t)$ for i=1,2, and $r_t = F_K(L_t^1, L_t^2, K_t; t)$ from equations (11) and (12), i.e. from the first-order conditions of the firm.

Proof of Proposition 1.

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We start by rewriting equation (A8) as

$$\sum_{i=1}^{2} \left[\frac{\partial W}{\partial (n_{t}^{i} u_{t}^{i})} n_{t}^{i} u_{t,c}^{i} MRS_{G,c}^{i,t} + \frac{\partial W}{\partial (n_{t-1}^{i} u_{t-1}^{i})} n_{t-1}^{i} u_{t-1,x}^{i} MRS_{G,x}^{i,t} \right]$$

$$+ \lambda_{t} \left[u_{t,G}^{2} - \hat{u}_{t,G}^{2} \right] + \lambda_{t-1} \left[u_{t-1,G_{t}}^{2} - \hat{u}_{t-1,G_{t}}^{2} \right] + \mu_{t+1} (1 - \xi) - \mu_{t} = 0$$
(A10)

Next we rewrite equations (A2), (A3), (A5), and (A6) as

$$\frac{\partial W}{\partial (n_t^1 U_t^1)} n_t^1 u_{t,c}^1 = \lambda_t \hat{u}_{t,c}^2 + \gamma_t n_t^1 - \frac{n_t^1}{N_t} \frac{\partial \mathcal{L}}{\partial \overline{c_t}}, \qquad (A11)$$

$$\frac{\partial W}{\partial (n_t^2 U_t^2)} n_t^2 u_{t,c}^2 = -\lambda_t u_{t,c}^2 + \gamma_t n_t^2 - \frac{n_t^2}{N_t} \frac{\partial \mathcal{L}}{\partial \overline{c_t}}, \qquad (A12)$$

$$\frac{\partial W}{\partial (n_{t-1}^{1}U_{t-1}^{1})} n_{t-1}^{1} u_{t-1,x}^{1} = \lambda_{t-1} \hat{u}_{t-1,x}^{2} + \gamma_{t} n_{t-1}^{1} - \frac{n_{t-1}^{1}}{N_{t}} \frac{\partial \mathcal{L}}{\partial \overline{c_{t}}}, \qquad (A13)$$

$$\frac{\partial W}{\partial (n_{t-1}^2 U_{t-1}^2)} n_{t-1}^2 u_{t,x}^2 = -\lambda_{t-1} v_{t,x}^2 + \gamma_t n_{t-1}^2 - \frac{n_{t-1}^2}{N_t} \frac{\partial \mathcal{L}}{\partial \overline{c_t}}.$$
 (A14)

Substituting equations (A11)-(A14) into equation (A10) we obtain

$$\begin{split} \gamma_{t} \Big[n_{t}^{1} MRS_{G,c}^{1,t} + n_{t}^{2} MRS_{G,c}^{2,t} + n_{t-1}^{1} MRS_{G,x}^{1,t} + n_{t-1}^{2} MRS_{G,x}^{2,t} \Big] \Big[1 - \frac{N_{t}}{\gamma_{t}} \frac{\partial \mathcal{L}}{\partial \overline{c}_{t}} \Big] \\ + \lambda_{t} \hat{u}_{t,c}^{2} \Big[MRS_{G,c}^{1,t} - M\hat{R}S_{G,c}^{2,t} \Big] + \lambda_{t-1} \hat{u}_{t-1,x}^{2} \Big[MRS_{G,x}^{1,t} - M\hat{R}S_{G,x}^{2,t} \Big] \quad .$$
(A15)
$$+ \mu_{t+1} \Big[1 - \xi \Big] - \mu_{t} = 0 \end{split}$$

Using the short notation

$$\Theta_{t} = \gamma_{t} \left[n_{t}^{1} MRS_{G,c}^{1,t} + n_{t}^{2} MRS_{G,c}^{2,t} + n_{t-1}^{1} MRS_{G,x}^{1,t} + n_{t-1}^{2} MRS_{G,x}^{2,t} \right] \left[1 - \frac{N_{t}}{\gamma_{t}} \frac{\partial \mathcal{L}}{\partial \overline{c_{t}}} \right] \\ + \lambda_{t} \hat{u}_{t,c}^{2} \left[MRS_{G,c}^{1,t} - M\hat{R}S_{G,c}^{2,t} \right] + \lambda_{t-1} \hat{u}_{t-1,x}^{2} \left[MRS_{G,x}^{1,t} - M\hat{R}S_{G,x}^{2,t} \right]$$

We have that $\mu_t = \Theta_t + \mu_{t+1} [1 - \xi]$ and hence $\mu_{t+1} = \Theta_{t+1} + \mu_{t+2} [1 - \xi]$ so that

$$\mu_{t} = \Theta_{t} + \left[\Theta_{t+1} + \mu_{t+2}\left[1-\xi\right]\right]\left[1-\xi\right]$$

$$= \Theta_{t} + \left[\Theta_{t+1} + \left[\Theta_{t+2} + \mu_{t+3}\left[1-\xi\right]\right]\left[1-\xi\right]\right]\left[1-\xi\right]$$

$$= \Theta_{t} + \Theta_{t+1}\left[1-\xi\right] + \Theta_{t+2}\left[1-\xi\right]^{2} + \dots = \sum_{\tau=0}^{\infty}\Theta_{t+\tau}\left[1-\xi\right]^{\tau} , \qquad (A16)$$

$$= \sum_{\tau=0}^{\infty} \gamma_{t+\tau} \left[MB_{t+\tau,G} + \Omega_{t+\tau} - \frac{MB_{t+\tau,G}}{N_{t+\tau}\gamma_{t+\tau}}\frac{\partial\mathcal{L}}{\partial\overline{c}_{t+\tau}}\right]\left[1-\xi\right]^{\tau}$$

where in the last step we have substituted back for Θ_t and used equations (17) and (18), i.e. the definitions of $MB_{t+\tau,G}$ and $\Omega_{t+\tau}$. Using finally that $\mu_t = \gamma_t$ from equation (A9), and dividing both sides of equation (A16) by γ_t , we obtain equation (19).

Proof of Corollary 1.

Since

$$\lim_{\xi \to 1} [1-\xi]^{\tau} = 0 \quad \text{for } \tau > 0 \text{ and } \lim_{\xi \to 1} [1-\xi]^{\tau} = 1 \quad \text{for } \tau = 0,$$

equation (20) follows immediately from equation (19).

Proof of Proposition 2.

We will here explore the positionality effect, and then substitute this into equation (19). The derivative of the Lagrangean with respect to \overline{c}_t is given by

$$\frac{\partial \mathcal{L}}{\partial \overline{c}_{t}} = \sum_{i=1}^{2} \frac{\partial W}{\partial (n_{t-1}^{i} U_{t-1}^{i})} n_{t-1}^{i} u_{t-1,\overline{c}_{t}}^{i} + \sum_{i=1}^{2} \frac{\partial W}{\partial (n_{t}^{i} U_{t}^{i})} n_{t}^{i} u_{t,\overline{c}_{t}}^{i} \\
+ \lambda_{t-1} \Big[u_{t-1,\overline{c}_{t}}^{2} - \hat{u}_{t-1,\overline{c}_{t}}^{2} \Big] + \lambda_{t} \Big[u_{t,\overline{c}_{t}}^{2} - \hat{u}_{t,\overline{c}_{t}}^{2} \Big]$$
(A17)

From equation (1) we have $u_{t,c}^{i} = v_{t,c}^{i} + v_{t,\Delta_{t}}^{i}$, $u_{t,\overline{c_{t}}}^{i} = -v_{t,\Delta_{t}}^{i}$, $u_{t,x}^{i} = v_{t,x}^{i} + v_{t,\Delta_{t+1}}^{i}$ and $u_{t,\overline{c_{t+1}}}^{i} = -v_{t,\Delta_{t+1}}^{i}$, so

$$u_{t,\overline{c}_{i}}^{i} = -\alpha_{t}^{i}u_{t,c}^{i}, \qquad (A18)$$

$$u_{t,\bar{c}_{t+1}}^{i} = -\beta_{t}^{i} u_{t,x}^{i} \,. \tag{A19}$$

Corresponding expressions hold for the mimicker. By combining equations (A17), (A18), and (A19), and the corresponding expressions for the mimicker, we obtain

$$\frac{\partial \mathcal{L}}{\partial \overline{c}_{t}} = -\sum_{i=1}^{2} \frac{\partial W}{\partial (n_{t-1}^{i} u_{t-1}^{i})} n_{t-1}^{i} \alpha_{t-1}^{i,x} u_{t-1,x}^{i} - \sum_{i=1}^{2} \frac{\partial W}{\partial (n_{t}^{i} u_{t}^{i})} n_{t}^{i} \alpha_{t}^{i,c} u_{t,c}^{i} \\ -\lambda_{t-1} \Big[\alpha_{t-1}^{2,x} u_{t-1,x}^{2} - \hat{\alpha}_{t-1}^{2,x} \hat{u}_{t-1,x}^{2} \Big] - \lambda_{t} \Big[\alpha_{t}^{2,c} u_{t,c}^{2} - \hat{\alpha}_{t}^{2,c} \hat{u}_{t,c}^{2} \Big]$$
(A20)

Substituting equations (A11)-(A14) into equation (A20) gives

$$\frac{\partial \mathcal{L}}{\partial \overline{c}_{t}} = -\frac{\overline{\alpha}_{t} \gamma_{t} N_{t}}{1 - \overline{\alpha}_{t}} + \frac{1}{1 - \overline{\alpha}_{t}} \left[\lambda_{t-1} \hat{u}_{t-1,x}^{2} \{ \hat{\alpha}_{t}^{2,x} - \alpha_{t}^{1,x} \} + \lambda_{t} \hat{u}_{t,c}^{2} \{ \hat{\alpha}_{t}^{2,c} - \alpha_{t}^{1,c} \} \right]
= N_{t} \gamma_{t} \frac{\alpha_{t}^{d} - \overline{\alpha}_{t}}{1 - \overline{\alpha}_{t}} , \quad (A21)$$

where in the last step we have used the definition of α_t^d . Substituting equation (A21) into equation (19) gives finally equation (21).

Proof of Proposition 3.

That the joint impact of present and future positionality effects increases the contribution to the public good in period t means that the benefit side is amplified by such effects compared to the optimal provision rule without such concerns, i.e. that

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} \frac{1-\alpha_{t+\tau}^d}{1-\overline{\alpha}_{t+\tau}} + \Omega_{t+\tau} \right] \left[1-\xi \right]^{\tau} > \sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} + \Omega_{t+\tau} \right] \left[1-\xi \right]^{\tau},$$

and hence

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} MB_{t+\tau,G} \frac{1-\alpha_{t+\tau}^d}{1-\overline{\alpha}_{t+\tau}} \left[1-\xi\right]^{\tau} - \sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} MB_{t+\tau,G} \left[1-\xi\right]^{\tau} > 0,$$

which directly implies the first inequality of Proposition 3. That the second inequality constitutes a sufficient condition for the first one is trivial since $\bar{\alpha}_t > 0$ for all *t*.

Proof of Proposition 4.

If leisure is weakly separable from private and public consumption as specified for all *t*, and where the sub-utility function $h_t(\cdot)$ is the same for both ability-types, then clearly $M\hat{R}S_{G,c}^{2,t} = MRS_{G,c}^{1,t}$ and $M\hat{R}S_{G,x}^{2,t} = MRS_{G,x}^{1,t}$, implying that $\Omega_t = 0$. Moreover, the positionality degrees will be the same for the mimicker and the low-ability type, implying that $\hat{\alpha}_t^{2,c} = \alpha_t^{1,c}$ and $\hat{\alpha}_t^{2,x} = \alpha_t^{1,x}$, so that $\alpha_t^d = 0$ for all *t*. Substituting $\alpha_t^d = 0$ and $\Omega_t = 0$ for all *t* into equation (21) implies equation (22).

Proof of Proposition 5.

Since $MRS_{G,c}^{i,t} = u_{t,G_t}^i / u_{t,c}^i$, $CMRS_{G,c}^{i,t} = v_{t,G_t}^i / v_{t,c}^i$ and $u_{t,G_t}^i = v_{t,G_t}^i$, it follows that $MRS_{G,c}^{i,t} = [v_{t,c}^i / u_{t,c}^i]CMRS_{G,c}^{i,t}$. Using that $u_{t,c}^i = v_{t,c}^i + v_{t,\Delta_t}^i$ then implies that

$$MRS_{G,c}^{i,t} = \frac{v_{t,c}^{i}}{v_{t,c}^{i} + v_{t,\Delta_{t}}^{i}} CMRS_{G,c}^{i,t} = (1 - \alpha_{t}^{i,c}) CMRS_{G,c}^{i,t}.$$
 (A22)

Similarly, when old we have

$$MRS_{G,x}^{i,t} = (1 - \alpha_t^{i,x})CMRS_{G,x}^{i,t}.$$
(A23)

Substituting equations (A22) and (A23) into equation (17) then implies

$$MB_{t,G} = \sum_{i} n_{t}^{i} (1 - \alpha_{t}^{i,c}) CMRS_{G,c}^{i,t} + \sum_{i} n_{t-1}^{i} (1 - \alpha_{t}^{i,x}) CMRS_{G,x}^{i,t}$$
$$= (1 - \overline{\alpha}_{t}) CMB_{t,G} \left[1 + \operatorname{cov} \left(\frac{1 - \alpha_{t}}{1 - \overline{\alpha}_{t}}, \frac{CMRS_{G,c}^{t}}{CMRS_{G,c}^{t}} \right) \right] , \qquad (A24)$$
$$= (1 - \overline{\alpha}_{t}) CMB_{t,G} \left[1 + \Psi_{t} \right]$$

where we have used the previously defined Ψ_t . Substituting equation (A24) into equation (21) implies equation (26).

Proof of Proposition 6.

From the conditions in Proposition 4 it follows that $\alpha_t^d = 0$ and $\Omega_t = 0$. Moreover, since the degree of positionality is the same for both types, we also have that $\Psi_t = 0$. From equation

(A9) it follows, since the interest rate is constant, that $\gamma_{t+1}/\gamma_t = 1/(1+r)$, and hence that $\gamma_{t+\tau}/\gamma_t = 1/(1+r)^{\tau}$. Substituting these conditions into equation (26) gives equation (27).

Proof of Corollary 2.

Once again, since

$$\lim_{\xi \to 1} [1-\xi]^{\tau} = 0 \quad \text{for } \tau > 0 \text{ and } \lim_{\xi \to 1} [1-\xi]^{\tau} = 1 \quad \text{for } \tau = 0,$$

equation (28) follows immediately from equation (27).

Proof of Proposition 7.

We will first derive the positionality effect in this more general case and then substitute this effect into equation (19). The derivative of the Lagrangean with respect to $\overline{c_t}$ can be written as

$$\frac{\partial \mathcal{L}}{\partial \overline{c}_{t}} = \sum_{i=1}^{2} \frac{\partial W}{\partial (n_{t-1}^{i} U_{t-1}^{i})} n_{t-1}^{i} u_{t-1,\overline{c}_{t}}^{i} + \sum_{i=1}^{2} \frac{\partial W}{\partial (n_{t}^{i} U_{t}^{i})} n_{t}^{i} u_{t,\overline{c}_{t}}^{i}
+ \sum_{i=1}^{2} \frac{\partial W}{\partial (n_{t+1}^{i} U_{t+1}^{i})} n_{t+1}^{i} u_{t+1,\overline{c}_{t}}^{i} + \lambda_{t-1} \Big[u_{t-1,\overline{c}_{t}}^{2} - \hat{u}_{t-1,\overline{c}_{t}}^{2} \Big] .$$
(A25)
$$+ \lambda_{t} \Big[u_{t,\overline{c}_{t}}^{2} - \hat{u}_{t,\overline{c}_{t}}^{2} \Big] + \lambda_{t+1} \Big[u_{t+1,\overline{c}_{t}}^{2} - \hat{u}_{t+1,\overline{c}_{t}}^{2} \Big]$$

From equation (1) we have

$$\begin{split} u_{t,c}^{i} &= v_{t,c}^{i} + v_{t,\Delta_{t}^{c}}^{i} + v_{t,\delta_{t}^{c}}^{i} = \frac{v_{t,\Delta_{t}^{c}}^{i}}{\alpha_{t}^{i,c}} = \frac{v_{t,\delta_{t}^{c}}^{i}}{\beta_{t}^{i,c}}, \\ u_{t,x}^{i} &= v_{t,x}^{i} + v_{t,\Delta_{t}^{x}}^{i} + v_{t,\delta_{t}^{x}}^{i} = \frac{v_{t,\Delta_{t}^{c}}^{i}}{\alpha_{t+1}^{i,x}} = \frac{v_{t,\delta_{t}^{c}}^{i}}{\beta_{t+1}^{i,x}}, \\ u_{t,\overline{c}_{t}}^{i} &= -v_{t,\Delta_{t}^{c}}^{i} - v_{t,\delta_{t}^{x}}^{i}, \\ u_{t,\overline{c}_{t-1}}^{i} &= -v_{t,\delta_{t}^{c}}^{i}, \\ u_{t,\overline{c}_{t+1}}^{i} &= -v_{t,\Delta_{t}^{c}}^{i}, \end{split}$$

so

$$u_{t,\bar{c}_{t}}^{i} = -\alpha_{t}^{i,c} u_{t,c}^{i} - \beta_{t+1}^{i,x} u_{t,x}^{i}, \qquad (A26)$$

,

$$u_{t,\bar{c}_{t-1}}^{i} = -\beta_{t}^{i,c} u_{t,c}^{i}, \qquad (A27)$$

$$u_{t,\overline{c}_{t+1}}^{i} = -\alpha_{t}^{i,x} u_{t,x}^{i},$$
(A28)

which substituted into equation (A25) imply

$$\frac{\partial \mathcal{L}}{\partial \overline{c}_{t}} = -\sum_{i=1}^{2} \frac{\partial W}{\partial (n_{t-1}^{i} U_{t-1}^{i})} n_{t-1}^{i} \alpha_{t-1}^{i,x} u_{t-1,x}^{i}
-\sum_{i=1}^{2} \frac{\partial W}{\partial (n_{t}^{i} U_{t}^{i})} n_{t}^{i} \Big[\alpha_{t}^{i,c} u_{t,c}^{i} + \beta_{t}^{i,x} u_{t,x}^{i} \Big]
\sum_{i=1}^{2} \frac{\partial W}{\partial (n_{t+1}^{i} U_{t+1}^{i})} n_{t+1}^{i} \beta_{t+1}^{i,c} u_{t+1,c}^{i} + \lambda_{t-1} \Big[-\alpha_{t}^{2,x} u_{t-1,x}^{2} + \hat{\alpha}_{t}^{2,x} u_{t-1,x}^{2} \Big].$$
(A29)

$$+\lambda_{t} \Big[-\alpha_{t}^{2,c} u_{t,c}^{2} - \beta_{t}^{2,x} u_{t+1,x}^{2} + \hat{\alpha}_{t}^{2,c} \hat{u}_{t,c}^{2} + \hat{\beta}_{t+1}^{2,x} \hat{u}_{t,x}^{2} \Big]
+\lambda_{t+1} \Big[-\beta_{t+1}^{2,c} u_{t+1,c}^{2} + \hat{\beta}_{t+1}^{2,c} \hat{u}_{t+1,c}^{2} \Big]$$

By substituting equations (A11)-(A14) into equation (A29), and collecting terms, we obtain

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \overline{c}_{t}} &= \frac{\partial \mathfrak{L}}{\partial \overline{c}_{t+1}} \frac{\overline{\beta}_{t+1}}{1 - \overline{\alpha}_{t}} - N_{t} \gamma_{t} \frac{\overline{\alpha}_{t}}{1 - \overline{\alpha}_{t}} - N_{t+1} \gamma_{t+1} \frac{\overline{\beta}_{t+1}}{1 - \overline{\alpha}_{t}} \\ &+ \frac{\lambda_{t-1} \hat{u}_{t-1,x}^{2}}{1 - \overline{\alpha}_{t}} \Big[\hat{\alpha}_{t}^{2,x} - \alpha_{t}^{1,x} \Big] + \frac{\lambda_{t} \hat{u}_{t,c}^{2}}{1 - \overline{\alpha}_{t}} \Big[\hat{\alpha}_{t}^{2,c} - \alpha_{t}^{1,c} \Big] \\ &+ \frac{\lambda_{t} \hat{u}_{t,x}^{2}}{1 - \overline{\alpha}_{t}} \Big[\hat{\beta}_{t+1}^{2,x} - \beta_{t+1}^{1,x} \Big] + \frac{\lambda_{t+1} \hat{u}_{t+1,c}^{2}}{1 - \overline{\alpha}_{t}} \Big[\hat{\beta}_{t+1}^{2,c} - \beta_{t+1}^{1,c} \Big] \\ &= \frac{1}{1 - \overline{\alpha}_{t}} \Bigg[\overline{\beta}_{t+1} \frac{\partial \mathfrak{L}}{\partial \overline{c}_{t+1}} + N_{t} \gamma_{t} [\alpha_{t}^{d} - \overline{\alpha}_{t}] + N_{t+1} \gamma_{t+1} [\beta_{t}^{d} - \overline{\beta}_{t+1}] \Bigg] \end{aligned}$$
(A30)

where we have used the short notations α_t^d and β_t^d as defined earlier. Using the definition for B_t and the short notation

$$\varphi_t = \frac{\overline{\beta}_{t+1}}{1 - \overline{\alpha}_t},$$

the recursive equation (A30) can more conveniently be rewritten as follows:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \overline{c}_{t}} &= B_{t} + \varphi_{t} \frac{\partial \mathfrak{L}}{\partial \overline{c}_{t+1}} = B_{t} + \varphi_{t} \left[B_{t+1} + \varphi_{t+1} \frac{\partial \mathfrak{L}}{\partial \overline{c}_{t+2}} \right] \\ &= B_{t} + \varphi_{t} \left[B_{t+1} + \varphi_{t+1} \left[B_{t+2} + \varphi_{t+2} \frac{\partial \mathfrak{L}}{\partial \overline{c}_{t+3}} \right] \right] \\ &= B_{t} + B_{t+1} \varphi_{t} + B_{t+2} \varphi_{t} \varphi_{t+1} + B_{t+3} \varphi_{t} \varphi_{t+1} \varphi_{t+2} \dots \\ &= B_{t} + \sum_{i=1}^{\infty} B_{t+i} \prod_{j=1}^{i} \varphi_{t+j-1} \end{aligned}$$
(A31)

Substituting back $\varphi_t = \overline{\beta}_{t+1} / (1 - \overline{\alpha}_t)$ into equation (A31) implies

$$\frac{\partial \mathcal{L}}{\partial \overline{c}_{t}} = B_{t} + \sum_{i=1}^{\infty} B_{t+i} \prod_{j=1}^{i} \frac{\overline{\beta}_{t+j}}{1 - \overline{\alpha}_{t+j-1}}.$$
(A32)

Substituting equation (A32) into equation (19) implies equation (29).

Proof of Proposition 2'.

Given Assumptions A, equation (A32) reduces to the geometric series

$$\frac{\partial \mathcal{L}}{\partial \overline{c}_{t}} = \frac{N\gamma_{t}}{1 - \overline{\alpha}} \left[\alpha^{d} - \overline{\alpha} + \frac{\beta^{d} - \overline{\beta}}{1 + r} \right] \sum_{i=0}^{\infty} \left[\frac{\overline{\beta}}{(1 - \overline{\alpha})(1 + r)} \right]^{i}, \quad (A33)$$

$$= N\gamma_{t} \frac{\alpha^{d} - \overline{\alpha} + (\beta^{d} - \overline{\beta})/(1 + r)}{1 - \overline{\alpha} - \overline{\beta}/(1 + r)}$$

where in the last step we have implicitly assumed that $0 < \overline{\beta} < (1 - \overline{\alpha})(1 + r)$ so that the series converges. Using the definitions for $\overline{\rho}$ and ρ^d imply further that

$$\frac{\partial \mathcal{L}}{\partial \overline{c}_t} = N \gamma_t \frac{\rho^d - \overline{\rho}}{1 - \overline{\rho}},\tag{A34}$$

which substituted into equation (A19) implies equation (30).

Proof of Proposition 3'.

That the joint impact of present and future positionality effects increases the contribution to the public good in period t means that the benefit side is amplified by such effects compared to the optimal provision rule without such concerns, i.e. that

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} \frac{1-\rho^d}{1-\overline{\rho}} + \Omega_{t+\tau} \right] \left[1-\xi\right]^{\tau} > \sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} + \Omega_{t+\tau} \right] \left[1-\xi\right]^{\tau}$$

Which is clearly true if and only if $\overline{\rho} - \rho^d > 0$, for which a sufficient condition (remember that $\overline{\rho} > 0$) is that $\rho^d < 0$.

Proof of Proposition 4'.

If leisure is weakly separable from private and public consumption as specified for all *t*, and where the sub-utility function $h_t(\cdot)$ is the same for both ability-types, then clearly $M\hat{R}S_{G,c}^{2,t} = MRS_{G,c}^{1,t}$ and $M\hat{R}S_{G,x}^{2,t} = MRS_{G,x}^{1,t}$, implying that $\Omega_t = 0$. Moreover, all positionality degrees will be the same for the mimicker and the low-ability type, implying that $\hat{\alpha}_t^{2,c} = \alpha_t^{1,c}$, $\hat{\alpha}_t^{2,x} = \alpha_t^{1,x}$, $\hat{\beta}_t^{2,c} = \beta_t^{1,c}$ and $\hat{\beta}_{t-1}^{2,x} = \beta_{t-1}^{1,x}$ for all *t*, and that $\alpha^d = \beta^d = 0$. Substituting $\alpha^d = 0$, $\beta^d = 0$ and $\Omega_t = 0$ for all *t* into equation (30) implies equation (31).

Proof of Proposition 5'.

Combining $MRS_{G,c}^{i,t} = [v_{t,c}^i / u_{t,c}^i] CMRS_{G,c}^{i,t}$ with $u_{t,c}^i = v_{t,c}^i + v_{t,\Delta_t^c}^i + v_{t,\Delta_t^c}^i$ implies

$$MRS_{G,c}^{i,t} = \frac{v_{t,c}^{i}}{v_{t,c}^{i} + v_{t,\Delta_{t}^{c}}^{i} + v_{t,\delta_{t}^{c}}^{i}} CMRS_{G,c}^{i,t} = (1 - \alpha_{t}^{i,c} - \beta_{t}^{i,c}) CMRS_{G,c}^{i,t}$$
(A35)

Similarly, when old we have

$$MRS_{G,x}^{i,t} = (1 - \alpha_t^{i,x} - \beta_t^{i,x}) CMRS_{G,x}^{i,t}.$$
(A36)

Substituting equations (A35) and (A36) into equation (29) implies

$$\begin{split} MB_{t,G} &= \sum_{i} n_{t}^{i} (1 - \alpha_{t}^{i,c} - \beta_{t}^{i,x}) CMRS_{G,c}^{i,t} + \sum_{i} n_{t-1}^{i} (1 - \alpha_{t}^{i,x} - \beta_{t}^{i,x}) CMRS_{G,x}^{i,t} \\ &= (1 - \overline{\alpha}_{t} - \overline{\beta}_{t}) CMB_{t,G} \left[1 + \operatorname{cov} \left(\frac{1 - \alpha_{t} - \beta_{t}}{1 - \overline{\alpha}_{t} - \overline{\beta}_{t}}, \frac{CMRS_{G,c}^{t}}{CMRS_{G,c}^{t}} \right) \right]$$

$$= (1 - \overline{\alpha}_{t} - \overline{\beta}_{t}) CMB_{t,G} \left[1 + \Lambda_{t} \right]$$
(A37)

where we have used the previously defined Λ_t . Substituting equation (A37) into equation (30) implies equation (32).

Proofs of Proposition 6' and Corollary 2'

Equivalent to the proofs of Proposition 6 and Corollary 2.

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