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Abstract: Did Pressler Understand how to Use the Indicator Per Cent
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A classical problem in forestry is the determination of the optimal rotation. This problem was solved during the 19th century, by German forest mathematicians. Martin Faustmann deserves some of the fame. However, he did not explicitly derive the conditions for an optimal solution. His contender is Max Robert Pressler. He invented the concept of Indicator Per Cent, which can be used to determine whether a stand is mature for harvesting or not. Did Pressler fully realize this? Having analyzed some of the relevant literature our answer is no.
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Did Pressler Understand how to Use the Indicator Per Cent?

by Peichen Gong and Karl-Gustaf Löfgren

One of the most discussed topics in forest economics is the “optimal rotation age”, i.e., at what age is it economically optimal to cut a stand of trees. This problem was essentially solved during the 19th century, by German forest mathematicians. Who of them made the most important contributions is, in our opinion, not easy to pin point. Most forest economists would probably say that Martin Faustmann wins the prize. He wrote at least three important papers that have contributed to the solution of the optimal rotation problem (Faustmann 1849a, 1849b, 1853). The 1849 papers were both published in Allgemeine Forst-und Jagd Zeitung (AFJZ), and the 1853 paper was published in Neue Jahrbücher der Forstkunde edited by Georg Wilhelm von Wedekind, who also was the Editor of AFJZ during Faustmann’s haydays. However, Faustmann did not explicitly derive the conditions for an optimal solution, although he criticized the wine aging solution (Geldreinergetragswirtschaft) suggested by Hartig (1833) and König (1835; first edition)[1]. He understood, of course, that it was a good idea to choose the rotation age that maximizes the land value, although to make this an objective optimal investment criterion presupposes a perfect credit market, where the

[1] An important observation made by Scorgie and Kennedy (1996) is that the idea behind “Jevons” wine aging formula was used as a solution of the optimal rotation period by the bishop of Lladaff already in 1794 in a paper addressed to the Board of Agriculture and Internal Improvement. Even more interesting is perhaps a comment made by William Marshall in 1808; an Agriculturalist who was a honorary member of the board of Agriculture. Marshall reviewed the reports submitted to the board and added comments here and there. In reviewing a report concerning the agricultural improvements in the county of Westmoreland, Marshall corrected Watson’s solution by adding in a footnote: “Together with the annual value of the land it grows upon-of more consideration than, perhaps, the interest of money. “

borrowing and lending rates coincide. Irving Fisher's separation theorem was not available until the beginning of the 20:th century[2].

Except for the omission of the exact first order conditions, Faustmann's papers contain an impressive number of novel observations, and he is an important candidate for the title the Founder of Forest Economics. In particular, he noted in Faustmann (1853), where he criticized Hartig and König, that the marginal value increase in the stand at the time of harvesting must exactly compensate for the interest on the value of the stand as well as the interest on the value of the land. He gives the correct intuition that the later the land becomes available for a new stand, the less becomes its present value (ceteris paribus), on account of the effect of discounting[3]. Hence, the opportunity cost of land has to be counted as a cost of delaying the harvesting of the stand. Faustmann also writes[4]:

"If you look at the stand from this economic standpoint- and you always have to do this by determining the economically most advantageous harvesting age; then it can occur at an earlier point in time than the one that is determined by König's opinion".

In other words, he also understood that the correct rotation age will be shorter than the solution of the wine aging formula (Geldreinertragswirtschaft) suggested by among others Hartig and König.

One of Faustmann's most important contenders is Max Robert Pressler. He invented the concept of Indicator Per Cent (Die Weiserprocent), which can be used to determine whether a stand is mature for harvesting or not. However, did Pressler really realize this? We can find three papers by Pressler (Pressler 1860a, 1860b, 1869) where the concept was developed. The last of these is even entitled "On the Indicator Percent" (Zum Weiserprocent). The concept Indicator Per Cent was first mentioned in Pressler (1860b). Stridsberg (1956) indicated that there was a proof of its usefulness in separating immature from mature stands. We do not

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[2] It tells us that under a perfect capital market and perfect foresight any investor will choose the investment that yields the highest present value.
[3] See Faustmann (1853) page 362-363. For the shape of the wine aging formula, see below.
[4] Authors' translation

quite agree. One reason is that if it is a proof of something, it is not a rule that tells us how to determine whether a stand is economically mature[5].

Optimal Rotation Age and the Indicator Per Cent

The analytical apparatus used by the German forest mathematicians were quite heavy discrete time formulae with a lot of Christmas tree decorations. We will to start with work in continuous time without such decorations like income from thinnings and annual expenditure for administration etc. Following current "stripped practice" we define:

$f(t) =$ *the stock of timber in a stand of age t*

$P(t) =$ *the stumpage price at age t*

$E(t) = P(t) f(t)$ *denotes the net revenue from harvesting the stand at age t*

C=the regeneration cost

$r =$ *the ruling interest rate in a perfect credit market*

The rotation problem can now be formulated as the maximization of net present value of all future harvests:

$$\pi(t) = \max_t \{[-Ce^{rt} + E(t)](e^{-rt} + e^{-r2t}...)\} = \max_t \frac{E(t) - Ce^{rt}}{e^{rt} - 1} \tag{1}$$

The first order condition for maximum can be written as

$$E'(t) = r\{E(t) + \frac{E(t) - Ce^{rt}}{e^{rt} - 1}\} \tag{2}$$

The interpretation of equation (2) is that it is optimal to harvest a forest stand at the age when the time rate of change in its value is equal to the interest on the value of the standing timber stock plus the interest on the value of the land. The wine aging formula will result if the value of land is deleted from equation (2).

Most German forest mathematician[6] were during the whole 19th century working with a discrete time framework and their discrete time slimmed version of the land value can be written as:

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[5] Our interpretation is that he proved that if the total capital (land +stand) increases with the ruling interest rate, then the present value of the stand remains constant.

[6] The continuous time approach formulation is used in a paper by A Klebsch (1869), AFJZ supplement in a paper on Ueber ein problem der Forstwissenschaft. It is about a relationship between the value of land and the production function. The continuous time approach was also used by Wicksell (1987) and Ohlin (1921).

$$\pi(t) = \frac{E(t) - C(1+p)^t}{(1+p)^t - 1},\ t = 1, 2, .. \tag{3}$$

where $p = e^r - 1$ is the annual rate of interest. The restriction that t can only assume integer numbers means that, even though we acknowledge the fact that the value of the harvest is time dependent, the usual calculus used to derive equation (2) is no longer valid due to the discrete time constraint. However, taking the derivative of (3) *as if t was a real number* yields:

$$\frac{\partial \pi(t)}{\partial t} = \frac{1}{(1+p)^t - 1}\left[E^{'}(t) - \ln(1+p)\{E(t) + \pi(t)\}\right] \tag{4}$$

To maximize the present value of land the expression in (4) should ideally be put equal to zero, but since time is discrete this will, loosely speaking, only happen with probability zero. In a discrete setting, we can approximate the term $E'(t)$ on the right hand side of equation (4) by $\Delta E(t) = E(t+1) - E(t) = P(t+1)f(t+1) - P(t)f(t)$. Substitute $\frac{\partial \pi(t)}{\partial t}$ in the left hand side of equation (4) by $\Delta\pi(t) = \pi(t+1) - \pi(t)$ and divide through by $E(t) + \pi(t)$ we obtain:

$$\frac{\Delta\pi(t)}{\pi(t) + E(t)} = \frac{1}{(1+p)^t - 1}\left[\frac{\Delta E(t)}{\pi(t) + E(t)} - \ln(1+p)\right] \tag{5}$$

Clearly land value increases (decreases) if $\Delta\pi(t)$ increases (decreases). This holds if and only if the first term in the square brackets on the right hand side is greater (less) than the second term. The first term is the Presslerian *Weiserprocent.* It can be interpreted as the rate of return of the capital (the value of bare land $\pi(t)$ *plus* the value of the standing timber $E(t)$) when kept in the forest. The second term is the interest rate (note that $\ln(1+p) \approx p$). Therefore, equation (5) can be used to distinguish mature stands from immature ones. If the indicator per cent is larger than the ruling interest rate, the stand is immature and should be kept, since we are told that the yield is larger in the forest than in the bank. If the opposite is true, the stand has reached maturity and should be harvested[7]. This is one obvious and important use of the indicator per cent. It has also been frequently used in practice, and it must be considered as one of the most important inventions in normative forestry.

[7] This presupposes of course that the growth function has adequate integer concavity properties. This is the case if the production possibility set is integer convex. For definition see Frank (1969).

Pressler's Contribution

Pressler wrote two articles on the economics of forestry in 1860, both were published in AFJZ. The first paper is entitled Zur Berständigung über der Reinertragswaldbau und dessen Betriebsideal (For the Comprehension of Net Revenue Silviculture and the Management Objectives Derived thereof). The sub-title is Aus und zu der forstlichen Finanzrechnung (On and Additions to Forestal Financial Accounting). The second paper has the same main title but Pressler adds a sub-headline Aus der Holzzuwachslehre (On the Science of Timber Growth).

The first paper (Pressler 1860a) contains few technicalities but the claim on page 53, which is cited by, among others, Stridsberg (1956), and contains the statement that the present value of the forest capital culminates, or rather does not change, when it grows at the same rate as the interest rate. More specifically, as long as $\pi(t+1)+E(t+1)=e^{r}[\pi(t)+E(t)]$ the present value of the total capital remains constant. This is however, of doubtful relevance for the determination of the rotation period.

In the second paper (Pressler 1860b) he developed in detail the concept of the indicator per cent (die Weiserprocent), which 150 years later still is a key ingredient in practical forestry . The derivation is quite elegant and Pressler's approach is more comprehensive than what we used in deriving Equation (5), in the sense that the increase in net yield from one year to another is decomposed into a first increment, i.e. the relative volume growth, and a second increment, i.e. the relative stumpage price increase (even a price change is considered). Using our notations, the first increment (a) equals

$$a=\frac{f(t+1)-f(t)}{f(t)}$$

and the second increment (b) is

$$b=\frac{P(t+1)-P(t)}{P(t)}$$

Thus, the increment in the value of the standing timber stock at age t is $\Delta E(t)=P(t+1)f(t+1)-P(t)f(t)=(1+b)P(t)(1+a)f(t)-P(t)f(t)=(a+b+ab)P(t)f(t)$ or $\Delta E(t)=(a+b+ab)E(t)\approx(a+b)E(t)$. Substitute into our expression of the Indicator per cent gives:

$$\frac{\Delta E(t)}{\pi(t)+E(t)}=\frac{(a+b)E(t)}{\pi(t)+E(t)}=(a+b)\frac{R}{R+1} \qquad (6)$$

where $R=\frac{E(t)}{\pi(t)}$ is the "relative timber value".

The right-hand side of Equation (6) is the Indicator per cent derived by Pressler (1860b). In connection with this derivation Pressler writes:

"For reasons which will be given in detail in the third article…..the indicator percent because of its decisive or indicating significance for a silviculture oriented towards the highest net revenue"[8]

We have cited another article which we will discuss below, but we know of no further article on the indicator per cent in AFJZ. A bit further down in the manuscript he continues:

"Surely, there is no need to draw my reader's attention to the why and wherefore that one of the main objectives of economic silviculture is to attain the highest net revenue, which demands *reaching and maintaining the highest indicator per cent of the timber."*

"It is therefore advisable to attempt, *right from the start, to work with the highest reduction factor's numerator or relative timber value* (r) *possible; i.e.* to establish the most valuable timber capital (*E*) on the smallest possible land capital (π); and further, trying with all economic efforts conceivable to maintain the first as well as the second increment (a+b) at the largest possible level".

The quotations might sound as good ideas, but they give no clue to how to use the indicator per cent to distinguish mature stands from immature. In other words, he did not explicitly suggest the use of the indicator per cent as a practical way to determine the optimal rotation age. Neither did he make it clear how can one use the indicator per cent to choose among different silvicultural options. It seems that Presseler suggested that one should adopt all silvicultural activities that could increase the indicator per cent. This, however, is generally incorrect because the indicator per cent, as it is defined in equation (6), does not take into account the silvicultural costs.

[8] Page 191. The citation has been translated by Dr Wilhelm Löwenstein and Jeanne R Wirkner, University of Göttingen, Germany

Maximizing the internal rate of return ρ is a possible way to arrive at the correct rotation period as shown by Wicksell (1987)[9]. To get the correct answer, however, the internal rate of return has to be calculated by including the land rent Π (or land value) which is determined in a competitive market. Given a rotation age *t*, the internal rate of return $\rho(t)$ is defined by the following equation

$$Ce^{\rho(t)t} + \int_0^t \Pi e^{\rho(t)z} dz = E(t)$$

which can be rewritten as

$$\rho(t)\frac{E(t) - Ce^{\rho(t)t}}{e^{\rho(t)t} - 1} = \Pi \tag{7}$$

Given a land rent Π, the first order condition for the maximization of $\rho(t)$ is that $d\rho(t)/dt = 0$. Let T be the rotation age which $\rho(t)$ reaches its maximum. From Equation (7), we can derive the following expression of the first order condition:

$$E'(T) = e^{\rho(T)T}(\rho(T)C + \Pi) \tag{8}$$

When the land rent Π is determined in a competitive market with the market interest rate equal to r, we know that the maximum internal rate of return on timber investment will equal the market rate of interest, i.e. $\rho(T) = r$. Substitute r for $\rho(T)$ in (7) we obtain the market equilibrium land rent

$$\Pi = r\frac{E(T) - Ce^{rT}}{e^{rT} - 1}$$

Substitute the right-hand side of the above equation into (8), we obtain the first order condition for the optimal rotation T

$$E'(T) = r\left[E(T) + \frac{E(T) - Ce^{rT}}{e^{rT} - 1}\right] \tag{9}$$

which is identical to the first-order condition (Equation (2)) for the rotation age that maximizes the land value[10].

Equation (9) means that the marginal indicator per cent at the optimal rotation age equals to the market interest rate. Since we know that the maximum internal rate of return $\rho(T)$, which

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[9] Published in Ekonomiska Samfundets Tidskrift based on an unpublished manuscript probably written around the start of the 20th century. Wicksell called ρ the yield energy.

[10] There are other economists that have recommended determining the rotation period by maximizing the rate of return. One of them is Kenneth Boulding (1935). As shown by Johansson and Löfgren (1985) the resulting rotation period will be longer than the wine ageing rotation period if max $\rho < r$, and equal to or greater than the wine ageing rotation period if $\max \rho \geq r$.

corresponds to the “average indicator per cent” during a whole rotation, should also equal the market rate of interest, we can conclude that the optimal rotation age is not characterized by maximization of the (marginal) indicator per cent.

Pressler wrote at least another paper which deals exclusively with the indicator per cent; Zum Weiserprocent (1869). This paper contains, as far as we can understand, different ways of computing it, not any discussion of how it should be used to determine when a stand is mature. More specifically, he produced formulas for the rate of return of a stand that grows from age *m* to age *n*+*m*. He defined the rate of return (Weiserprocent) *w* as:

$$(1.0w)^n = \frac{E(n+m)+\pi}{E(m)+\pi} \tag{10}$$

where $E(n)$ and $E(n+m)$ are the timber value of the stand at age *m* and age *n*+*m*, respectively, and π is the value of the land.

The left hand side is the compound yield of one thaler over n years and this is put equal to the total capital (land and stand) at *n*+*m* divided by the corresponding total capital at *m*. To determine the value of land at the optimal rotation he uses a market interest rate (Wirthscaftszinsfuss), which at the time was typically set at 3 per cent in applications to forestry. How he finds the optimal rotation, which is necessary to determine π, is not clear. By taking the *n-th* root he can find the indicator per cent *w*:

$$(1+0.w) = \sqrt[n]{\frac{E(n+m)+\pi}{E(m)+\pi}} \tag{11}$$

which is a practical “approximation”[11] of the (marginal) indicator per cent derived in equation (6) above.

The paper ended with a numerical example[12] where he used a 3 per cent interest rate for the calculation of land value and *n* =10. He ends up with an indicator per cent equals 2.74 per cent. He did not comment on whether the stand should be harvested at the beginning, during, or at the end of the 10-year period, however.

[11] For people who did not have access to mathematical tables he presents a simple formula which gives a surprising good approximation to the solution of formula (11).
[12] Page 324.

Who was the first to use the indicator per cent as a ranking device?

It is an interesting coincidence that in the same issue of Tharandter Jahrbuch where Pressler's article Zum Weiserprocent was published there was a paper by Friedrich Judeich, who criticized the Professor of Economics at Münich, Helferich, who in an article Die Waldrente in Zeitschrift für Staatswissenschaft (Helferich 1867) claimed that there was no reason to account for (the interest on) the value of land when you want to determine the optimal rotation period. Judeich is not very technical and draws rather heavily on Pressler's earlier work on the indicator per cent. He also made some correct observations about the optimal rotation age; e.g., that it is typically lower than what you would get if one cuts the stand when average value growth culminates. He was, however, unable to convince the reader that he understood that the indicator percent is part of a "first order condition" to determine the optimal rotation period.

Stridsberg (1956) also mentions a short paper by von Seckendorf (1867) AFJZ claiming that the latter has seen the light. In his reference list he changes to von Seckendorf (1868) AFJZ supplement pp 164-168. However, there is no supplement to the 1868 volume of AFJZ. There is a supplement to the 1869 issue, but in the only volume available in Sweden pages 161-168 are missing[13].

Stridsberg mentions in particular the "proof" by Kraft (1885). And that is a good idea since Kraft handles the problem completely correct and ends up with a formula similar equivalent to equation (5) above, which he calls an approximation formula (ein näherungsformel), perhaps since he knows that he is dealing with a discrete time problem. In Kraft's approach there is no longer any doubt that he knows how to use the indicator per cent to distinguish between mature and immature stands.

Conclusion

The way Pressler discussed the indicator per cent seems to indicate that its main use is to keep the yield in a stand as high as possible by silvicultural means. However, using the indicator per cent to choose among silvicultural options is generally incorrect. It is hard to prove, not to say impossible, that Pressler did not realize that his invention of the indicator per cent can be used to determine when it is time to cut down a forest stand and obtain the highest profit. In

[13] Heyer (1871) is another reference in Stridsberg (1956), bur we have not yet been able to find the book.

one of his papers (Pressler 1860b), there is, however, a line indicating that clear–cutting is one activity where formula (6) above matters. He writes[14]:

"As long as a timber producer or a forester is in the dark about the indicator increment of his timber in the various age-classes, under different treatments and especially in regard to clear cutting- he will bear resemblance to a producer, user and manager of forces and capital who, in this respect tends to deal rather scientifically and systematically with matters of secondary importance without considering the essential point, i.e. the virtual mode of operation of these values, forces and capitals".

On the other hand, the example in his (1869) paper, where the indicator per cent ends up at 2.74 per cent, while the discount rate determining the value of land is 3 per cent, and where there is no comment from Pressler about maturity seems to indicate that he is the one that misses an essential point.

It should be mentioned that, although one can use the indicator per cent to determine at which age a stand should be harvested, the rotation age determined this way is optimal only if one knows the correct value of the land. In other words, from a theoretical point of view, one must know the optimal rotation age first (for calculating the correct land value) in order to use the indicator per cent to determine the optimal rotation age. Nevertheless, the indicator per cent (calculated using an ad hoc or market value of the land) is a practically useful "indicator" of the maturity of a forest stand, because in most realistic situations, an error in the land value would not have any significant effect on the "optimal rotation age" determined using the indicator per cent.

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