Dynamic cost-benefit analysis of large projects: The role of capital and investment costs

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Abstract

Based on an ideal index for deflating after-project prices, we derive a dynamic cost-benefit rule for evaluating large projects. We show that, in addition to the conventional income and consumer surplus meaures, the rule also entails an extra term involving capital and investment cost changes.
Keywords: cost-benefit rules, large projects, and capital costs

1 Introduction

Structural transformations occur frequently in real life, such as the replacement of an old road passing through a large number of cities by a large-capacity freeway moving the heavy traffic away from the urban areas. National environmental policy may change by discrete increases in emission taxes of pollutants and decreases in taxes on labor. One intention of such a green tax reform is to create large changes in equilibrium prices. Every time a project is large enough to considerably affect the prices in an economy, the dynamic theory of cost-benefit analysis for a marginal variation (see Starrett, 1988; and Li and Löfgren, 2007) has to be modified. The general idea behind the necessary modification is not new as it dates back to the French economist Dupuit. However, a satisfactory theory in a growth theoretic context has not been available until recently.

A rigorous theory for dynamic welfare comparisons has been developed by Weitzman (2001) who shows that the difference in intertemporal welfare between two economies or two points in time of the same economy can be exactly measured by the difference in real national income plus a consumer surplus term. In addition, he mentions that the theory may also be used to conduct social cost-benefit analysis by comparing the welfare levels generated by "twin economies" with identical preferences and technology but different initial capital stocks. This paper explores this issue further. We show that while the theory is valid for this special case, the cost-benefit rule for a more general, dynamic project also entails an extra term reflecting the change in capital cost as well as the change in the value of investment during the project period. To arrive at our main results, we start by introducing a multisector Ramsey growth model and state the generic cost-benefit rules.

2 The model and generic cost-benefit rules

In order to derive our dynamic cost-benefit rule in its most general form, we consider a multi-sector growth model with all possible consumption and investment goods taken into account. Let $C = (C_1, C_2, ..., C_m)$ be a n-dimensional vector of consumption flows at a given time t, which is supposed to exhaust all possible goods and services that are relevant to social welfare, or to the standard of living of a representative individual. In addition to the usual market commodities, environmental services such as forest amenities, biodiversity and ecosystem functions, in flow terms, are also considered to be part of the consumption vector. This means that the prices of these services are rental prices. The utilitarian measure of intertemporal welfare at time $t = 0$ can be expressed as

$$W = \int_0^\infty U(C(t)) \exp(-\theta t) dt \tag{1}$$

where $U(C)$ is a given concave, non-decreasing, instantaneous utility function with continuous second order derivatives defined for $C \geq 0$, and θ is the utility rate of discount. Let $K = (K_1, K_2, ..., K_n)$ be a vector of capital goods, which is assumed to contain all types of capital goods in the economy including natural resources such as minerals, forests, air, water, and even human capital in the form of technological knowledge. Net investments are, by definition, the change in capital stocks, i.e. $I_i = \dot{K}_i$, $i = 1, 2, ..., n$, which, in a vector form can be expressed as $I = \dot{K}$, given $K(0) = K_0 > 0$. At each point in time t, consumption $C(t)$ and investment $I(t)$ are allocated within the $(m+n)-$dimensional attainable-possibility set $S(K(t); \alpha)$, conditional on a collection of "parameters", α, (Drèze and Stern, 1987), such that $(C(t), I(t)) \in S(K(t); \alpha)$ which is assumed to be strictly convex.

The parameters α may represent any premise that modifies the feasible set for consumption and investment allocations. This includes aspects such as a given property right regime, a given taxation system, or an inherent public infrastructure, which are not optimized in the economic system. Conditional on the parameters, a social planner is assumed to maximize the current-value Hamiltonian at each point in time t, i.e. $H(t) = U(C(t)) + \Psi(t)I(t)$ with respect to $\{C(t), I(t)\}$ subject to the initial condition and the attainability set, where $\Psi(t)$ is the n-dimensional vector of the utility shadow prices of capital satisfying the following equation of motion $\dot{\Psi} = \theta\Psi - \nabla H_K \big|_{*(t)}$, where the notion $\big|_{*(t)}$ means evaluation along the optimal trajectory at time t. Note that the feasible set for the optimization problem $S(K(t); \alpha)$ contains a collection of governance parameters in addition to the resource constraints. Thus, the optimal trajectories of consumption, investment and capital stocks depend on the parameter α. Let $\{C(\alpha, t), I(\alpha, t), K(\alpha, t)\}$ denote the conditional optimum trajectory, then the maximized intertemporal welfare can be expressed as

$$\hat{W}(\alpha) \equiv \int_0^\infty U\left[C(\alpha, t)\right] \exp(-\theta t) dt \tag{2}$$

Now, consider a project involving a change in the parameter set from a_0 to α_1 over a period of time $[0, \tau]$, which results in changes in the stream of consumption both within the project period and beyond. Then, the generic cost benefit rule can be stated as

Lemma 1 *If a project* $\Delta\alpha = \alpha_1 - \alpha_0$ *over* $t \in [0, \tau]$ *leads to a positive change in the intertemporal welfare in (2) i.e.* $\Delta\hat{W} = \hat{W}(\alpha_1) - \hat{W}(\alpha_0) > 0$, *then the project is socially profitable; otherwise not.*

Alternatively, this rule can be expressed in terms of net social profits. Let $C_a(\alpha, t) = \partial C(\alpha, t)/\partial\alpha$, $I_\alpha(\alpha, t) = \partial I(\alpha, t)/\partial\alpha$ and $K_\alpha(\alpha, t) = \partial K(\alpha, t)/\partial\alpha$ for $t \in [0, \tau]$ denote the marginal change in consumption, investment and capital, respectively, caused by an infinitesimal change in the parameter α within the project period. The net social profits at time t from a marginal project $d\alpha$ can then be expressed as

$$B(\alpha, t) = \nabla U\left[C(\alpha, t)\right] \cdot C_\alpha(\alpha, t) + \Psi(\alpha, t) \cdot I_\alpha(\alpha, t) + \Omega(\alpha, t) \cdot K_\alpha(\alpha, t) \tag{3}$$

where $\nabla U\left[C(\alpha, t)\right]$ denotes the utility prices of consumption and $\Omega(\alpha, t) = \dot{\Psi}(\alpha, t) - \theta\Psi(\alpha, t)$ represents the costs-of-holding capital. For a large project $\Delta\alpha = \alpha_1 - \alpha_0$, the rule can be restated as

Lemma 2 *If the present discounted value of social profits within the project period i.e.*

$$\Delta\hat{W} = \int_0^\tau \int_{\alpha_0}^{\alpha_1} B(\alpha, t) \exp(-\theta t) d\alpha dt \tag{4}$$

is positive, then the project is socially profitable; otherwise not.

The rational for this rule is that the present value of the within-period-changes in investment and capital stocks completely captures the welfare effect of the changes in consumption beyond the period. Since the within-period-changes in consumption are already there, the integral in (4) corresponds to the present value of all future changes in consumption caused by the project (see Dixit et al, 1982; Arrow et al. 2003; Li and Löfgren, 2007 for more details).

3 The new results

Since the social profit in (3) is measured in utility metrics, we need to convert it into monetary units in order to arrive at a more operational cost-benefit rule. Let $P(\alpha, t) = \nabla U(C(\alpha, t))/\lambda(\alpha, t)$, $Q(\alpha, t) = \Psi(\alpha, t)/\lambda(\alpha, t)$ and $R(\alpha, t) = \Omega(\alpha, t)/\lambda(\alpha, t)$ denote the nominal (money) prices for consumption, investment and capital rental, respectively, where $\lambda(\alpha, t)$ as the marginal utility of income satisfying the no-arbitrage condition $\dot{\lambda}(\alpha, t) = \lambda(\alpha, t)(r(\alpha, t) - \theta)$ with $r(\alpha, t)$ as the interest rate, all conditional on a parameter value α. Then, the expressions in (3) and (4) can be written as

$$b(\alpha, t) = P(\alpha, t) \cdot C_\alpha(\alpha, t) + Q(\alpha, t) \cdot I_\alpha(\alpha, t) + R(\alpha, t) \cdot K_\alpha(\alpha, t) \tag{5}$$

and

$$\Delta \hat{W} = \int_0^\tau \int_{\alpha_0}^{\alpha_1} b(\alpha, t)\lambda(\alpha, t) \exp(-\theta t) d\alpha dt \tag{6}$$

respectively. Although the social profit is now expressed in monetary terms, the presence of the parameter-dependent marginal utility of income makes it difficult to convert the final result into a money-metric measures. One way to get around this problem is to assume that the marginal utility of income is invariant with respect to α such that the term can be moved outside the inner integral, and then integrate over time. However, as shown by Starrett (1988), this is in general not consistent with utilitarian theory where marginal utility depends on income that, in turn, depends on the parameter. In this paper, we use a recent idea in Weitzman (2001) to define an ideal consumer price index to resolve the dilemma. The index is defined by

$$\pi(\alpha, t) = \frac{\tilde{P}(\alpha, t)C(\alpha_0, t)}{P(\alpha_0, t)C(\alpha_0, t)} \tag{7}$$

where $\tilde{P}(\alpha, t)$ denotes the would-be market clearing prices conditional on parameter α for consuming the pre-project quantities $C(\alpha_0, t)$. Since $\nabla U(C(\alpha_0, t)) = \lambda(\alpha_0, t)P(\alpha_0, t) = \lambda(\alpha, t)\tilde{P}(\alpha, t)$, we have

$$\lambda(\alpha_0, t) = \pi(\alpha, t)\lambda(\alpha, t) \tag{8}$$

This index would enable us to lift the term $\lambda(\alpha, t)$ out of the inner integral in (6) such that

$$\Delta\hat{W} = \int_0^\tau \left[\int_{\alpha_0}^{\alpha_1} \bar{b}(\alpha, t)d\alpha\right] \lambda(\alpha_0, t)\exp(-\theta t)dt \tag{9}$$

where

$$\bar{b}(\alpha, t) = \bar{P}(\alpha, t) \cdot C_\alpha(\alpha, t) + \bar{Q}(\alpha, t) \cdot I_\alpha(\alpha, t) + \bar{R}(\alpha, t) \cdot K_\alpha(\alpha, t) \tag{10}$$

denotes the deflated social profit with $\bar{P}(\alpha, t) = P(\alpha, t)/\pi(\alpha, t)$, $\bar{Q}(\alpha, t) = Q(\alpha, t)/\pi(\alpha, t)$ and $\bar{R}(\alpha, t) = R(\alpha, t)/\pi(\alpha, t)$ as the "deflated prices" with the pre-project price level as numeraire. Note also that $\lambda(\alpha, t)\exp(-\theta t)$ in (9) can be written as

$$\lambda(\alpha_0, t)\exp(-\theta t) = \lambda(\alpha_0, 0)\exp(-\int_0^t r(\alpha_0, s)ds) \tag{11}$$

by the no-arbitrage relationship between utility and money rate of discounts. By normalizing $\lambda(\alpha_0, 0) = 1$, and integrating (9) by parts, we arrive at the following money-metric measure

$$\Delta\hat{W}_m = \int_0^\tau [\Delta Y(t) + CS(t) + \kappa(t)] \exp\left[-\textstyle\int_0^t r(\alpha_0, s)ds\right] dt \tag{12}$$

where $\Delta Y(t) = \left(\bar{P}(\alpha,t)C(\alpha,t) + \bar{Q}(\alpha,t)I(\alpha,t)\right)\big|_{\alpha_0}^{\alpha_1}$ denotes the discrete change in income, $CS(t) = -\int_{\bar{P}(\alpha_0,t)}^{\bar{P}(\alpha_1,t)} D(\bar{P},t)d\bar{P}$ a consumer surplus term at time t, and $\kappa(t) = \int_{\alpha_0}^{\alpha_1}\left[\bar{R}(\alpha,t)K_\alpha(\alpha,t) - I(\alpha,t)\bar{Q}_\alpha(\alpha,t)\right]d\alpha$ an extra cost term with $K_\alpha = \partial K/\partial\alpha$ and $\bar{Q}_\alpha = \partial\bar{Q}/\partial\alpha$. The vector function $D(\bar{P},t)$ denotes the compensated demand functions with the pre-project concurrent utility or income as a reference, and thereby the surplus measure also corresponds to the compensating or equivalent variations. Compared to the welfare-comparison result between "twin-economies" in Weitzman (2001), we obtain an extra term $\kappa(t)$ in the dynamic cost-benefit rule. The reason is that the project involves changes in net investment and capital stocks which need to be valued. In Weitzman's welfare comparison of twin-economies, optimal differences in capital stocks over time are generated by differences in initial conditions. We are analyzing a structural reform, which is not necessarily optimal, in an economy with given initial capital stocks. We summarize our main finding in the following proposition:

Proposition 1 *For a project represented by a parameter shift from $\alpha = \alpha_0$ to α_1 over the interval $[0,\tau]$, which may cause discrete changes in consumption, investment and capital stocks, as well as their accounting prices, the money measure in (12) can tell us whether or not the project is socially profitable. If it is positive, then the project is socially profitable; otherwise not.*

4 Conclusion

This paper has derived a dynamic cost-benefit rule for large projects from a multi-sector growth model, conditionally optimal for a given parameter set. As in Dreze and Stern (1987), we define a project as the change in the parameter value in a finite time period, involving changes in consumption, investment and capital stocks over time. By examining the change in the present value of net social profits, we find that the dynamic cost-benefit rule entails an extra term involving the cost of capital acquisition and the change in the value of investment, in addition to the conventional income plus consumer surplus terms as in Weitzman (2001).

5 References

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