Optimal Taxation and Asymmetric Information in an Economy with Second-Hand Trade*

Thomas Aronsson, Tomas Sjögren and Mikael Witterblad
Department of Economics, Umeå University
SE - 901 87 Umeå, Sweden

February 2008

Abstract

This paper concerns optimal income and commodity taxation in a two-type overlapping generations model, where used durable goods are traded in a second-hand market. As second-hand transactions are difficult to observe, we assume that the government is unable to directly control second-hand transactions via commodity taxation. A basic question is how the government in this case may use the second-hand market as a channel for relaxation of the self-selection constraint. We

*The authors would like to thank David Granlund and Magnus Wikström for helpful comments and suggestions. Research grants from FORMAS and The Swedish Research Council (Linneaus project: Ageing and Living Conditions) are also gratefully acknowledged.

show how the appearance of a second-hand market for used durable goods affects the optimal use of labor income and capital income taxation as well as the optimal use of commodity taxation on new durable goods.

Keywords: Optimal taxation, Intertemporal Choice, Durable Goods.

JEL Classification: D91, H 21, H23.

1 Introduction

There is a large literature dealing with redistribution under asymmetric information, in which the optimal use of nonlinear income taxation and the optimal mix of income and commodity taxation are two important issues. Although this research has produced insights of considerable value, most studies are based on (at least) two important simplifications: the economy is described by a static model, and (in case of a mixed tax system) there is typically a separate tax instrument for each commodity (except the numeraire). A consequence of this reference-model is a tendency to abstract from durable goods and, in particular, that used durable goods might be traded in second-hand markets; transactions which for obvious reasons are difficult to observe (and tax) by the government. In real world market economies, this type of trade may refer to a variety of durable goods including vehicles[1] such as boats and snowmobiles, kitchen equipment such as stoves, refrigerators and freezers, gardening tools such as lawnmowers, as well as different kinds of furnitures: commodities of importance for the standard of living and, therefore, also relevant from the perspective of redistribution.

[1]The second-hand market for used cars, on the other hand, is not an equally good example, as governments often keep car registers (which include all cars in traffic).

This paper deals with optimal income and commodity taxation in an overlapping generations (OLG) model with asymmetric information, where used durable goods are traded in a second-hand market[2,3]. The argument that second-hand transactions are difficult to observe will be taken to its extreme point, as we assume that the government is unable to directly control second-hand transactions via commodity taxation. Therefore, a basic question is how the appearance of a second-hand market (where the transactions are untaxed) affects the optimal use of income taxation as well as the optimal use of other commodity taxes.

Our paper is based on, and tries to combine, insights from two (related) strands of literature on redistribution under asymmetric information; (i) the study of mixed taxation and (ii) the study of optimal income taxation in dynamic economies. The former literature typically uses static models, and a major question is whether, and how, commodity taxes ought to supplement the income tax. Earlier research in this area shows that, if the redistribution is designed to benefit low-income agents (which is the common assumption), one may relax the self-selection constraint by encouraging the consumption of commodities which are substitutable for leisure, and discouraging the consumption of commodities which are complementary with leisure (e.g., Ed-

[2]The existing literature on optimal taxation and durable goods focuses on issues other than redistribution under asymmetric information. Much of the earlier research is found in the area of environmental economics; see, e.g., Runkel (2004) who analyzes taxation of a pollution-generating durable good in an economy where the producers operate under imperfect competition. There is also a large literature on environmental policy in intertemporal models (discussed elsewhere), in which the environmental quality has durable good properties.

[3]A major issue in the literature on second-hand trade is the trade with used cars, and several studies are based on the adverse selection approach developed by Akerlof (1970). See, e.g, Hendel and Lizzeri (1999), Porter and Sattler (1999) and Gilligan (2004).

wards et al. 1994)[4]. Furthermore, there is an incentive for the government to discourage the consumption of a commodity, if an increase in the consumer price of this particular commodity leads to less wage inequality, while the opposite policy incentive applies if an increase in the consumer price leads to more wage inequality (see Naito 1999 for a similar argument).

The literature dealing with redistribution under asymmetric information in dynamic economies has so far mainly focused on income taxation; more precisely, a major question is whether capital income taxation ought to supplement the labor income tax. The seminal contribution is a paper by Ordover and Phelps (1979), which is based on a model with a continuum of ability-types and fixed before tax wage rates. The results show that the marginal capital income tax rate should be zero for each ability-type, if leisure is separable from private consumption in the utility function. Brett (1997) considers an OLG framework with two ability-types (an extension of the two type model developed by Stern 1982 and Stiglitz 1982) and relaxes the separability assumption mentioned above. He finds that the marginal capital income tax rate of the low-ability type (who is the mimicked agent in his study) can be either positive or negative, whereas the marginal capital income tax rate of the high-ability type remains equal to zero. Pirttilä and Tuomala (2001) also use an OLG framework while adding endogenous gross wages rates. Their results show (in addition to mechanism emphasized by Brett) that intertemporal production inefficiency at the second best opti-

[4]If leisure is weakly separable from the other goods in the utility function - and with fixed gross wage rates - a government that is able to use a nonlinear income tax has no need for differentiated commodity taxation; see Atkinson and Stiglitz (1976).

mum may justify capital income taxation[5,6], whereas the labor income tax structure corresponds to that derived by Stiglitz (1982)[7] in a static model.

The present study is based on a two-type model, where one of the consumption goods is a durable good. We assume that new and used units of the durable good - to be referred to as the new and used durable good, respectively, - are imperfect substitutes in consumption. In addition, as our model does not restrict the analysis to a linear technology, it follows that the before tax wage rates are endogenous. The government redistributes via nonlinear labor income and capital income taxation supplemented by a linear commodity tax on the new durable good. As indicated above, there is no commodity tax on the used durable good (when sold on the second-hand market), meaning that the government has fewer effective policy instruments than the number of variables it wishes to control[8].

Our study contributes to the literature in primarily two ways. First, and foremost, we add a durable private good and a second-hand market to the framework for studying redistribution under asymmetric information. For

[5] By a similar argument, Aronsson et al. (in press) show that the appearance of equilibrium unemployment (caused by non-competitive wage formation) may justify capital income taxation, as it implies intertemporal production inefficiency.

[6] Other arguments for using capital income taxation are, e.g., that capital taxes may serve as an indirect instrument to tax unobservable wealth (Boadway et al. 2000) and paternalistic motives caused by differences in the utility discount rate between the consumers and the social planner (Tenhunen and Tuomala 2007).

[7] This means of positive marginal labor income tax rate for the low-ability type and a negative marginal labor income tax rate for the high-ability type.

[8] Although in contexts different from ours, certain aspects of optimal taxation in economies with 'too few' tax instruments have been addressed in earlier literature; e.g., national tax policies under transboundary environmental problems (Aronsson and Blomquist 2003, Aronsson et al. 2006) and alcohol taxation in economies with cross-border shopping (Christiansen 2003, Aronsson and Sjögren 2007).

reasons mentioned above, this extension has practical relevance as well as adds a new theoretical dimension by comparison with earlier literature. Second, and to the best of our knowledge, there are no earlier studies analyzing the optimal combination of labor income taxation, capital income taxation and commodity taxation in a dynamic economy with asymmetric information. As a consequence, our paper also contributes to the study of mixed taxation more generally.

The results show how and why the second-hand market for the used durable good influences the incentives underlying the commodity tax implemented on the new durable good as well as the incentives underlying income taxation. The basic mechanism that our paper emphasizes is that a change in the second-hand price (which can be accomplished via tax policy) may contribute to relax the self-selection constraint. Therefore, the second-hand market constitutes in a sense an additional channel of relevance for redistribution. As the government is unable to directly tax the second-hand trade, the income taxes and the commodity tax on the new durable good will, in our framework, partly serve as (imperfect) instruments for influencing the second-hand market. As we will argue below, the underlying motive for modifying the commodity tax on the new durable good is similar to the arguments for commodity taxation described above: we ought to encourage (discourage) the consumption of commodities which are substitutable for (complementary with) leisure, with the addition that, in our case, the commodity tax on the new durable good will depend on whether leisure is complementary with, or substitutable for, both the new <u>and</u> used durable good. The modification of the income tax structure caused by the second-hand trade depends on (among other things) how the consumption of the used durable good correlates with the labor supply and savings behavior, which are the aspects of behavior targeted by income taxation.

The outline of the paper is as follows. Section 2 presents the model and the outcome of private optimization as well as describes the optimal tax and expenditure problem. The policy-outcome in terms of optimal taxation is discussed in Section 3. Section 4 provides a summary and concluding remarks.

2 The Model

This section describes the decision-problems facing the consumers, firms and the government. The outcome in terms of optimal taxation will be analyzed in the next section.

2.1 Consumers

Let us assume a closed economy, where each consumer generation lives for two periods. There are two types of consumers; a low-ability type (denoted by superindex 1) and a high-ability type (denoted by superindex 2). The distinction between ability-types refers to productivity, meaning that the high-ability type is more productive and faces a higher gross wage rate than the low-ability type. To simplify the calculations, we assume that the number of consumers of each ability-type is constant across generations. Without loss of generality, therefore, we normalize the number of consumers of each ability-type and generation to one.

Consider a consumer of ability-type i born in the beginning of period t, who is young in period t and old in period $t+1$. The consumer derives utility form the consumption of a nondurable good and a durable good when young and old, respectively, as well as from leisure. Let c_{1t}^i and c_{2t+1}^i denote the consumption of the nondurable good when young and old, respectively, and z_t^i the amount of leisure consumed when young. Leisure is, in turn, defined as $z_t^i = h - l_t^i$, where h is the total time available in a given period and l_t^i the hours of work supplied when young. Following earlier related literature, we assume that the consumer does not work when old.

The durable good has a life-length of two time periods. When young, the consumer buys x_{1t}^i new units of the durable good and η_{1t}^i used units of the durable good. The used units are bought in a second-hand market and has a remaining life-length of one period. It is assumed that wear and tear means that new and used durable goods are imperfect substitutes. When the consumer becomes old, the amount of the new durable good bought in the previous period, x_{1t}^i, becomes units of a used durable good, which can be sold in the second-hand market. The units sold when old will be denoted χ_{2t+1}^i.

Therefore, the utility function of ability-type i born in period t can be written as

$$U_t^i = U\left(c_{1t}^i, x_{1t}^i, \eta_{1t}^i, z_t^i, c_{2t+1}^i, x_{1t}^i - \chi_{2t+1}^i\right), \tag{1}$$

in which the consumption of leisure when old (which is fixed) has been suppressed. The function $U(\cdot)$ is increasing in each argument and strictly concave. We assume that all goods are normal. Note also that the supply of the used durable good by the old consumer is constrained by the amount of the new durable good bought when young. Therefore, the expression $x_{1t}^i - \chi_{2t+1}^i$ denotes the consumption of the used durable good when old[9].

[9] As there are no bequests in the model, we assume that the old consumer does not buy the new durable good. In addition, note that the consumer is a net supplier of used durable goods when old.

Since the optimal tax problem set out below is formulated in terms of a conditional indirect utility function, it is convenient to start by solving the utility maximization problem conditional on the levels of leisure and saving, respectively. This decision-problem means choosing c_{1t}^{i}, x_{1t}^{i}, η_{1t}^{i}, c_{2t+1}^{i} and χ_{2t+1}^{i} to maximize the utility function given by equation (1) subject the budget constraint

$$b_{1t}^{i} = c_{1t}^{i} + q_{t,x}x_{1t}^{i} + q_{t,\eta}\eta_{1t}^{i} \tag{2}$$

$$b_{2t+1}^{i} = c_{2t+1}^{i} - q_{t+1,\eta}\chi_{2t+1}^{i} \tag{3}$$

where b_{1t}^{i} and b_{2t+1}^{i} are fixed budgets to be allocated for consumption in each period, and the nondurable good is used as the numeraire. The consumer price of the new durable good in period t, $q_{t,x}$, is given by $q_{t,x} = p_{t,x} + \tau_{t,x}$, where $p_{t,x}$ is the producer price and $\tau_{t,x}$ the commodity tax, while $q_{t,\eta}$ and $q_{t+1,\eta}$ are the (second-hand) prices of the used durable good in periods t and $t+1$, respectively. Solving this conditional utility maximization problem gives the conditional demand functions

$$c_{1t}^{i} = c_1\left(b_{1t}^{i}, b_{2t+1}^{i}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z_t^{i}\right) \tag{4}$$

$$x_{1t}^{i} = x_1\left(b_{1t}^{i}, b_{2t+1}^{i}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z_t^{i}\right) \tag{5}$$

$$\eta_{1t}^{i} = \eta_1\left(b_{1t}^{i}, b_{2t+1}^{i}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z_t^{i}\right) \tag{6}$$

$$c_{2t+1}^{i} = c_2\left(b_{1t}^{i}, b_{2t+1}^{i}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z_t^{i}\right) \tag{7}$$

and a conditional supply function

$$\chi_{2t+1}^{i} = \chi_2\left(b_{1t}^{i}, b_{2t+1}^{i}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z_t^{i}\right) \tag{8}$$

By substituting equations (4) - (8) into equation (1), we obtain the conditional indirect utility function

$$V_t^i = V\left(b_{1t}^i, b_{2t+1}^i, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z_t^i\right) \tag{9}$$

We can derive the hours of work and saving, respectively, by maximizing the conditional indirect utility function subject to the budget constraint

$$b_{1t}^i = w_t^i l_t^i - T_t\left(y_t^i\right) - s_t^i \tag{10}$$

$$b_{2t+1}^i = \left(1 + r_{t+1}\right) s_t^i - \Phi_{t+1}\left(I_{t+1}^i\right) \tag{11}$$

where w_t^i is the gross wage rate, s_t^i saving, r_{t+1} the interest rate, $y_t^i = w_t^i l_t^i$ the labor income and $I_{t+1}^i = r_{t+1} s_t^i$ the capital income. The function $T_t(\cdot)$ determines the labor income tax payment by the young generation in period t and $\Phi_{t+1}(\cdot)$ the capital income tax payment by the old generation in period $t+1$. In the analysis below, we will use the following short notations for income tax payments and marginal income tax rates

$$T_t^i = T_t\left(y_t^i\right), \quad T_{t,y}^i = \frac{\partial T_t\left(y_t^i\right)}{\partial y_t^i}$$

$$\Phi_{t+1}^i = \Phi_{t+1}\left(I_{t+1}^i\right), \quad \Phi_{t+1,I}^i = \frac{\partial \Phi_{t+1}\left(I_{t+1}^i\right)}{\partial I_{t+1}^i}$$

The first order conditions for the hours of work and saving, respectively, can then be written as

$$\frac{\partial V_t^i}{\partial b_{1t}^i} w_t^i (1 - T_{t,y}^i) - \frac{\partial V_t^i}{\partial z_t^i} = 0 \tag{12}$$

$$-\frac{\partial V_t^i}{\partial b_{1t}^i} + \frac{\partial V_t^i}{\partial b_{2t+1}^i}\left[1 + r_{t+1} - r_{t+1}\Phi_{t+1,I}^i\right] = 0 \tag{13}$$

2.2 Production

There are two competitive private production sectors; sector c produces the nondurable good and sector x the durable good. In each sector, identical firms produce a homogenous good by using a constant returns to scale technology, and we abstract from possible entry into, and exit our of, the goods markets. Given these characteristics, the number of firms in each sector is not important and will be normalized to one.

The firm in sector j, $j = c, x$, uses three variable production factors; labor of both ability-types, $l_{t,j}^1$ and $l_{t,j}^2$, and capital, $K_{t,j}$. The production function is given by $F_j\left(l_{t,j}^1, l_{t,j}^2, K_{t,j}\right)$, which is increasing and strictly concave in each argument. Normalizing with respect to $l_{t,j}^1$, we have $f_j(n_{t,j}, k_{t,j}) = F_j(l_{t,j}^1, l_{t,j}^2, K_{t,j})/l_{t,j}^1$, where $n_{t,j} = l_{t,j}^2/l_{t,j}^1$ and $k_{t,j} = K_{t,j}/l_{t,j}^1$. We assume that labor and capital are perfectly mobile across sectors, meaning that the factor prices will be the same in both sectors. The first order conditions can then be written as

$$w_t^1 = p_{t,j} f_j\left(n_{t,j}, k_{t,j}\right) - n_{t,j} p_{t,j} \frac{\partial f_j\left(n_{t,j}, k_{t,j}\right)}{\partial n_{t,j}} - k_{t,j} p_{t,j} \frac{\partial f_j\left(n_{t,j}, k_{t,j}\right)}{\partial k_{t,j}} \tag{14}$$

$$w_t^2 = p_{t,j} \frac{\partial f_j\left(n_{t,j}, k_{t,j}\right)}{\partial n_{t,j}} \tag{15}$$

$$r_t = p_{t,j} \frac{\partial f_j\left(n_{t,j}, k_{t,j}\right)}{\partial k_{t,j}} \tag{16}$$

for $j = c, x$, where $p_{t,j}$ is the producer price in sector j (recall that $p_{t,c} = 1$ from normalization).

2.3 Equilibrium

The factor market equilibrium is defined by the equilibrium conditions

$$l_t^i = \sum_j l_{t,j}^i \text{ for } i = 1, 2 \tag{17}$$

$$K_t = \sum_j K_{t,j} \tag{18}$$

where K_t is the aggregate capital stock in period t. Combining equations (14)-(18), we can solve for w_t^1, w_t^2, $l_{t,c}^1$, $l_{t,c}^2$, $l_{t,x}^1$, $l_{t,x}^2$ r_t, $K_{t,c}$ and $K_{t,x}$ as functions of $p_{t,x}$, l_t^1, l_t^2 and K_t. Substituting into the production function gives 'the equilibrium supply function'

$$S_{t,j} = S_j \left(p_{t,x}, l_t^1, l_t^2, K_t\right) \tag{19}$$

for $j = c, x$.

The equilibrium condition characterizing the market for the new durable good in period t becomes

$$S_{t,x}(\cdot) = \sum_i x_{1t}^i(\cdot) \tag{20}$$

Equation (20) implicitly defines the equilibrium producer price of the new durable good, i.e.

$$p_{t,x} = p\left(l_t^1, l_t^2, b_{1t}^1, b_{2t+1}^1, b_{1t}^2, b_{2t+1}^2, \tau_{t,x}, q_{t,\eta}, q_{t+1,\eta}, K_t\right) \tag{21}$$

Let us finally turn to the second-hand market, where the old generation sells the used durable good to the young generation. The equilibrium condition for the second-hand market in period t is given by

$$\sum_{i=1}^{2}\left(\chi_{2t}^{i}-\eta_{1t}^{i}\right)=0 \tag{22}$$

Note also that Walras' law implies that, as long as equations (20) and (22) hold, the market for the nondurable good will also be in equilibrium.

2.4 The Public Decision-Problem

The objective function facing the government is represented by a general social welfare function

$$W=W(V_0^1,V_0^2,V_1^1,V_1^2,V_2^1,V_2^2,...), \tag{23}$$

which allows for a separate welfare weight attached to the utility function of each ability-type in each generation.

The government redistributes via income and commodity taxation. Note that $T_t(\cdot)$ is a general labor income tax, which can be used to implement any desired combination of l_t^1, b_{1t}^1, l_t^2 and b_{1t}^2 (with the savings held constant). It is, therefore, convenient to follow the approach in earlier comparable literature by using l_t^1, b_{1t}^1, l_t^2 and b_{1t}^2, instead of the parameters of the function $T_t(\cdot)$, as direct decision-variables facing the government. Similarly, the general capital income tax function $\Phi_{t+1}(\cdot)$ can be used to implement any desired combination of b_{1t}^1, b_{2t+1}^1, b_{1t}^2 and b_{2t+1}^2 (with the labor income held constant); therefore, instead of using the parameters of the function $\Phi_{t+1}(\cdot)$ as decision-variables in the optimal tax problem, we add b_{2t+1}^1 and b_{2t+1}^2 to the set of direct decision-variables mentioned above. Note that, as the government effectively controls the saving behavior, it also possesses perfect control of the capital stock.

If we use equations (20) and (22), the resource constraint can be written in terms of the numeraire good as[10]

$$S_{t,c} + K_t - K_{t+1} - \sum_i c^i_{1t} - \sum_i c^i_{2t} = 0 \tag{24}$$

As in most earlier studies dealing with redistribution under asymmetric information, we assume that the government wants to redistribute from the high-ability type to the low-ability type. The government can observe the labor and capital income of each individual, although ability is private information. Therefore, it will be important to prevent the high-ability type from pretending to be a low-ability type, i.e. from becoming a mimicker, in order to gain from the redistribution policy. Note that the hours of work that the high-ability type needs to supply in order to reach the same labor income as the low-ability type is given by $\hat{l}^2_{1t} = (w^1_t/w^2_t)l^1_{1t} = \phi_t l^1_t$, where $\phi_t = \phi(p_{t,x}, l^1_t, l^2_t, K_t)$ is the wage ratio (or relative wage rate) in period t. Denoting the conditional indirect utility function of the mimicker by $\hat{V}^2_t$, the self-selection constraint that may bind becomes

$$\begin{aligned} V^2_t &= V\left(b^2_{1t}, b^2_{2t+1}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, z^2_t\right) \\ &\geq V\left(b^1_{1t}, b^1_{2t+1}, q_{t,x}, q_{t,\eta}, q_{t+1,\eta}, h - \phi_t l^1_t\right) = \hat{V}^2_t \end{aligned} \tag{25}$$

The expression on the left hand side of the weak inequality is the utility intended for the high-ability type, whereas the expression on the right hand

[10] As the government is able to transfer resources lump-sum across generations in our framework and possesses perfect control of the capital stock, we only need to consider one from the resource constraint and the government budget constraint; see Atkinson and Sandmo (1980) and Pirttilä and Tuomala (2001). Note also that we have assumed that the nondurable good can be freely transformed into physical capital.

side is the utility of the mimicker (in this case a high-ability type pretending to be a low-ability type). Note that the mimicker and low-ability type faces the same levels[11] of before-tax and disposable income both when young and old. However, the mimicker is more productive and, as a consequence, spends more time on leisure than the low-ability-type.

Finally, note that the government uses that equation (21) determines the producer price of the new durable good, $p_{t,x}$, and that the equilibrium condition for the used durable good is given by equation (22).

The Lagrangean corresponding to the government's optimization problem can be written as

$$\mathcal{L} = W + \sum_t \lambda_t[V_t^2 - \hat{V}_t^2] + \sum_t \gamma_t[S_{t,c} + K_t - K_{t+1} - \sum_i c_{1t}^i - \sum_i c_{2t}^i]$$
$$+ \sum_t \mu_t \sum_i [\chi_{2t}^i - \eta_{1t}^i]$$

where λ_t and γ_t are the Lagrange multipliers associated with the self-selection constraint and resource constraint, respectively, in period t. The constraints in the second row represent the equilibrium condition for the second-hand market for the used durable good in each period, and μ_t is the Lagrange multiplier associated with the equilibrium condition in period t. As we are focusing on the role of the second-hand market in the context of redistributive policy, this formulation is convenient as it enables us to identify the determinants of the shadow price that the government attaches to the used durable good. The additional (and artificial) direct decision-variable corresponding to this constraint is $q_{t,\eta}$. In summary, therefore, the decision-variables are l_t^1,

[11] To be a mimicker, the high-ability type must mimic the point chosen on each tax function (both the labor income tax and the capital income tax) by the low-ability type.

l_t^2, b_{1t}^1, b_{1t}^2, b_{2t}^1, b_{2t}^2, $\tau_{t,x}$, K_t and $q_{t,\eta}$ for all t. The first order conditions are presented in the Appendix.

Note finally that there is a potential time inconsistency problem. The reason is that when each consumer reveals his/her true type at the end of the first period, there may be an incentive for the government to change the structure of capital income taxation. Although this potential problem is recognized, we follow Pirttilä and Tuomala (2001) by assuming that the government can credibly commit to the announced tax and expenditure policies.

3 Optimal Taxation

The shadow price of the used durable good over the shadow price of the resource constraint, μ_t/γ_t, will play an important role for the optimal tax structure. This variable represents the marginal value that the government attaches to increased supply of the used durable good measured in terms of tax revenues. As increased supply means a lower price, ceteris paribus, one may also interpret μ_t/γ_t as indirectly reflecting the marginal cost that the government attaches to a price increase in the second-hand market.

It is instructive to begin by analyzing how μ_t/γ_t is determined. Let the symbol "˜" indicate compensated demand and/or supply. Recall from the properties of compensated supply and demand functions that

$$\frac{\partial \tilde{\chi}_{2t}^i}{\partial q_{t,\eta}} > 0 \text{ and } \frac{\partial \tilde{\eta}_{1t}^i}{\partial q_{t,\eta}} < 0, \tag{26a}$$

i.e. an increase in the price of the used durable good in period t, ceteris paribus, leads to increased compensated supply of, and decreased compensated demand for, the used durable good in period t. For purposes of interpretation, we also make the following assumptions;

$$\frac{\partial \tilde{x}^i_{1t}}{\partial q_{t,\eta}} > 0, \; \frac{\partial \tilde{\chi}^i_{2t+1}}{\partial q_{t,\eta}} > 0, \; \frac{\partial \tilde{x}^i_{1t-1}}{\partial q_{t,\eta}} > 0 \text{ and } \frac{\partial \tilde{\eta}^i_{1t-1}}{\partial q_{t,\eta}} < 0, \tag{26b}$$

To give the basic intuition behind the first and second parts of (26b), note that an increase in the price of the used durable good in period t provides an incentive for the consumer to buy more of the new durable good in period t (because new and used durable goods are substitutes). As a consequence, when the consumer becomes old (in period $t+1$), he/she has more of the used durable good at his/her disposal and, therefore, more of the used durable good to supply in the second-hand market. The intuition behind the third and fourth parts of (26b) is that the higher $q_{t,\eta}$, ceteris paribus, the stronger will be the incentive to buy the new[12] (instead of the used) durable good in period $t-1$ in order to supply the used durable good in period t.

To simplify the notations, let L_μ denote the lag operator on μ, so $L_\mu \mu_t = \mu_{t-1}$, and define

$$\sigma_t = \sum_i \left(\frac{\partial \tilde{\chi}^i_{2t}}{\partial q_{t,\eta}} - \frac{\partial \tilde{\eta}^i_{1t}}{\partial q_{t,\eta}} \right) + \sum_i \frac{\partial \tilde{\chi}^i_{2t+1}}{\partial q_{t,\eta}} L_\mu^{-1} - \sum_i \frac{\partial \tilde{\eta}^i_{1t-1}}{\partial q_{t,\eta}} L_\mu > 0, \tag{26}$$

as measuring how an increase in $q_{t,\eta}$ affects the intertermporal net supply of the used durable good. This effect is, in turn, decomposable into in three components. The first is a positive direct effect on the net supply in period t. The second and third components reflect that an increase in $q_{t,\eta}$ gives rise to intertemporal behavioral responses, as is leads to increased supply of the used durable good in period $t+1$ and reduced demand for the used durable good in period $t-1$ according to the assumptions made above.

[12] This is interpretable to mean that the purchase of the new durable good contains a savings-component, as an increase in $q_{t,\eta}$ means a higher 'return' on the investment in the new durable good made at time $t-1$.

Consider the following result;

Proposition 1: *At the second best optimum, the shadow price of the used durable good over the shadow price of the government's budget constraint can be written as*

$$
\begin{aligned}
\frac{\mu_t}{\gamma_t} &= \frac{1}{\sigma_t \gamma_t}\left[\lambda_t \frac{\partial \hat{V}_t^2}{\partial b_{1t}^1}[\eta_{1t}^1 - \hat{\eta}_{1t}^2] + \lambda_{t-1}\frac{\partial \hat{V}_{t-1}^2}{\partial b_{2t}^1}[\hat{\chi}_{2t}^2 - \chi_{2t}^1]\right] \\
&\quad - \frac{1}{\sigma_t \gamma_t}\sum_{k=0}^{1}\sum_i \frac{\partial \tilde{x}_{1t-k}^i}{\partial q_{t,\eta}}\left[\frac{\rho_{t-k}\lambda_{t-k}}{D_t}\frac{\partial \phi_{t-k}}{\partial p_{t-k,x}} + \tau_{t-k,x}\gamma_{t-k}\right]
\end{aligned}
$$

where

$$
\rho_{t-k} = \frac{l_{t-k}^1 \partial \hat{V}_{t-k}^2 / \partial \hat{z}_{t-k}^2}{(1 + \partial p_{t-k,x}/\partial q_{t,\eta})} > 0 \text{ and } D_t = \frac{\partial S_{t,x}}{\partial p_{t,x}} - \sum_i \frac{\partial x_{1t}^i}{\partial q_{t,x}} > 0
$$

Proof: See the Appendix.

To interpret Proposition 1, note first that if the self-selection constraint does not bind (so $\lambda_{t-1} = \lambda_t = 0$), and if the commodity tax on the new durable good is always equal to zero (so $\tau_{t-1,x} = \tau_{t,x} = 0$), then $\mu_t/\gamma_t = 0$. This means that, in the absence of asymmetric information or other distortions, the constraint associated with the second-hand market becomes redundant.

With a binding self-selection constraint, on the other hand, the second-hand market is no longer redundant; instead, the second-hand market provides an additional channel via which the government may relax the self-selection constraint. The first row of the expression for μ_t/γ_t is directly related to the self-selection constraint; the two components reflect differences between the low-ability type and the mimicker with respect to the demand for the used durable good when young (the first term) and the supply of the used durable good when old (the second term). To understand why such differences in the consumption pattern may arise, recall that although the low-ability type and the mimicker face the same levels of before-tax and disposable income both when young and old, the mimicker is more productive and spends more time on leisure than the low-ability type. If leisure is substitutable for the used durable good in the sense that the young low-ability type consumes more of the used durable good than the young mimicker in period t, so $\eta_{1t}^1 - \hat{\eta}_{1t}^2 > 0$, the first term on the right hand side contributes to increase the marginal value that the government attaches to the used durable good. The intuition is that a lower price of the used durable good means a larger utility gain for the low-ability type than it does for the mimicker. As we will argue below, this mechanism provides an incentive for the government to try to reduce the price of the used durable good, i.e. making it less scarce, which contributes to relax the self-selection constraint. The argument goes the other way around if leisure is complementary with the used durable good in the sense that the young low-ability type consumes less of the used durable good than the young mimicker in period t.

The second term in the first row is an intertemporal analogue to the first term. If the old mimicker supplies more of the used durable good than the old low-ability type in period t, so $\hat{\chi}_{2t}^2 - \chi_{2t}^1 > 0$, the second term on the right hand side contributes to increase the marginal value that the government attaches to the used durable good in period t. The intuition behind this example is based on the assumption that leisure is substitutable for the used durable good in the sense that the young low-ability type consumes more than the mimicker in period $t-1$, in which case the mimicker (who bought more of the new durable good in period $t-1$) is likely to be able to sell more of the used durable good in period t. Again, the argument goes the other way around if leisure is complementary with the used durable good in the sense that the young low-ability type consumes less of the used durable than the mimicker in period $t-1$.

The first part of the second row is also related to the self-selection constraint although for a different reason; the relative wage rate is endogenous and will, therefore, respond to a change in the price of the used durable good. There are two mechanisms involved (an atemporal and an intertemporal). First, as the new and used durable good are substitutes in consumption, we have $\partial\tilde{x}_{1t}^{i}/\partial q_{t,\eta} > 0$, suggesting that an increase in $q_{t,\eta}$ causes upward pressure on $p_{t,x}$. If $\partial\phi_t/\partial p_{t,x} > 0$, it follows that an increase in the price of the used durable good in period t leads to an increase in the wage ratio in period t, which makes mimicking less attractive and contributes to relax the self-selection constraint. Therefore, as a higher $q_{t,\eta}$ makes mimicking less attractive, this provides an incentive for the government to attach a lower marginal value to the used durable good (or, equivalently, attach a lower marginal cost to an increase in the price of the used durable good) than it would otherwise have done. The argument will be the opposite if $\partial\phi_t/\partial p_{t,x} < 0$. Second, to interpret the relationship between $q_{t,\eta}$ and ϕ_{t-1}, recall our assumption that an increase in $q_{t,\eta}$ leads to increased compensated demand for the new durable good in period $t-1$, i.e. $\partial\tilde{x}_{1t-1}^{i}/\partial q_{t,\eta} > 0$. This suggests that an increase in $q_{t,\eta}$ also causes upward pressure on $p_{t-1,x}$. Therefore, the interpretation of the relationship between $q_{t,\eta}$ and ϕ_{t-1} is analogous to the interpretation of the relationship between $q_{t,\eta}$ and ϕ_t.

The second part of the second row contains tax base effects, as an increase in the price of the used durable good affects the demand for the new durable good and, therefore, also the tax revenues. As $\partial\tilde{x}_{1t-1}^{i}/\partial q_{t,\eta} > 0$ and $\partial\tilde{x}_{1t}^{i}/\partial q_{t,\eta} > 0$ by assumption, the tax base effects provide an argument for the government to attach a lower marginal value to the used durable good (i.e. it attaches a lower marginal cost to an increase in the price of the used durable good) than it would otherwise have done, as an increase in the price leads to increased tax revenues.

3.1 Commodity Taxation

Let us now turn to the commodity tax on the new durable good and, in particular, how the appearance of a second-hand market for the used durable good modifies the use of commodity taxation. For notational convenience, we introduce the short notation;

$$H_t = \sum_i \frac{\partial \tilde{x}_{1t}^i}{\partial q_{t,x}} < 0$$

Consider Proposition 2;

Proposition 2: *At the second best optimum, the commodity tax on the new durable good can be written as*

$$\begin{aligned} \tau_{t,x} = & \frac{\lambda_t \partial \hat{V}_t^2 / \partial b_{1t}^1}{\gamma_t H_t} [x_{1t}^1 - \hat{x}_{1t}^2] - \frac{\rho_t \lambda_t}{\gamma_t D_t} \frac{\partial \phi_t}{\partial p_{t,x}} \\ & + \frac{\mu_t}{\gamma_t} \left[\sum_i \frac{\partial \tilde{\eta}_{1t}^i}{\partial q_{t,x}} - \sum_i \frac{\partial \tilde{\chi}_{2t+1}^i}{\partial q_{t,x}} L_\mu^{-1} \right] \frac{1}{H_t} \end{aligned}$$

Proof: See the Appendix.

The first row of the formula in Proposition 2 reflects the self-selection constraint. If leisure is complementary with the new durable good in the sense that the young low-ability type consumes less than the young mimicker, so $x_{1t}^1 - \hat{x}_{1t}^2 < 0$, then the first term on the right hand side contributes to increase the commodity tax on the new durable good. The intuition is that an increase in the consumer price of the new durable good will, in this case, cause a greater utility loss for the mimicker than it does for the low-ability type, which relaxes the self-selection constraint. Therefore, a higher commodity tax creates room for additional redistribution. By an analogous argument, if leisure is substitutable for the new durable good, so $x_{1t}^1 - \hat{x}_{1t}^2 > 0$, the first term on the right hand side contributes to a lower commodity tax. This effect is well understood from earlier literature on redistribution under asymmetric information based on static models; e.g. Edwards et al. (1994).

The second term in the first row appears because a higher tax on the new durable good leads to a lower producer price which, in turn, affects the wage ratio. If $\partial \phi_t / \partial p_{t,x} > 0$ (< 0), this reduction in $p_{t,x}$ makes mimicking more (less) attractive, as the mimicker needs to supply less (more) hours of work to reach the same labor income as the low-ability type. This provides an incentive for the government to implement a lower (higher) tax on the new durable good than it would otherwise have done. An analogous argument for commodity taxation is derived by Naito (1999) in the context of a static model.

The second row of the formula in Proposition 2 is novel and arises because the government attaches a nonzero marginal value to the used durable good. We will refer to the expression summarized by the second row as the 'direct effect' of the second-hand market, as its qualitative contribution is measured with the other terms (i.e. in the first row) held constant. This direct effect may either contribute to increase or decrease the commodity tax. To begin with, note that the expression within the square bracket in the second row is positive. First, as new and used durables are substitutes in consumption, we have $\partial \tilde{\eta}_{1t}^i / \partial q_{t,x} > 0$. Second, as an increase in the consumer price of the new durable good in period t reduces the consumption of the new durable good in period t, ceteris paribus, it will (according to our earlier assumptions) also contribute to reduce the supply of the used durable good in period $t+1$, i.e. $\partial \tilde{\chi}^i_{2t+1}/\partial q_{t,x} < 0$. Therefore, the following result is a direct consequence of Proposition 2;

Corollary 1: *If $\mu_t/\gamma_t > 0$ (< 0), the direct effect of the second-hand market is to decrease (increase) the commodity tax on the new durable good.*

The intuition is that a lower commodity tax on the new durable good contributes to reduce the price of the used durable good in period t and increase the supply of the used durable good in period $t+1$. This leads to higher welfare if $\mu_t/\gamma_t > 0$ (in which case it is desirable to make the used durable good less scarce) and lower welfare if $\mu_t/\gamma_t < 0$.

Note finally that the direct effect of the second-hand market is, in a sense, similar to the effect of the first term on the right hand side of the tax formula in Proposition 2, as the sign of each of them is dependent on whether leisure is complementary with, or substitutable for, the durable good. As such, they will under certain conditions reinforce each other. If leisure is substitutable for the new and used durable good, and if the latter effect is strong enough to imply that $\mu_t/\gamma_t > 0$[13], then these terms represent two sides of the same argument for subsidizing the new durable good. The analogous (joint) argument for taxation may apply if leisure is complementary with the new and used durable good.

[13] Recall from Proposition 1 that $\eta^1_{1t} > \hat{\eta}^2_{1t}$ contributes to increase μ_t/γ_t.

3.2 Labor Income Taxation

As the government has fewer effective policy instruments than the number of variables it wishes to control, the appearance of a second-hand market is also important from the point of view of the income tax structure. Let us begin by characterizing the marginal labor income tax rates. Define

$$MRS_{t,zb}^{i} = \frac{\partial V_t^i/\partial z_t^i}{\partial V_t^i/\partial b_{1t}^i} \text{ and } M\hat{R}S_{t,zb}^{2} = \frac{\partial \hat{V}_t^2/\partial \hat{z}_t^2}{\partial \hat{V}_t^2/\partial b_{1t}^1},$$

where $\hat{z}_t^2 = h - \phi_t l_t^1$, to be the marginal rate of substitution between leisure and private income for ability-type i and the mimicker, respectively, in period t. To simplify the expressions for the marginal labor income tax rates, we shall also use the short notation

$$\Delta_{t,l}^{i} = \frac{\partial \phi_t}{\partial l_t^i} - \frac{1}{(1+\partial p_{t,x}/\partial \tau_{t,x})} \frac{\partial \phi_t}{\partial p_{t,x}} \frac{\partial S_{t,x}/\partial l_t^i + \partial \tilde{x}_{1t}^i/\partial z_t^i}{D_t}, \tag{27}$$

where $1 + \partial p_{t,x}/\partial \tau_{t,x} > 0$, for how an increase in the hours of work supplied by ability-type i affects the wage ratio (which involves a direct effect and an indirect effect via the producer price of the new durable good). Consider Proposition 3;

Proposition 3: *At the second best optimum, the marginal labor income tax rates can be written as*

$$T^1_{t,y} = \frac{1}{w^1}\{\lambda_{t,b}[MRS^1_{t,zb} - M\hat{R}S^2_{t,zb}\phi_t] - \lambda_{t,l}\Delta^1_{t,l} + \tau_{t,x}\frac{\partial \tilde{x}^1_{1t}}{\partial z^1_t}$$
$$- \frac{\mu_t}{\gamma_t}[\frac{\partial \tilde{\eta}^1_{1t}}{\partial z^1_t} - \frac{\partial \tilde{\chi}^1_{2t+1}}{\partial z^1_t}L^{-1}_{\mu}]\}$$

$$T^2_{t,y} = \frac{1}{w^2}\{-\lambda_{t,l}\Delta^2_{t,l} + \tau_{t,x}\frac{\partial \tilde{x}^2_{1t}}{\partial z^1_t}$$
$$- \frac{\mu_t}{\gamma_t}[\frac{\partial \tilde{\eta}^2_{1t}}{\partial z^2_t} - \frac{\partial \tilde{\chi}^2_{2t+1}}{\partial z^2_t}L^{-1}_{\mu}]\}$$

where $\lambda_{t,b} = [\lambda_t/\gamma_t][\partial \hat{V}^2_t/\partial b^1_{1t}] > 0$ and $\lambda_{t,l} = [\lambda_t/\gamma_t][\partial \hat{V}^2_t/\partial \hat{z}^2_{1t}]l^1_t > 0$.

Proof: See the Appendix.

The marginal labor income tax rates in Proposition 3 reflect a combination of three underlying motives for influencing the hours of work; (i) a desire to relax the self-selection constraint via channels that do not directly depend on the second-hand market, (ii) a desire to offset distortions caused by the commodity tax, and (iii) a desire to use the second-hand market as an additional channel for redistribution. The latter follows because the government lacks a tax instrument by which it may directly control the second-hand trade (meaning that the use of the other tax instruments will be modified accordingly). We shall discuss each of these incentives below.

The first two motives mentioned above are captured by the first row in each tax formula in Proposition 3. In the formula for the low-ability type, the first term on the right hand side is positive and appears because the low-ability type needs to forego more leisure than the mimicker in order to accomplish a given increase the disposable income; in our model, this implies $MRS_{t,zb}^{1} - M\hat{R}S_{t,zb}^{2}\phi_t > 0$. This incentive to tax the labor income of the low-ability type at the margin is well understood from earlier work by Stiglitz (1982), and the intuition is that a higher marginal labor income tax rate for the low-ability type makes mimicking less attractive which, in turn, contributes to relax the self-selection constraint.

The remaining two terms in the first row of the tax formula for the low-ability type have the same structure as their counterparts in the tax formula for the high-ability type. As we saw above, the variable $\Delta_{t,l}^{i}$ is decomposable into two parts, both of which serve to relax the self-selection constraint in period t via the wage ratio (recall that $\Delta_{t,l}^{i}$ is multiplied by λ_t in the tax formulas in Proposition 3). First, l_t^1 and l_t^2 directly affect the wage ratio; if $\partial\phi_t/\partial l_t^1 < 0$ and $\partial\phi_t/\partial l_t^2 > 0$, which is arguably realistic and in line with results derived by Stiglitz (1982), the direct effect of the hours of work on the wage ratio will contribute to increase the marginal labor income tax rate of the low-ability type and decrease the marginal labor income tax rate of the high-ability type. Second, in our framework, the hours of work also affect the wage ratio indirectly via the producer price of the new durable good, which is seen from the second term on the right hand side of equation (27). In general, as an increase in the hours of work[14] leads to increased supply of the new

[14]Note that the marginal labor income tax rate for ability-type i is calculated by using the first order conditions for l_t^i and b_{1t}^i. Therefore, when we change the hours of work, the private disposable income will also change in order to maintain budget balance for the government. This also explains why the change in the demand for the new durable good (which, in part, determines how an increase in the hours of work affects the producer price of the new durable good) is measured in terms of the compensated demand; see the second part of the expression for $\Delta_{t,l}^{i}$.

durable good, whereas the demand for the new durable good may change in either direction, the sign of the corresponding change in the producer price is ambiguous. To begin with, suppose that the supply response dominates in the sense that an increase in the hours of work leads to a lower producer price of the new durable good. In this case, therefore, if $\partial\phi_t/\partial p_{t,x} > 0$ (< 0), the indirect effect of l_t^i on ϕ_t constitutes an incentive for the government to implement a higher (lower) marginal labor income tax rate on ability-type i than it would otherwise have done. The intuition is that the resulting change in the producer price makes mimicking less attractive, which contributes to relax the self-selection constraint. If, on the other hand, an increase in the hours of work leads to a higher producer price of the new durable good, the desire to relax the self-selection constraint implies policy incentives opposite to those just described.

The final term in the first row of each tax formula appears because the income tax might be used to offset distortions caused by the commodity tax. To see this, note that the higher the optimal $\tau_{t,x}$, the higher (lower) will be the marginal labor income tax rate facing ability-type i, if leisure is complementary with (substitutable for) the new durable good in the sense that $\partial\tilde{x}_{1t}^i/\partial z_t^i > 0$ (< 0). For instance, if leisure is complementary with the new durable good, the increase in the marginal income tax rate caused by the final term in the first row leads to increased demand for leisure and, therefore, increased consumption of the new durable. By analogy, if leisure is substitutable for the new durable good, a lower marginal income tax rate can be used to increase the consumption of the new durable. The intuition is that the government has no incentive to distort the consumption of the new durable good per se (there is no externality or other market failure associated with the new durable good); only to carry out redistribution in the most efficient way. Another (yet equivalent) interpretation is that the final term in the first row measures a tax base effect, suggesting if leisure complementary with (substitutable for) the new durable good, then the government will implement a higher (lower) marginal labor income tax rate than it would otherwise have done in order to gain revenues from the commodity tax.

The second row of each tax formula captures the effect of the second-hand market on the marginal labor income tax rate. This effect depends on (i) whether the marginal value that the government attaches to the used durable good is positive or negative, and (ii) whether leisure is complementary with, or substitutable for, the new and used durable goods. The latter means that the government may use the labor income tax structure in period t to affect the price of the used durable good in periods t <u>and</u> $t+1$.

Consider the role played by the two terms in the square bracket in the second row of each formula in Proposition 3. If leisure is complementary with the used durable good in the sense that $\partial\tilde{\eta}_{1t}^{i}/\partial z_t^i > 0$, and if $\mu_t/\gamma_t > 0$ (< 0), the first term within the square contributes to decrease (increase) the marginal labor income tax rate facing ability-type i. The intuition is that the government may use the marginal labor income tax rates in period t to affect the price of the used durable good in period t; to be more specific, there is an incentive to reduce the price via a lower marginal labor income tax rate if $\mu_t/\gamma_t > 0$, and an incentive to increase the price via a higher marginal labor income tax rate if $\mu_t/\gamma_t < 0$. The policy incentives will be opposite to those just described if leisure is substitutable for the used durable good. By analogy, if leisure is complementary with the new durable good, increased consumption of leisure in period t leads to increased supply of the used durable good in period $t+1$, so $\partial\tilde{\chi}_{2t+1}^{i}/\partial z_t^i > 0$. In this case, therefore, and if $\mu_t/\gamma_t > 0$ (< 0), then the second term within the square bracket contributes to increase (decrease) the marginal labor income tax rate facing ability-type i, whereas opposite incentives apply if leisure is substitutable for the new durable good. The basic intuition is that the government can use the marginal income tax rates in period t to affect the supply and, therefore, the price of the used durable good in period $t+1$.

As the contribution of the second-hand market to the labor income tax structure is complex, and depends on several components whose qualitative effects cannot be determined unambiguously, let us also discuss a specific example. Consistent with the idea that new durable goods are to a greater extent consumed by the high-ability type, while used durable goods are consumed to a greater extent by the low-ability type, suppose that leisure is complementary with the new durable good and substitutable for the used durable good in the sense that $\partial \tilde{\chi}^i_{2t+1}/\partial z^i_t > 0$ and $\partial \tilde{\eta}^i_{1t}/\partial z^i_t < 0$. In addition, suppose that the tendency to consume the used durable good by the low-ability type is strong enough to imply that the government attaches a positive marginal value to the used durable good. These additional assumptions mean that the second-hand market will contribute to higher marginal labor income tax rates for both ability-types. The intuition is that increased marginal labor income taxation will, in this case, unambiguously contribute to a lower price of the used durable good, which by our assumptions is in line with the distributional profile of the public policy.

3.3 Capital Income Taxation

Let us now turn to the marginal capital income tax rates. The main concern will be to analyze how the appearance of a second-hand market modifies the use of capital income taxation. This question has clear practical relevance, since the government (by assumption) is unable to control the second-hand market via a direct instrument.

To shorten the notations as much as possible, let

$$MRS^i_{t,b_1b_2} = \frac{\partial V^i_t/\partial b^i_{1t}}{\partial V^i_t/\partial b^i_{2t+1}} \text{ and } M\hat{R}S^2_{t,b_1b_2} = \frac{\partial \hat{V}^2_t/\partial b^1_{1t}}{\partial \hat{V}^2_t/\partial b^1_{2t+1}}$$

denote the marginal rate of substitution between present and future disposable income for ability-type i and the mimicker, respectively, of generation t. In addition, we shall use the following short notations for terms proportional to the marginal utility of private disposable income and leisure, respectively, facing the mimicker

$$\lambda^*_{t,b_2} = \frac{\lambda_t}{\gamma_t}\frac{\partial \hat{V}^2_t}{\partial b^1_{2t+1}} > 0,\ \kappa_t = \frac{\lambda_t l^1_t}{\gamma_t H_t}\frac{\partial \hat{V}^2_t/\partial \hat{z}^2_t}{(1+\partial p_{t,x}/\partial \tau_{t,x})} < 0,$$
$$\check{\kappa}_{t+1} = \frac{\lambda_{t+1} l^1_{t+1}}{\gamma_t}\frac{\partial \hat{V}^2_{t+1}}{\partial \hat{z}^2_{t+1}} > 0 \text{ and } \check{\kappa}^p_{t+1} = \frac{\check{\kappa}_{t+1}}{1+\partial p_{t+1,x}/\partial \tau_{t+1,x}} > 0$$

as well as use the expression

$$\Lambda^i_{t,t+1} = \kappa_t \frac{\partial \phi_t}{\partial p_{t,x}}\frac{\partial x^i_{1,t}}{\partial s^i_t} + \check{\kappa}_{t+1}\frac{\partial \phi_{t+1}}{\partial K_{t+1}} + \check{\kappa}^p_{t+1}\frac{\partial \phi_{t+1}}{\partial p_{t+1,x}}\frac{\partial p_{t+1,x}}{\partial K_{t+1}} \tag{28}$$

to denote how the self-selection constraints contribute to the marginal capital income tax rate of ability-type i via the wage ratio. Consider Proposition 4;

Proposition 4: *At the second best optimum, the marginal capital income tax rates can be written as*

$$\Phi^1_{t,I} = \frac{\gamma_t}{\gamma_{t+1} r_{t+1}} \{ \lambda^*_{t,b_2} [MRS^1_{t,b_1 b_2} - M\hat{R}S^2_{t,b_1 b_2}] - \Lambda^1_{t,t+1} - \tau_{t,x} \frac{\partial x^1_{1t}}{\partial s^1_t}$$
$$+ \frac{\mu_t}{\gamma_t} [\frac{\partial \eta^1_{1t}}{\partial s^1_t} - \frac{\partial \chi^1_{2t+1}}{\partial s^1_t} L^{-1}_{\mu}]\}$$

$$\Phi^2_{t,I} = \frac{\gamma_t}{\gamma_{t+1} r_{t+1}} \{ -\Lambda^2_{t,t+2} - \tau_{t,x} \frac{\partial x^2_{1t}}{\partial s^2_t}$$
$$+ \frac{\mu_t}{\gamma_t} [\frac{\partial \eta^2_{1t}}{\partial s^2_t} - \frac{\partial \chi^2_{2t+1}}{\partial s^2_t} L^{-1}_{\mu}]\}$$

Proof: See the Appendix.

The marginal capital income tax rates are structured in a way similar to the marginal labor income tax rates. Therefore, and by analogy to the discussion in the previous subsection, the marginal capital income tax rates reflect a combination of three underlying motives for influencing savings and capital formation; (i) a desire to relax the self-selection constraint via channels that do not directly depend on the second-hand market, (ii) a desire to offset distortions caused by the commodity tax, and (iii) a desire to use the second-hand market as an additional channel for redistribution.

The first two motives are captured by the first row in each tax formula. Consider first the formula for the low-ability type, where the difference between the two marginal rates of substitution reflects a difference in relative valuation of current consumption between the low-ability type and the mimicker. If the low-ability type attaches a higher relative value to the current consumption than the mimicker, so $MRS^1_{t,b_1b_2} - M\hat{R}S^2_{t,b_1b_2} > 0$, meaning that the relative valuation of current consumption decreases with the use of leisure, the government may relax the self-selection constraint by implementing a higher marginal capital income tax rate for the low-ability type than it would otherwise have done. An analogous motive for a lower marginal capital income tax rate applies if $MRS^1_{t,b_1b_2} - M\hat{R}S^2_{t,b_1b_2} < 0$. This argument for taxing or subsidizing the low-ability type at the margin is well understood from earlier research[15]; e.g. Brett (1997) and Pirttilä and Tuomala (2001).

The terms $\Lambda^1_{t,t+1}$ and $\Lambda^2_{t,t+1}$ are also associated with the self-selection constraint, although for another reason: the saving behavior of each ability-type in period t affects the wage ratio in periods t and $t+1$. According to equation (28), three separate effects are distinguishable, of which only the second has been discussed in earlier literature (Pirttilä and Tuomala, 2001). To interpret how the first part of the expression for $\Lambda^i_{t,t+1}$ contributes to the marginal capital income tax rate facing ability-type i, let us make the (reasonable) assumption that increased saving in period t reduces the demand for the new durable good in period t which, in turn, implies that the producer price of the durable good decreases. In this case, and if $\partial\phi_t/\partial p_{t,x} > 0$ (< 0), a lower produce price contributes to decrease (increase) the wage ratio, which makes mimicking more (less) attractive. As a consequence, this

[15] Note also that the result derived by Ordover and Phelps (1979), explaining when the marginal capital income tax rate should be zero for each ability-type, applies as a special case here as well. To see this, let us assume, as they did, that the gross wage rates are fixed. For the purpose of the argument, we shall also assume away the use of commodity taxation as well as the second-hand market. In this case, and if leisure is weakly separable from the other goods in the utility function, meaning that the consumption pattern of the mimicker coincides with that of the low-ability type, so $MRS^1_{t,b_1b_2} = M\hat{R}S^2_{t,b_1b_2}$, Proposition 4 would imply that the marginal capital income tax rate is zero for each ability-type.

provides an incentive for the government to relax the self-selection constraint by implementing a higher (lower) marginal capital income tax rate than it would otherwise have done. The interpretations of the second and third parts of $\Lambda^i_{t,t+1}$ are analogous with the exception that they refer to the wage ratio in period $t+1$; the intuition is, of course, that a change in the saving behavior in period t affects the capital stock and, therefore, the wage ratio in period $t+1$.

By analogy to the way in which the labor income tax is used, the government may also use the capital income tax structure to offset distortions caused by the commodity tax on the new durable good; an aspect captured by the final term in the first row. The intuition is that lower saving (caused by a higher marginal capital income tax rate) leads to increased consumption of the new durable good and contributes, therefore, to counteract the distortion caused by the commodity tax. Again, an alternative interpretation is in terms of a tax base effect.

The second row of each formula in Proposition 4 reflects the policy incentives created by the second-hand market. As our discussion serves to give the basic intuition behind the results, let us once again assume that increased saving in period t means decreased consumption of durable goods in period t. There are two distinct effects in the second row of each tax formula. First, as increased saving by ability-type i means reduced demand for the used durable good, i.e. $\partial\eta^i_{1t}/\partial s^i_t < 0$, increased saving causes downward pressure on $q_{t,\eta}$. As a consequence, if $\mu_t/\gamma_t > 0$, this mechanism provides an incentive for the government to increase the saving via a lower marginal capital income tax rate. The intuition is that a decrease in the price of the used durable good is, in this case, associated with a social benefit; therefore, increased saving contributes to higher welfare via a lower price of the used durable good.

The policy incentive will be the opposite if $\mu_t/\gamma_t < 0$. Second, as increased saving in period t means reduced consumption of the new durable good, it will also imply that the supply of the used durable good decreases in period $t+1$, i.e. $\partial\chi^i_{2t+1}/\partial s^i_t < 0$: in other words, increased saving in period t causes upward pressure on $q_{t+1,\eta}$. If $\mu_t/\gamma_t > 0$, this is detrimental for welfare and provides incentive for the government to reduce savings via higher marginal capital income tax rates, meaning that the second term within the square bracket contributes to reduce the marginal capital income tax rates. Again, the opposite applies if $\mu_t/\gamma_t < 0$.

In summary, the effect of the second-hand market on the marginal capital income tax rates is ambiguous. The reason is that a change in the saving behavior in period t, ceteris paribus, tends to affect the prices of used durable goods in periods t and $t+1$ in opposite directions. Therefore, whether the government wants to increase or decrease savings in order to relax the self-selection constraint via the second-hand market depends on the relative size of these two components.

4 Summary and Discussion

This paper deals with redistribution under asymmetric information by means of optimal income and commodity taxation in an OLG model with durable goods, where used durable goods are traded in a second-hand market. The type of transactions carried our in second-hand markets may, in terms of real world market economies, involve a variety of used durables including vehicles such as boats and snowmobiles, kitchen equipment such as stoves, refrigerators and freezers, and gardening tools such as lawnmowers: transactions which for obvious reasons are difficult to observe by the government.

The basic question is how the government may use the second-hand market as an additional mechanism for redistribution, even if it lacks a direct tax instrument to influence the market outcome. We assume that the set of tax instruments contains nonlinear taxes on labor income and capital income as well as a commodity tax on new durable goods.

In our framework, the government attaches a marginal value (positive or negative) to the used durable good, because its scarcity affects the possibilities of redistribution. If the government attaches a positive marginal value to the used durable good, it follows that the appearance of a second-hand market for the used durable good provides an incentive for the government to choose a lower commodity tax on the new durable good than it would otherwise have done. The intuition is that a decrease in the consumer price of the new durable good contributes to reduce the price of the used durable good, as new and used durable goods are substitutes in consumption, and because increased consumption of the new durable good leads to increased supply of the used durable good in the next period. The effects of the second hand market on the marginal labor income tax rates depend on whether leisure is complementary with, or substitutable for, the new and used durable good. For instance, if leisure is substitutable for the used durable, this will provide an incentive for the government to increase the marginal labor income tax rates relative to the outcome without a second-hand market, as increased consumption of leisure, in this case, contributes to reduce the demand for and, therefore, the price of the used durable good. Note also that, if the government, instead, were to attach a negative marginal value to the used durable good, policy incentives opposite to those just described will apply. Finally, the effect of the second-hand market on the marginal capital income tax rates is ambiguous. The intuition is that a change in the saving behavior tends to affect the current and future prices of the used durable good in opposite directions; a result which holds independently of whether the government attaches a positive or negative marginal value to the used durable good.

Future research in this area may take several directions, and we shall point of two of them. First, the assumption that the transactions in second-hand markets are not observable is clearly a simplification; another alternative is, of course, that this information can be collected at a cost, in which case the government would be able to directly control (although imperfectly) the outcome of the second-hand trade. Second, the contribution of the second-hand market to the tax structure is complex, and very few results hold unambiguously. Therefore, it would be interesting to complement the theoretical study set out here with numerical simulations, in which case the most likely qualitative effects may be assessed. We hope to carry out both these extensions in future research.

5 Appendix

The First Order Conditions

To simplify the analysis, let us first use equations (2) - (3) to rewrite the resource constraint, implying that the Lagrangean can be rewritten to read

$$\mathcal{L} = W + \sum_{t=0}^{\infty} \lambda_t \left[V_t^2 - \hat{V}_t^2 \right] + \sum_{t=0}^{\infty} \mu_t \sum_{i=1}^{2} \left[\chi_{2t}^i - \eta_{1t}^i \right]$$
$$+ \sum_{t=0}^{\infty} \gamma_t \left[S_{t,c} + p_{t,x} S_{t,x} + \tau_{t,x} \sum_i x_{1t}^i + K_t - K_{t+1} - \sum_i b_{1t}^i - \sum_i b_{2t}^i \right]$$

Define $G_t = \sum_i [\chi^i_{2t} - \eta^i_{1t}]$. The first order conditions can then be written as

$$\begin{aligned} b^1_{1t} : 0 = {} & \frac{\partial W}{\partial V^1_t}\frac{\partial V^1_t}{\partial b^1_{1t}} - \lambda_t \frac{\partial \hat{V}^2_t}{\partial b^1_{1t}} + \gamma_t \left[\tau_{t,x}\frac{\partial x^1_{1t}}{\partial b^1_{1t}} - 1\right] \\ & + \mu_t \frac{\partial G_t}{\partial b^1_{1t}} + \mu_{t+1}\frac{\partial G_{t+1}}{\partial b^1_{1t}} + \frac{\partial \mathcal{L}}{\partial p_{t,x}}\frac{\partial p_{t,x}}{\partial b^1_{1t}} \end{aligned} \tag{A.1}$$

$$\begin{aligned} l^1_t : 0 = {} & -\frac{\partial W}{\partial V^1_t}\frac{\partial V^1_t}{\partial z^1_t} + \lambda_t \frac{\partial \hat{V}^2_t}{\partial \hat{z}^2_t}\left(\phi_t + l^1_t \frac{\partial \phi_t}{\partial l^1_t}\right) + \gamma_t \left[w^1_t - \tau_{t,x}\frac{\partial x^1_{1t}}{\partial z^1_t}\right] \\ & + \mu_t \frac{\partial G_t}{\partial l^1_t} + \mu_{t+1}\frac{\partial G_{t+1}}{\partial l^1_t} + \frac{\partial \mathcal{L}}{\partial p_{t,x}}\frac{\partial p_{t,x}}{\partial l^1_t} \end{aligned} \tag{A.2}$$

$$\begin{aligned} b^1_{2t+1} : 0 = {} & \frac{\partial W}{\partial V^1_t}\frac{\partial V^1_t}{\partial b^1_{2t+1}} - \lambda_t \frac{\partial \hat{V}^2_t}{\partial b^1_{2t+1}} + \gamma_t \tau_{t,x}\frac{\partial x^1_{1t}}{\partial b^1_{2t+1}} - \gamma_{t+1} \\ & + \mu_t \frac{\partial G_t}{\partial b^1_{2t+1}} + \mu_{t+1}\frac{\partial G_{t+1}}{\partial b^1_{2t+1}} + \frac{\partial \mathcal{L}}{\partial p_{t,x}}\frac{\partial p_{t,x}}{\partial b^1_{2t+1}} \end{aligned} \tag{A.3}$$

$$\begin{aligned} b^2_{1t} : 0 = {} & \left(\frac{\partial W}{\partial V^2_t} + \lambda_t\right)\frac{\partial V^2_t}{\partial b^2_{1t}} + \gamma_t \left[\tau_{t,x}\frac{\partial x^2_{1t}}{\partial b^2_{1t}} - 1\right] \\ & + \mu_t \frac{\partial G_t}{\partial b^2_{1t}} + \mu_{t+1}\frac{\partial G_{t+1}}{\partial b^2_{1t}} + \frac{\partial \mathcal{L}}{\partial p_{t,x}}\frac{\partial p_{t,x}}{\partial b^2_{1t}} \end{aligned} \tag{A.4}$$

$$\begin{aligned} l^2_t : 0 = {} & -\left(\frac{\partial W}{\partial V^2_t} + \lambda_t\right)\frac{\partial V^t_2}{\partial z^2_t} + \lambda_t \frac{\partial \hat{V}^2_t}{\partial \hat{z}^2_t} l^1_t \frac{\partial \phi_t}{\partial l^1_t} + \gamma_t \left[w^2_t - \tau_{t,x}\frac{\partial x^2_{1t}}{\partial z^2_t}\right] \\ & + \mu_t \frac{\partial G_t}{\partial l^2_t} + \mu_{t+1}\frac{\partial G_{t+1}}{\partial l^2_t} + \frac{\partial \mathcal{L}}{\partial p_{t,x}}\frac{\partial p_{t,x}}{\partial l^2_t} \end{aligned} \tag{A.5}$$

$$
\begin{aligned}
b_{2t+1}^{2} : 0 = & \left(\frac{\partial W}{\partial V_t^2}+\lambda_t\right)\frac{\partial V_2^t}{\partial b_{2t+1}^2}+\gamma_t\tau_{t,x}\frac{\partial x_2^t}{\partial b_{2t+1}^2}-\gamma_{t+1} \\
& +\mu_t\frac{\partial G_t}{\partial b_{2t+1}^2}+\mu_{t+1}\frac{\partial G_{t+1}}{\partial b_{2t+1}^2}+\frac{\partial \mathcal{L}}{\partial p_{t,x}}\frac{\partial p_{t,x}}{\partial b_{2t+1}^2}
\end{aligned} \tag{A.6}
$$

$$
K_t : 0 = \lambda_t\frac{\partial \hat{V}_t^2}{\partial z_{1t}^1}l_{1t}^1\frac{\partial \phi_t}{\partial K_t}+\gamma_t\left(1+r_t\right)-\gamma_{t-1}+\frac{\partial \mathcal{L}}{\partial p_{t,x}}\frac{\partial p_{t,x}}{\partial K_t} \tag{A.7}
$$

$$
\begin{aligned}
\tau_{t,x} : 0 = & -x_{1t}^1\frac{\partial V_t^1}{\partial b_{1t}^1}\frac{\partial W}{\partial V_t^1}-x_{1t}^2\left(\frac{\partial W}{\partial V_t^2}+\lambda_t\right)\frac{\partial V_2^t}{\partial b_{1t}^2}+\lambda_t\hat{x}_{1t}^2\frac{\partial \hat{V}_t^2}{\partial b_{1t}^1} \\
& +\gamma_t\left[x_{1t}^1+x_{1t}^2+\tau_{t,x}\left(\frac{\partial x_{1t}^1}{\partial q_{t,x}}+\frac{\partial x_{1t}^2}{\partial q_{t,x}}\right)\right] \\
& +\mu_t\frac{\partial G_t}{\partial q_{t,x}}+\mu_{t+1}\frac{\partial G_{t+1}}{\partial q_{t,x}}+\frac{\partial \mathcal{L}}{\partial p_{t,x}}\frac{\partial p_{t,x}}{\partial \tau_{t,x}}
\end{aligned} \tag{A.8}
$$

$$
\begin{aligned}
q_{t,\eta} : 0 = & -\eta_{1t}^1\frac{\partial V_t^1}{\partial b_{1t}^1}\frac{\partial W}{\partial V_t^1}-\eta_{1t}^2\left(\frac{\partial W}{\partial V_t^2}+\lambda_t\right)\frac{\partial V_t^2}{\partial b_{1t}^2}+\lambda_t\hat{\eta}_{1t}^2\frac{\partial \hat{V}_t^2}{\partial b_{1t}^1} \\
& +\chi_{2t}^1\frac{\partial V_{t-1}^1}{\partial b_{2t}^1}\frac{\partial W}{\partial V_{t-1}^1}+\chi_{2t}^2\left(\frac{\partial W}{\partial V_{t-1}^2}+\lambda_{t-1}\right)\frac{\partial V_{t-1}^2}{\partial b_{2t}^2}-\lambda_{t-1}\hat{\chi}_{2t}^2\frac{\partial \hat{V}_{t-1}^2}{\partial b_{2t}^1} \\
& +\gamma_t\left[\tau_{t,x}\left(\frac{\partial x_{1t}^1}{\partial q_{t,\eta}}+\frac{\partial x_{1t}^2}{\partial q_{t,\eta}}\right)\right]+\gamma_{t-1}\left[t_x^{t-1}\left(\frac{\partial x_{1t-1}^1}{\partial q_{t,\eta}}+\frac{\partial x_{1t-1}^2}{\partial q_{t,\eta}}\right)\right] \\
& +\mu_{t-1}\frac{\partial G_{t-1}}{\partial q_{t,\eta}}+\mu_t\frac{\partial G_t}{\partial q_{t,\eta}}+\mu_{t+1}\frac{\partial G_{t+1}}{\partial q_{t,\eta}}+\frac{\partial \mathcal{L}}{\partial p_{t-1,x}}\frac{\partial p_{t-1,x}}{\partial q_{t,\eta}}+\frac{\partial \mathcal{L}}{\partial p_{t,x}}\frac{\partial p_{t,x}}{\partial q_{t,\eta}}
\end{aligned} \tag{A.9}
$$

where

$$\frac{\partial \mathcal{L}}{\partial p_{t,x}} = -x_{1t}^1 \frac{\partial V_t^1}{\partial b_{1t}^1} \frac{\partial W}{\partial V_t^1} - x_{1t}^2 \left(\frac{\partial W}{\partial V_t^2} + \lambda_t \right) \frac{\partial V_t^2}{\partial b_{1t}^2} + \lambda_t \hat{x}_{1t}^2 \frac{\partial \hat{V}_t^2}{\partial b_{1t}^1}$$
$$+ \gamma_t \left[x_{1t}^1 + x_{1t}^2 + \tau_{t,x} \left(\frac{\partial x_{1t}^1}{\partial q_{t,x}} + \frac{\partial x_{1t}^2}{\partial q_{t,x}} \right) \right]$$
$$+ \mu_t \frac{\partial G_t}{\partial q_{t,x}} + \mu_{t+1} \frac{\partial G_{t+1}}{\partial q_{t,x}} + \lambda_t \frac{\partial \hat{V}_t^2}{\partial \hat{z}_{1t}^2} l_t^1 \frac{\partial \phi_t}{\partial p_{t,x}} \tag{A.10}$$

To simplify the expression for $\partial \mathcal{L} / \partial p_{t,x}$, let us define

$$A_t = -x_{1t}^1 \frac{\partial V_t^1}{\partial b_{1t}^1} \frac{\partial W}{\partial V_t^1} - x_{1t}^2 \left(\frac{\partial W}{\partial V_t^2} + \lambda_t \right) \frac{\partial V_t^2}{\partial b_{1t}^2} + \lambda_t \hat{x}_{1t}^2 \frac{\partial \hat{V}_t^2}{\partial b_{1t}^1}$$
$$+ \gamma_t \left[x_{1t}^1 + x_{1t}^2 + \tau_{t,x} \left(\frac{\partial x_{1t}^1}{\partial q_{t,x}} + \frac{\partial x_{1t}^2}{\partial q_{t,x}} \right) \right]$$
$$+ \mu_t \frac{\partial G_t}{\partial q_{t,x}} + \mu_{t+1} \frac{\partial G_{t+1}}{\partial q_{t,x}} \tag{A.11}$$

which can be used to rewrite equation (A.8) as

$$0 = A_t + \left(A_t + \lambda_t \frac{\partial \hat{V}_t^2}{\partial \hat{z}_t^2} l_t^1 \frac{\partial \phi_t}{\partial p_{t,x}} \right) \frac{\partial p_{t,x}}{\partial \tau_{t,x}} = 0 \tag{A.12}$$

Adding and subtracting $\lambda_t \left(\partial \hat{V}_t^2 / \partial \hat{z}_t^2 \right) l_t^1 \left(\partial \phi_t / \partial p_{t,x} \right)$ in equation (A.12), while using that the expression within the parenthesis in equation (A.12) equals $\partial \mathcal{L} / \partial p_{t,x}$, we obtain

$$\frac{\partial \mathcal{L}}{\partial p_{t,x}} = \frac{\lambda_t l_t^1 \partial \hat{V}_t^2 / \partial \hat{z}_t^2}{(1 + \partial p_{t,x} / \partial \tau_{t,x})} \frac{\partial \phi_t}{\partial p_{t,x}} \tag{A.13}$$

Proof of Proposition 1

Use equations (A.1), (A.3), (A.4) and (A.6) to derive expressions for $(\partial W / \partial V_t^1)(\partial V_t^1 / \partial b_{1t}^1)$, $(\partial W / \partial V_{t-1}^1)/(\partial V_{t-t}^1 / \partial b_{2t}^1)$, $(\partial W / \partial V_t^2 + \lambda_t)(\partial V_t^2 / \partial b_{1t}^2)$ and $(\partial W / \partial V_{t-1}^2 + \lambda_{t-1})(\partial V_{t-1}^2 / \partial b_{2t}^2)$, respectively, and substitute into equation (A.9). Then, by using

$$\frac{\partial G_t}{\partial q_{t,\eta}} = \sum_i [\frac{\partial \chi^i_{2t}}{\partial q_{t,\eta}} - \frac{\partial \eta^i_{1t}}{\partial q_{t,\eta}}]$$
$$\frac{\partial G_{t-1}}{\partial q_{t,\eta}} = -\sum_i \frac{\partial \eta^i_{1t-1}}{\partial q_{t,\eta}}]$$
$$\frac{\partial G_{t+1}}{\partial q_{t,\eta}} = \sum_i \frac{\partial \chi^i_{2t+1}}{\partial q_{t,\eta}}$$
$$\frac{\partial G_t}{\partial b^i_{1t}} = -\frac{\partial \eta^i_{1t}}{\partial b^i_{1t}}$$
$$\frac{\partial G_t}{\partial b^i_{2t}} = \frac{\partial \chi^i_{2t}}{\partial b^i_{2t}}$$
$$\frac{\partial G_{t+1}}{\partial b^i_{1t}} = \frac{\partial \chi^i_{2t+1}}{\partial b^i_{1t}}$$
$$\frac{\partial G_{t-1}}{\partial b^i_{2t}} = -\frac{\partial \eta^i_{1t-1}}{\partial b^i_{2t}}$$

together with the Slutsky condition, we have

$$\begin{aligned}
&- \lambda_t \frac{\partial \hat{V}^2_t}{\partial b^1_{1t}} [\eta^1_{1t} - \hat{\eta}^2_{1t}] + \lambda_{t-1} \frac{\partial \hat{V}^2_{t-1}}{\partial b^1_{2t}} [\chi^1_{2t} - \hat{\chi}^2_{2t}] + \gamma_t \tau_{t,x} \frac{\partial \tilde{x}^i_{1t}}{\partial q_{t,\eta}} + \gamma_{t-1} \tau_{t-1,x} \frac{\partial \tilde{x}^i_{1t-1}}{\partial q_{t,\eta}} \\
&+ \mu_t [-\sum_i \frac{\partial \tilde{\eta}^i_{1t-1}}{\partial q_{t,\eta}} L_\mu + \sum_i (\frac{\partial \tilde{\chi}^i_{2t}}{\partial q_{t,\eta}} - \frac{\partial \tilde{\eta}^i_{1t}}{\partial q_{t,\eta}}) + \sum_i \frac{\partial \tilde{\chi}^i_{2t+1}}{\partial q_{t,\eta}} L^{-1}_\mu] \\
&+ \sum_i \frac{\partial \tilde{x}^i_{1t}}{\partial q_{t,\eta}} \frac{\rho_t \lambda_t}{D_t} \frac{\partial \phi_t}{\partial p_{k,x}} + \sum_i \frac{\partial \tilde{x}^i_{1t-1}}{\partial q_{t,\eta}} \frac{\rho_{t-1} \lambda_{t-1}}{D_t} \frac{\partial \phi_{t-1}}{\partial p_{t-1,x}} = 0 \qquad \text{(A.14)}
\end{aligned}$$

Rearranging gives the formula in Proposition 1.■

Proof of Proposition 2

Use equations (A.1) and (A.4) to derive expressions for $(\partial W/\partial V^1_t)(\partial V^1_t/\partial b^1_{1t})$ and $(\partial W/\partial V^2_t + \lambda_t)(\partial V^2_t/\partial b^2_{1t})$, respectively, and substitute into equation

(A.8). Then, by using the derivatives of G_t and G_{t+1} derived above together with

$$\frac{\partial G_t}{\partial q_{t,x}} = -\frac{\partial \eta_{1t}^1}{\partial q_{t,x}} - \frac{\partial \eta_{1t}^2}{\partial q_{t,x}}$$
$$\frac{\partial G_{t+1}}{\partial q_{t,x}} = \frac{\partial \chi_{2t+1}^1}{\partial q_{t,x}} + \frac{\partial \chi_{2t+1}^2}{\partial q_{t,x}}$$

and the Slutsky condition, we have

$$\begin{aligned} 0 = & \left(\hat{x}_{1t}^2 - x_{1t}^1\right) \frac{\lambda_t}{\gamma_t} \frac{\partial \hat{V}_t^2}{\partial b_{1t}^1} + \tau_{t,x} \left(\frac{\partial \tilde{x}_{1t}^1}{\partial q_{t,x}} + \frac{\partial \tilde{x}_{1t}^2}{\partial q_{t,x}} \right) \\ & + \frac{\mu_t}{\gamma_t} \left(-\frac{\partial \tilde{\eta}_{1t}^1}{\partial q_{t,x}} - \frac{\partial \tilde{\eta}_{1t}^2}{\partial q_{t,x}} \right) + \frac{\mu_{t+1}}{\gamma_t} \left(\frac{\partial \tilde{\chi}_{2t+1}^1}{\partial q_{t,x}} + \frac{\partial \tilde{\chi}_{2t+1}^2}{\partial q_{t,x}} \right) \\ & + \frac{1}{\gamma_t} \frac{\partial \mathcal{L}}{\partial p_{t,x}} \left(\frac{\partial p_{t,x}}{\partial \tau_{t,x}} + x_{1t}^1 \frac{\partial p_{t,x}}{\partial b_{1t}^1} + x_{1t}^2 \frac{\partial p_{t,x}}{\partial b_{1t}^2} \right) \end{aligned} \tag{A.15}$$

To evaluate the terms within the parenthesis in the third row, we differentiate equation (20) to obtain

$$\frac{\partial p_{t,x}}{\partial \tau_{t,x}} = \frac{\sum_i \partial x_{1t}^i / \partial q_{t,x}}{\partial S_{t,x} / \partial p_{t,x} - \sum_i \partial x_{1t}^i / \partial q_{t,x}} \tag{A.16}$$
$$\frac{\partial p_{t,x}}{\partial b_{1t}^i} = \frac{\partial x_{1t}^i / \partial b_{1t}^i}{\partial S_{t,x} / \partial p_{t,x} - \sum_i \partial x_{1t}^i / \partial q_{t,x}} \tag{A.17}$$

Substituting equations (A.16) and (A.17) into equation (A.15), we obtain the formula in Proposition 2.■

Proof of Proposition 3

Let us begin with the marginal labor income tax rate of the low-ability type. By combining equations (A.1) and (A2), while using equation (A.13), we have

$$
\begin{aligned}
&MRS^1_{t,zb}[\lambda_t \frac{\partial \hat{V}^2_t}{\partial b^1_{1t}} - \gamma_t(\tau_{t,x}\frac{\partial x^1_{1t}}{\partial b^1_{1t}} - 1) - \mu_t \frac{\partial G_t}{\partial b^1_{1t}} - \mu_{t+1}\frac{\partial G_{t+1}}{\partial b^1_{1t}} \\
&- \frac{\lambda_t l^1_t \partial \hat{V}^2_t/\partial \hat{z}^2_t}{(1+\partial p_{t,x}/\partial \tau_{t,x})}\frac{\partial \phi_t}{\partial p_{t,x}}\frac{\partial p_{t,x}}{\partial b^1_{1t}}] \\
&= \lambda_t \frac{\partial \hat{V}^2_t}{\partial \hat{z}^2_t}[\phi_t + l^1_t \frac{\partial \phi_t}{\partial l^1_t}] + \gamma_t(w^1_t - \tau_{t,x}\frac{\partial x^1_{1t}}{\partial z^1_t}) \\
&+ \mu_t \frac{\partial G_t}{\partial l^1_t} + \mu_{t+1}\frac{\partial G_{t+1}}{\partial l^1_t} + \frac{\lambda_t l^1_t \partial \hat{V}^2_t/\partial \hat{z}^2_t}{(1+\partial p_{t,x}/\partial \tau_{t,x})}\frac{\partial \phi_t}{\partial p_{t,x}}\frac{\partial p_{t,x}}{\partial l^1_t} \qquad \text{(A.18)}
\end{aligned}
$$

Then, by substituting $w^1_t - MRS^1_{t,zb} = w^1_t T^1_{t,y}$ from equation (12) into equation (A.18), while using

$$
\begin{aligned}
\frac{\partial p_{t,x}}{\partial l^1_t} &= -\frac{\partial S_{t,x}/\partial l^1_t + \partial x^1_{1t}/\partial z^1_t}{D_t} \\
\frac{\partial G_t}{\partial l^1_t} &= \frac{\partial \eta^1_{1t}}{\partial z^1_t} \\
\frac{\partial G_t}{\partial b^1_{1t}} &= -\frac{\partial \eta^1_{1t}}{\partial b^1_{1t}} \\
\frac{\partial G_{t+1}}{\partial l^1_t} &= -\frac{\partial \chi^1_{2t+1}}{\partial z^1_t} \\
\frac{\partial G_{t+1}}{\partial b^1_{1t}} &= \frac{\partial \chi^1_{2t+1}}{\partial b^1_{1t}} \\
\frac{\partial \tilde{x}^1_{1t}}{\partial z^1_t} &= \frac{\partial x^1_{1t}}{\partial z^1_t} - \frac{\partial x^1_{1t}}{\partial b^1_{1t}} MRS^1_{t,zb} \\
\frac{\partial \tilde{\eta}^1_{1t}}{\partial z^1_t} &= \frac{\partial \eta^1_{1t}}{\partial z^1_t} - \frac{\partial \eta^1_{1t}}{\partial b^1_{1t}} MRS^1_{t,zb} \\
\frac{\partial \tilde{\chi}^1_{2t+1}}{\partial z^1_t} &= \frac{\partial \chi^1_{2t+1}}{\partial z^1_t} - \frac{\partial \chi^1_{2t+1}}{\partial b^1_{1t}} MRS^1_{t,zb}
\end{aligned}
$$

gives the expression for the marginal labor income tax rate of the low-ability type in Proposition 3. The expression for the marginal labor income tax rate of the high-ability type can be derived in an analogous way.■

Proof of Proposition 4

Consider the marginal capital income tax rate of the low-ability type. By combining equations (A.1) and (A.3), we can derive

$$
\begin{aligned}
&MRS^1_{t,b_1b_2}[\lambda_t \frac{\partial \hat{V}^2_t}{\partial b^1_{2t+1}} - \gamma_t \tau_{t,x} \frac{\partial x^1_{1t}}{\partial b^1_{2t+1}} + \gamma_{t+1} - \mu_t \frac{\partial G_t}{\partial b^1_{2t+1}} - \mu_{t+1} \frac{\partial G_{t+1}}{\partial b^1_{2t+1}} - \frac{\partial \mathcal{L}}{\partial p_{t,x}} \frac{\partial p_{t,x}}{\partial b^1_{2t+1}}] \\
&= \lambda_t \frac{\partial \hat{V}^2_t}{\partial b^1_{1t}} - \gamma_t \tau_{t,x} \frac{\partial x^1_{1t}}{\partial b^1_{1t}} + \gamma_t - \mu_t \frac{\partial G_t}{\partial b^1_{1t}} - \mu_{t+1} \frac{\partial G_{t+1}}{\partial b^1_{1t}} - \frac{\partial \mathcal{L}}{\partial p_{t,x}} \frac{\partial p_{t,x}}{\partial b^1_{1t}} \qquad \text{(A.19)}
\end{aligned}
$$

Next, rewrite equation (A.7) to read

$$
\frac{\gamma_t}{\gamma_{t+1}} = \frac{\lambda_{t+1}}{\gamma_{t+1}} \frac{\partial \hat{V}^2_{t+1}}{\partial \hat{z}^2_{t+1}} l^1_{t+1} \frac{\partial \phi_{t+1}}{\partial K_{t+1}} + (1 + r_{t+1}) + \frac{1}{\gamma_{t+1}} \frac{\partial \mathcal{L}}{\partial p_{t+1,x}} \frac{\partial p_{t+1,x}}{\partial K_{t+1}} \qquad \text{(A.20)}
$$

By substituting equation (A.20) into equation (A.19), while using equation (13), we have

$$
\begin{aligned}
\Phi^1_{t,I} = &\frac{\lambda_t}{\gamma_{t+1} r_{t+1}} \frac{\partial \hat{V}^2_t}{\partial b^1_{2t+1}} \left(MRS^1_{t,b_1b_2} - M\hat{R}S^2_{t,b_1b_2} \right) \\
&+ \frac{\gamma_t \tau_{t,x}}{\gamma_{t+1} r_{t+1}} \left(\frac{\partial x^1_{1t}}{\partial b^1_{1t}} - \frac{\partial x^1_{1t}}{\partial b^1_{2t+1}} MRS^1_{t,b_1b_2} \right) \\
&- \frac{\lambda_{t+1}}{r_{t+1} \gamma_{t+1}} \frac{\partial \hat{V}^2_{t+1}}{\partial \hat{z}^2_{t+1}} l^1_{t+1} \frac{\partial \phi_{t+1}}{\partial K_{t+1}} - \frac{1}{r_{t+1} \gamma_{t+1}} \frac{\partial \mathcal{L}}{\partial p_{t+1,x}} \frac{\partial p_{t+1,x}}{\partial K_{t+1}} \\
&+ \frac{\mu_t}{\gamma_{t+1} r_{t+1}} \left(\frac{\partial G_t}{\partial b^1_{1t}} - \frac{\partial G_t}{\partial b^1_{2t+1}} MRS^1_{t,b_1b_2} \right) \\
&+ \frac{\mu_{t+1}}{\gamma_{t+1} r_{t+1}} \left(\frac{\partial G_{t+1}}{\partial b^1_{1t}} - \frac{\partial G_{t+1}}{\partial b^1_{2t+1}} MRS^1_{t,b_1b_2} \right) \\
&+ \frac{1}{\gamma_{t+1} r_{t+1}} \frac{\partial \mathcal{L}}{\partial p_{t,x}} \left(\frac{\partial p_{t,x}}{\partial b^1_{1t}} - \frac{\partial p_{t,x}}{\partial b^1_{2t+1}} MRS^1_{t,b_1b_2} \right) \\
&- \frac{1}{r_{t+1} \gamma_{t+1}} \frac{\partial \mathcal{L}}{\partial p_{t+1,x}} \frac{\partial p_{t+1,x}}{\partial K_{t+1}} \qquad \text{(A.21)}
\end{aligned}
$$

Then, substituting

$$
\begin{aligned}
\frac{\partial G_t}{\partial b_{1t}^1} &= -\frac{\partial \eta_{1t}^1}{\partial b_{1t}^1} \\
\frac{\partial G_t}{\partial b_{2t+1}^1} &= -\frac{\partial \eta_{1t}^1}{\partial b_{2t+1}^1} \\
\frac{\partial G_{t+1}}{\partial b_{1t}^1} &= \frac{\partial \chi_{2t+1}^1}{\partial b_{1t}^1} \\
\frac{\partial G_{t+1}}{\partial b_{2t+1}^1} &= \frac{\partial \chi_{2t+1}^1}{\partial b_{2t+1}^1} \\
\frac{\partial x_{1t}^1}{\partial s_t^1} &= \frac{\partial x_{1t}^1}{\partial b_{2t+1}^1} MRS_{t,b_1 b_2}^1 - \frac{\partial x_{1t}^1}{\partial b_{1t}^1} \\
\frac{\partial \eta_{1t}^1}{\partial s_t^1} &= \frac{\partial \eta_{1t}^1}{\partial b_{2t+1}^1} MRS_{t,b_1 b_2}^1 - \frac{\partial \eta_{1t}^1}{\partial b_{1t}^1} \\
\frac{\partial \chi_{2t+1}^1}{\partial s_t^1} &= \frac{\partial \chi_{2t+1}^1}{\partial b_{2t+1}^1} MRS_{t,b_1 b_2}^1 - \frac{\partial \chi_{2t+1}^1}{\partial b_{1t}^1} \\
\frac{\partial p_{t,x}}{\partial b_{2t+1}^1} &= \frac{\partial x_{1t}^1 / \partial b_{2t+1}^1}{D_t}
\end{aligned}
$$

into equation (A.21) and rearranging gives the expression for the marginal capital income tax rate of the low-ability type in Proposition 4. The expression for the marginal capital income tax rate of the high-ability type can be derived in an analogous way.■

References

[1] Akerlof, G. (1970) The Market for 'Lemons': Quality Uncertainty and the Market Mechanism. *Quarterly Journal of Economics* **84**, 488-500.

[2] Aronsson, T. and Blomquist, S. (2003) Optimal Taxation, Global Externalities and Labor Mobility. *Journal of Public Economics* **87**, 2749-2764.

[3] Aronsson, T., Sjögren, T. and Dalin, T. (in press) Optimal Taxation and Redistribution in an OLG Model with Unemployment. *International Tax and Public Finance.*

[4] Aronsson, T., Jonsson, T. and Sjögren, T. (2006) Environmental Policy and Optimal Taxation in a Decentralized Economic Federation. *FinanzArchiv* **62**, 437-454.

[5] Aronsson, T., and Sjögren, T. (2007) An Optimal Tax Approach to Alcohol Policy. Mimeo.

[6] Atkinson, A. B., and Stiglitz, J. E. (1976) The Design of Tax Structure: Direct versus Indirect Taxation. *Journal of Public Economics* **1**, 55-75.

[7] Atkinson, A. B. (1977) Optimal Taxation and the Direct versus Indirect Tax Controversy. *Canadian Journal of Economics* **10**, 590-606.

[8] Boadway, R., Marchand, M., and Pestieau, P. (2000) Redistribution with Unobservable Bequests: A Case for Taxing Capital Income. *The Scandinavian Journal of Economics* **102**, 253-267.

[9] Brett, C. (1997) A Note on Nonlinear Taxation in an Overlapping Generations Model. Mimeo.

[10] Christiansen, V. (2003) Cross-border Shopping and Tax Structure. ERPU working paper 03-04.

[11] Edwards, J., M. J. Keen, and Tuomala, M. (1994) Income Tax, Commodity Taxes and Public Good Provision. *FinanzArchiv* **51**, 472-497.

[12] Gilligan, T. W. (2004) Lemons and Leases in the Used Business Aircraft Market. *Journal of Political Economy* **112**, 1157-80.

[13] Hendel, I. and Lizzeri, A. (1999) Adverse Selection in Durable Goods Markets. *The American Economic Review* **89**, 1097-1115.

[14] Naito, H. (1999) Re-examination of Uniform Commodity Taxes under a Non-linear Income Tax System and its Implication for Production Efficiency. *Journal of Public Economics* **71**, 165-188.

[15] Ordover, J. A., and Phelps, E. S. (1979) The Concept of Optimal Taxation in the Overlapping-generations Model of Capital and Wealth. *Journal of Public Economics* **12**, 1-26.

[16] Pirttilä J., and Tuomala, M. (2001) On Optimal Non-linear Taxation and Public Good Provision in an Overlapping Generations Economy. *Journal of Public Economics* **79**, 485-501.

[17] Porter, R. H. and Sattler, P. (1999) Patterns of Trade in the Market for Used Durables: Theory and Evidence. NBER Working Paper Series.

[18] Runkel, M. (2004) Efficient Tax-Subsidy Schemes for a Polluting Durable-Good Industry without Commitment Ability. *FinanzArchiv* **60**, 494-514.

[19] Stern, N. H. (1982) Optimum Taxation with Errors in Administration. *Journal of Public Economics* **17**, 181-211.

[20] Stiglitz, J. E. (1982) Self-Selection and Pareto-Efficient Taxation. *Journal of Public Economics* **17**, 213-240.

[21] Tenhunen, S. and Tuomala, M. (2007) On Optimal Lifetime Redistribution Policy. Tampere Economic Working Papers No 50.