Public Goods and Optimal Paternalism under Present-Biased Preferences*

Thomas Aronsson[1] and David Granlund[1,2]

[1]Department of Economics, Umeå University,
SE-901 87 Umeå, Sweden

[2]The Swedish Retail Institute (HUI),
SE-103 29 Stockholm, Sweden.

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Abstract

This paper deals with the optimal provision of a state-variable public good in a two-type model, when the consumers have present-biased preferences due to quasi-hyperbolic discounting. The results show that the preference for immediate gratification facing the (mimicking) high-ability type weakens the incentive to adjust the public provision in response to the self-selection constraint.

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1 Introduction

Over the last decades, numerous studies have reported strong evidence suggesting that people and animals have "present-biased" preferences, i.e. a tendency to give less weight to the future welfare consequences of today's actions than would be optimal for the individual himself/herself in a longer time-perspective (see, e.g., Thaler, 1981; Mazur, 1987; Kirby, 1997; Viscusi, Huber and Bell, 2008; Brown, Chua and Camerer, 2009). Present-biased preferences might be exemplified by quasi-hyperbolic discounting, where the individual, at any time t, attaches a higher utility discount rate to tradeoffs between periods t and $t+1$ than to similar tradeoffs in the more distant future. Viscusi, Huber and Bell (2008) have studied discounting of the benefits attached to a public good, exemplified by water quality. Based on a representative U.S. sample of 2,914

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respondents, they estimate the "quasi-hyperbolic discounting parameter" (referred to as "β" below) to be in the interval 0.48-0.61. This suggests that the weight given to benefits in period $t+1$, relative to benefits in period t, is roughly half of the weight that consumers in period t give to benefits in period $t+2$ relative to benefits in period $t+1$.

The purpose of this short paper is to examine how a paternalistic government would modify the policy rule for public provision in response to quasi-hyperbolic discounting. We focus on a state-variable public good, as many real world public goods, such as, e.g., different aspects of environmental quality, have this particular character. Our study is based on an overlapping generations (OLG) model with two ability-types, where each individual lives for three periods (the minimum number of periods to address quasi-hyperbolic discounting). The government is assumed to carry out redistribution under asymmetric information by means of nonlinear labor and capital income taxation as well as provide the state-variable public good referred to above. Therefore, our concern will be to study the supplemental role of public provision when the income taxes are optimal.

2 The Model and Main Results

Consider an OLG economy where each consumer lives for three periods; works in the first two and becomes a pensioner in the third. Each generation consists of two consumer-types: a low-ability type earning wage rate w_t^l and a high-ability type earning wage rate $w_t^h > w_t^l$ in period t. The instantaneous utility faced by ability-type i of age a in period t is written

$$u_{a,t}^i = u(c_{a,t}^i, z_{a,t}^i, G_t), \tag{1}$$

where c denotes consumption of a numeraire good, z leisure and G the public good. The age indicator, a, takes the value 0, 1 and 2, if the consumer is young, middle-aged and old, respectively. When young and middle-aged, leisure is given by a time endowment less the hours of work, i.e. $z_{0,t}^i = H - \ell_{0,t}^i$ and $z_{1,t+1}^i = H - \ell_{1,t+1}^i$, whereas all time is spent on leisure when old, so $z_{2,t+2}^i = H$, for $i = l, h$. The intertemporal objective of ability-type i of generation t is given by

$$U_t^i = u_{0,t}^i + \beta^i \sum_{j=1}^{2} \Theta_j u_{j,t+j}^i, \tag{2}$$

where $\Theta^t = 1/(1+\theta)^t$ is a conventional exponential discount factor with utility discount rate θ (the same for everybody), whereas $\beta^i \in (0,1)$ is a type-specific and time-inconsistent preference for immediate gratification.[1]

Let s denote saving and r the interest rate. We abstract from bequests, meaning that the intial wealth of each consumer is zero. The budget constraint faced by ability-type i of generation t can then be written

[1] It would add no important insight into the consequences of quasi-hyperbolic discounting if we were to assume that the conventional utility discount factor differs between ability-types.

as follows;

$$w_{0,t}^i \ell_{0,t}^i - T_{0,t}^i - s_{0,t}^i = c_{0,t}^i \quad (3)$$

$$s_{0,t}^i[1 + r_{t+1}] + w_{1,t+1}^i \ell_{1,t+1}^i - T_{1,t+1}^i - s_{1,t+1}^i = c_{1,t+1}^i \quad (4)$$

$$s_{1,t+1}^i[1 + r_{t+2}] - T_{2,t+2}^i = c_{2,t+2}^i \quad (5)$$

where the price of the consumption good has been normalized to one. The variables $T_{0,t}^i$, $T_{1,t+1}^i$ and $T_{2,t+2}^i$ represent the income tax payments made when young, middle-aged and old, respectively, which are nonlinear functions of income. Although the optimal use of income taxation will not be examined here, we assume that the income tax system is flexible in the sense of allowing the government to control, the consumption, labor supply and savings behavior of each ability-type.[2]

To simplify, we follow much earlier literature in assuming that output is produced by a linear technology, which is interpreted to mean that the factor prices (wage rates and interest rate) are exogenous.

The public good evolves according to the following difference equation;

$$G_t = g_t + \delta G_{t-1}, \quad (6)$$

where g_t is the incremental provision (or investment in the public good) in period t, while $\delta \in (0, 1)$ represents the depreciation factor.

Turning to public policy, our concern is to analyze the optimal provision of the state-variable public good when decided upon by a paternalistic government; therefore, we assume that $\beta^l = \beta^h = 1$ from the point of view of the government.[3] The objective of the government is represented by a utilitarian social welfare function. The contribution of ability-type i of generation t to this social welfare function becomes

$$V_{0,t}^i = u_{0,t}^i + \sum_{j=1}^{2} \Theta^j u_{j,t+j}^i, \quad (7)$$

and the social welfare function is written as

$$W = \sum_t \sum_i \Theta^j V_{0,t}^i. \quad (8)$$

The informational assumptions are conventional: the government can observe labor and capital income, whereas ability is private information. We focus on the "normal case", where the government attempts to redistribute from the high-ability to the low-ability type. As a consequence, the government must prevent

[2] Aronsson and Sjögren (2009) analyze the optimal use of income and commodity taxation by a paternalistic government when the consumers apply quasi-hyperbolic discounting.

[3] This assumption is in line with earlier comparable literature on optimal paternalism; see, e.g., O'Donoghue and Rabin (2003, 2006) and Aronsson and Thunström (2008).

the high-ability type from becoming a mimicker. This can be formalized by introducing a self-selection constraint

$$U_{0,t}^h = u_{0,t}^h + \beta^h \sum_{j=1}^{2} \Theta^j u_{j,t+j}^h \geq \widehat{U}_{0,t}^h = \widehat{u}_{0,t}^h + \beta^h \sum_{j=1}^{2} \Theta^j \widehat{u}_{j,t+j}^h, \tag{9}$$

where $\widehat{U}_{0,t}^h$ denotes the utility of the mimicker. We assume that an individual who reveals himself/herself to be a high-ability type when young cannot credibly pretend to be a low-ability type when middle-aged, which means that the decision of whether or not to become a mimicker is taken by the young high-ability type. The mimicker faces the same income-consumption combinations as the low-ability type; however, as the mimicker is more productive, he/she spends more time on leisure than the low-ability type.

For all t, the resource constraint is written

$$\sum_i \left[w_{0,t}^i l_{0,t}^i + w_{1,t}^i l_{1,t}^i - c_{0,t}^i - c_{1,t}^i - c_{2,t}^i \right] + K_t(1+r_t) - K_{t+1} - p_t g_t = 0, \tag{10}$$

where K_t is the capital stock at the beginning of period t, and p_t is a fixed marginal cost of public provision interpretable as the marginal rate of transformation between the incremental public good and the private consumption good in period t.

The decision-problem facing the government is to maximize the social welfare function presented in equation (8), subject to the accumulation equation for the public good, the self-selection constraint and the resource constraint given by equations (6), (9) and (10), respectively. The Lagrangean corresponding to this optimization problem becomes

$$\begin{aligned} L \;=\; & W + \sum_t \mu_t \left\{ \delta G_{t-1} + g_t - G_t \right\} \\ & + \sum_t \gamma_t \left\{ \sum_i \left[w_{0,t}^i l_{0,t}^i + w_{1,t}^i l_{1,t}^i - c_{0,t}^i - c_{1,t}^i - c_{2,t}^i \right] - p_t g_t + K_t(1+r_t) - K_{t+1} \right\} \\ & + \sum_t \lambda_t \left\{ u_{0,t}^h + \beta^h \sum_{j=1}^{2} \Theta^j u_{j,t+j}^h - \widehat{u}_{0,t}^h - \beta^h \sum_{j=1}^{2} \Theta_j \widehat{u}_{j,t+j}^h \right\} \end{aligned} \tag{11}$$

where μ, γ and λ are Lagrange multipliers. In this second best problem, the decision-variables are $l_{0,t}^i$, $c_{0,t}^i$, $l_{1,t}^i$, $c_{1,t}^i$, $c_{2,t}^i$ (for $i = 1, 2$), g_t and K_t for all t.[4]

Let

$$MRS_{a,t}^i = \frac{\partial u_{a,t}^i / \partial G_t}{\partial u_{a,t}^i / \partial c_{a,t}^i} \text{ and } \widehat{MRS}_{a,t}^i = \frac{\partial \hat{u}_{a,t}^2 / \partial G_t}{\partial \hat{u}_{a,t}^2 / \partial c_{a,t}^2}$$

[4] Since the government can control the private consumption and work hours by each ability-type via the tax system, it is convenient to write the second best problem as a direct decision-problem (where the government decides upon private consumption and work hours instead of tax parameters). This approach is standard in the literature on optimal nonlinear taxation. See also earlier literature on optimal income taxation in dynamic economies; e.g., Pirttilä and Tuomala (2001) and Aronsson and Johansson-Stenman (in press).

denote the marginal rate of substitution between the public good and private consumption faced by ability-type i of age a in period t and the corresponding marginal rate of substitution faced by the mimicker, respectively. We can then present our main result as follows;[5]

Proposition 1 *When the consumers have present-biased preferences, and the labor and capital income taxes are optimal, the policy rule for the state-variable public good is given by*

$$p_t = \sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \delta^{\tau} SMBG_{\tau} \tag{12}$$

where

$$\begin{aligned} SMBG_t &= \sum_i \sum_a MRS^i_{a,t} + \frac{\lambda_t}{\gamma_t} \frac{\partial \widehat{u}^h_{0,t}}{\partial c^1_{0,t}} \left[MRS^l_{0,t} - \widehat{MRS}^h_{0,t} \right] \\ &\quad + \beta^h \Theta \frac{\lambda_{t-1}}{\gamma_t} \frac{\partial \widehat{u}^h_{1,t}}{\partial c^1_{1,t}} \left[MRS^l_{1,t} - \widehat{MRS}^h_{1,t} \right]. \end{aligned} \tag{13}$$

Equation (12) means that the marginal rate of transformation between the public good and the private consumption good in period t, p_t, should equal the sum of social marginal benefits that this investment gives rise to over time, which reflect the marginal willingness to pay for the public good by the consumers as well as effects via the self-selection constraint. This corresponds to the results derived by Pirttilä and Tuomala (2001), yet with the exception that the self-control problem to be discussed below was absent in their study. Note that the instantaneous social marginal benefit in period t, $SMBG_t$, reflects the marginal willingness to pay by all three age-groups, i.e. the young, middle-aged and old, respectively, in period t. Also, the future marginal benefits of an incremental contribution to the public good in period t are not discounted directly by the utility discount rate; instead, the quotient of present value shadow prices attached to the government's budget constraint, $\gamma_{t+\tau}/\gamma_t$, serves this purpose, meaning that the weight that the government attaches to future marginal benefits will both reflect the pure rate of time preference, θ, and the relative marginal cost of public funds (i.e. relative to the base year value γ_t).

We can see from equation (13) that the self-control problem facing the consumers affects the policy rule for the public good via the self-selection constraint; more precisely via the contribution to this constraint by the middle-aged generation.[6] The reason is that the consumer preference for immediate gratification enters the social optimization problem via the self-selection constraint only, because the paternalistic government faces no such preference by assumption. Two interesting observations follow immediately from equation (13). First, the preference for immediate gratification weakens the contribution that the self-selection constraint has on the policy rule for public provision. In a sense, therefore, quasi-hyperbolic discounting brings us closer to (a dynamic analogue to) the first best policy rule that would apply without asymmetric information. Second,

[5] Note that Proposition 1 applies irrespective of whether the consumers are naive (erroneously expect not to suffer from the self-control problem in future periods) or sophisticated (in which case a consumer implements a plan that his/her future selves will follow). For a more thorough discussion of naivety versus sophistication, see, e.g., O'Donoghue and Rabin (1999).

[6] Since $MRS^l_{2,t} = \widehat{MRS}^h_{2,t}$, the corresponding effect for the old generation vanishes. The reason is that the old consumers spend all available time on leisure regardless of ability.

it is only the preference for immediate gratification facing the high-ability type, β^h, that affects the policy rule directly; there is no corresponding effect of β^l. We summarize these observations in the following two corollaries to Proposition 1;

Corollary 1. *All other things equal, the preference for immediate gratification faced by the high-ability type, $\beta^h \in (0,1)$, weakens the policy incentive associated with the self-selection constraint. The smaller β^h, ceteris paribus, the weaker will be the incentive created by the second row of equation (13) to overprovide (underprovide) the public good relative to the Samuelson rule if $MRS^l_{1,t} > (<)\ \widehat{MRS}^h_{1,t}$.*

Corollary 2. *The preference for immediate gratification faced by the low-ability type, β^l, does not affect the policy rule for the public good.*

When the young high-ability type decides whether or not to become a mimicker, he/she attaches less weight to the future utility consequences of today's actions than he/she would have done in the absence of the self-control problem (as the utilities facing the young consumer's middle-aged and old selves are multiplied by $\beta^h < 1$). As a consequence, the welfare contribution of public provision that goes via $MRS^l_{1,t} - \widehat{MRS}^h_{1,t}$ is only a fraction of the corresponding effect that would follow without the preference for immediate gratification, which explains Corollary 1. Corollary 2 follows because the preference for immediate gratification facing the low-ability type does not enter the social optimization problem directly. In other words, self-control problems facing mimicked agents (who are not potential mimickers themselves) will not modify the policy rule for the public good. However, this does not mean that the level of β^l is unimportant; the low-ability type's self-control problem may still influence his/her choice of consumption-savings profile and, therefore, have important indirect effects on the public good via the marginal rates of substitution in equation (13).

In the special case where $\delta = 0$, in which the public good becomes a flow variable, equation (12) reduces to read $p_t = SMBG_t$, meaning that the forward-looking benefit measure reduces to a static measure. The qualitative effects of quasi-hyperbolic discounting will, nevertheless, remain as in Corollaries 1 and 2: β^h still affects the policy rule via the self-selection constraint faced by the middle-aged, and β^l does not modify the policy rule for a flow-variable public good.

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