Simultaneity and Asymmetry of Returns and Volatilities in the Emerging Baltic State Stock Exchanges

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Abstract

The paper suggests a nonlinear and multivariate time series model framework that enables the study of simultaneity in returns and in volatilities, as well as asymmetric effects arising from shocks and an outside stock exchange. Using daily data 2000-2006 for the Baltic state stock exchanges and that of Moscow we find recursive structures with Riga directly depending in returns on Tallinn and Vilnius, and Tallinn on Vilnius. For volatilities both Riga and Vilnius depend on Tallinn. In addition, we find evidence of asymmetric effects arising in Moscow and in Baltic state shocks on both returns and volatilities.

Key Words: Time series, nonlinear, multivariate, finance, value at risk, portfolio allocation.

JEL Classification: C32, C51, G11, G12, G14, G15.

Umeå Economic Studies 725, 2007

1. Introduction

This paper studies the joint evolution of returns and volatilities in the indices of the Baltic States stock exchanges, Riga (Latvia), Tallinn (Estonia), and Vilnius (Lithuania). These relatively small emerging marketplaces are geographically closely located. Besides sharing a common owner, many of the largest traders are common to all three marketplaces. Using three daily volume indices, Brännäs and Soultanaeva (2006) detected asymmetric effects in the series. Moreover, they demonstrated that good or bad news arriving from Russia (Moscow) have asymmetric impacts on the volatility transmissions for all indices under study. The model adopted was a univariate extension of an asymmetric ARMA (ARasMA) model introduced by Brännäs and De Gooijer (1994). Thus, each series was analyzed separately. Here, our main focus will be the joint modelling of, and the allowance for, simultaneity in both returns and volatilities along with asymmetry, and "Moscow" effects.

A lesson from the current within-day trading literature concerning some other marketplaces is that information processing is very fast (e.g., Engle and Russell, 1998). Given the institutional setup of the Baltic state marketplaces it is likely that information transmission between these markets is virtually instantaneous. Even if there are unidirectional causations within the day, a study based on a daily sampling frequency cannot but find an average effect that may go both ways. The sampling frequency scenario is in fact a main motivation in macro-econometrics for employing structural systems which can incorporate simultaneous endogenous effects. Only recently has there been some model-based financial studies allowing for simultaneity in returns (e.g., Rigobon and Sack, 2003, De Wet, 2006, Lee, 2006).

Obviously, and perhaps more interestingly from a risk management point of view, there is also reason to expect simultaneous effects in volatilities. Rigobon and Sack (2003) were the first ones to find simultaneity in volatilities. But, as in the studies of De Wet (2006) and Lee (2006), the simultaneity arises in a very restrictive way, and only as a consequence of the simultaneity in returns. Gannon (2004, 2005) detects simultaneity for some Asian markets using realized volatilities. Engle and Kroner (1995) suggested a related framework but focus theoretically on simultaneity in returns only.

The model platform for the current study is the univariate ARasMA model of Brännäs and De Gooijer (1994) combined with the asymmetric and quadratic GARCH of Brännäs and De Gooijer (2004). Brännäs and Soultanaeva (2006) extended this model class to allow for explanatory variables. The model is here to be given its first multivariate form and to allow for simultaneity in returns and volatilities separately. Notably, extensions of this type introduce additional parameters into an already richly parameterized model. Kroner and Ng (1998), De Goeij and Marquering (2005) and others discussed ways of parameterizing, in particular, the volatility functions for models to be estimable. To allow for simultaneity we will have to be restrictive in terms of correlation structure, lag lengths, and asymmetric effects.

The paper is organized as follows. In Section 2 we introduce the model and discuss some of its properties. In particular, we discuss the identifiability or uniqueness of estimation. Section 3 presents the estimator along with the employed stepwise model specification procedure. The section discusses testing against simultaneous, asymmetric, and Moscow effects. In addition, the use of the model for portfolio allocation and value at risk (VaR) studies are outlined. Section 4 presents the data-set. The empirical findings are given in Section 5. The final section concludes and relates our findings to other studies.

2. A Structural Vector ARasMA-asQGARCH Model

2.1 The Model

Consider an *m*-dimensional time series $\mathbf{y}_t = (y_{1t}, \ldots, y_{mt})'$. In this study $\{\mathbf{y}_t\}$ contains the variables of interest, i.e. the returns at time *t* of *m* stock market indices. The vector time series process $\{\mathbf{y}_t\}$ is assumed to be weakly stationary. Let $\mathbf{x}_t = (x_{1t}, \ldots, x_{kt})'$ denote a vector of exogenous variables that may affect the process $\{\mathbf{y}_t\}$ like, within the context of this paper, the impact of news of the Russian stock exchange (RTS). To introduce the asymmetric structure of the proposed model we first need to define an *m*-dimensional vector discrete-time stochastic process generated by $\mathbf{u}_t = (u_{1t}, \ldots, u_{mt})'$ defined by

$$\mathbf{u}_t = \mathbf{H}_t^* \boldsymbol{\varepsilon}_t,$$

where $\{\varepsilon_t\} \sim WN(\mathbf{0}, \mathbf{I}), \mathbf{H}_t^* = \{h_{ij,t}^*\}$ (i, j = 1, 2, ..., m), and \mathcal{F}_{t-1} denotes the history of the time series up to and including time t - 1. Hence, the conditional variance is $V(\mathbf{u}_t | \mathcal{F}_{t-1}) = \mathbf{H}_t^* \mathbf{H}_t^{*'} \equiv \mathbf{H}_t$. Then, asymmetries in the vector error process can be introduced as follows

$$\mathbf{u}_t^+ = \max(\mathbf{0}, \mathbf{u}_t) = \mathbf{H}_t^* \boldsymbol{\varepsilon}_t^+ \text{ and } \mathbf{u}_t^- = \min(\mathbf{0}, \mathbf{u}_t) = \mathbf{H}_t^* \boldsymbol{\varepsilon}_t^-$$

where $\varepsilon_t^+ = \max(\mathbf{0}, \varepsilon_t)$ and $\varepsilon_t^- = \min(\mathbf{0}, \varepsilon_t)$. Now a simultaneous or structural vector ARasMA model can be defined as

$$\mathbf{A}_{0}\mathbf{y}_{t} = \sum_{i=1}^{p} \mathbf{A}_{i}\mathbf{y}_{t-i} + \mathbf{u}_{t} + \sum_{i=1}^{q} \left(\mathbf{B}_{i}^{+}\mathbf{u}_{t-i}^{+} + \mathbf{B}_{i}^{-}\mathbf{u}_{t-i}^{-} \right) + \mathbf{c}_{0} + \sum_{i=0}^{r} \left(\mathbf{C}_{i}^{+}\mathbf{x}_{t-i}^{+} + \mathbf{C}_{i}^{-}\mathbf{x}_{t-i}^{-} \right), \quad (1)$$

where $\mathbf{x}_t^+ = \max(\mathbf{0}, \mathbf{x}_t)$, and $\mathbf{x}_t^- = \min(\mathbf{0}, \mathbf{x}_t)$. Model (1) accounts for asymmetric effects unless for all i, $\mathbf{B}_i^+ = \mathbf{B}_i^-$ and $\mathbf{C}_i^+ = \mathbf{C}_i^-$. If appropriate, the threshold level for the process $\{\mathbf{x}_t\}$ may be set at another value than **0**. Within the context of the present paper, the time series processes $\{\mathbf{x}_t^+\}$ and $\{\mathbf{x}_t^-\}$ represent positive and negative returns at time t in the RTS index. It is easy to see that the threshold levels in $\{\mathbf{u}_t^+\}$ and $\{\mathbf{u}_t^-\}$ can be accommodated by the vector of constants \mathbf{c}_0 .

The $m \times m$ non-symmetric matrix \mathbf{A}_0 in (1) contains the simultaneity parameters,

$$\mathbf{A}_{0} = \begin{pmatrix} 1 & a_{12}^{0} & \cdots & a_{1m}^{0} \\ a_{21}^{0} & 1 & \cdots & a_{2m}^{0} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}^{0} & a_{m2}^{0} & \cdots & 1 \end{pmatrix},$$

where an assumption of normalization has been imposed, i.e. coefficients along the diagonal are equal to 1. Assume \mathbf{A}_0 is nonsingular. Then the conditional mean (return) of $\{\mathbf{y}_t\}$ follows

directly from the conditional reduced form of (1) as

$$E(\mathbf{y}_{t}|\mathcal{F}_{t-1}) = \sum_{i=1}^{p} \mathbf{A}_{0}^{-1} \mathbf{A}_{i} \mathbf{y}_{t-i} + \sum_{i=1}^{q} \mathbf{A}_{0}^{-1} \left(\mathbf{B}_{i}^{+} \mathbf{u}_{t-i}^{+} + \mathbf{B}_{i}^{-} \mathbf{u}_{t-i}^{-} \right) + \mathbf{A}_{0}^{-1} \mathbf{c}_{0} + \sum_{i=0}^{r} \mathbf{A}_{0}^{-1} \left(\mathbf{C}_{i}^{+} \mathbf{x}_{t-i}^{+} + \mathbf{C}_{i}^{-} \mathbf{x}_{t-i}^{-} \right).$$
(2)

Similarly, the conditional variance (volatility or risk) is given by

$$V(\mathbf{y}_t|\mathcal{F}_{t-1}) = \mathbf{A}_0^{-1}\mathbf{H}_t(\mathbf{A}_0^{-1})'$$

from which, e.g., the conditional correlation matrix can be obtained. Various options are available to specify an asymmetric model for \mathbf{H}_t ; see De Goeij and Marquering (2005). The specifications for \mathbf{H}_t suggested by these authors contain off-diagonal elements. Thus there are conditional and possibly unconditional correlations among the elements of $\{\mathbf{u}_t\}$, and consequently among those of $\{\mathbf{y}_t\}$. There is no simultaneity in conditional volatility behavior in the sense that the conditional variance of, say, u_{it} would be a direct function of the corresponding conditional variance of u_{jt} ($i \neq j$) in the same time period.

As we wish to have simultaneity in conditional volatility as an integral part of the model we need to consider an extension of the univariate asQGARCH model. One avenue that appears feasible is to view the structures of De Goeij and Marquering (2005) as "reduced forms". Note that structural forms may make economic sense but that only the reduced form gives the conditional variance interpretation. The situation resembles closely that of the simultaneous and reduced forms in classical macro-econometrics. Similarly, we view simultaneity to arise mainly due to the relatively low sampling frequency of one day while real trading occurs in continuous time, and partly due to identical actors on different stock exchanges.

Our general simultaneous specification for the conditional variance is very much in the same spirit as model (1). Given a vector time series process $\{\mathbf{z}_t\}$ of exogenous variables, the vector asQGARCH model is given by

$$\mathbf{D}_{0}\mathbf{h}_{t} = \sum_{i=1}^{P} \mathbf{D}_{i}\mathbf{h}_{t-i} + \sum_{i=1}^{Q} \left(\mathbf{F}_{i}^{+}\mathbf{u}_{t-i}^{+} + \mathbf{F}_{i}^{-}\mathbf{u}_{t-i}^{-}\right) + \sum_{i=1}^{Q} \mathbf{K}_{i}\mathbf{u}_{t-i}^{*,2} + \mathbf{g}_{0} + \sum_{i=0}^{R} \left(\mathbf{G}_{i}^{+}\mathbf{z}_{t-i}^{+} + \mathbf{G}_{i}^{-}\mathbf{z}_{t-i}^{-}\right), \qquad (3)$$

where \mathbf{g}_0 is an $\frac{1}{2}m(m+1) \times 1$ vector of constants, $\mathbf{z}_t^+ = \max(\mathbf{0}, \mathbf{z}_t)$, $\mathbf{z}_t^- = \min(\mathbf{0}, \mathbf{z}_t)$, and the vector $\mathbf{u}_t^{*,2}$ has elements u_{it}^2 (i = 1, ..., m). Within the context of the empirical analysis, the series $\{\mathbf{z}_t\}$ will enter (3) as the demeaned moving variance series of the RTS index; see Section 4 for more details on the construction of this series.

The reduced form of (3) is

$$\mathbf{h}_{t} = \sum_{i=1}^{P} \mathbf{D}_{0}^{-1} \mathbf{D}_{i} \mathbf{h}_{t-i} + \sum_{i=1}^{Q} \mathbf{D}_{0}^{-1} \left(\mathbf{F}_{i}^{+} \mathbf{u}_{t-i}^{+} + \mathbf{F}_{i}^{-} \mathbf{u}_{t-i}^{-} \right) + \sum_{i=1}^{Q} \mathbf{D}_{0}^{-1} \mathbf{K}_{i} \mathbf{u}_{t-i}^{*,2} + \mathbf{D}_{0}^{-1} \mathbf{g}_{0} + \sum_{i=0}^{R} \mathbf{D}_{0}^{-1} \left(\mathbf{G}_{i}^{+} \mathbf{z}_{t-i}^{+} + \mathbf{G}_{i}^{-} \mathbf{z}_{t-i}^{-} \right)$$

$$(4)$$

from which the corresponding \mathbf{H}_t matrix can be obtained. The matrix \mathbf{D}_0 captures simultaneity, whereas the matrices \mathbf{D}_i $(i \ge 1)$ are useful to represent persistence and possible cyclical features in the process $\{\mathbf{h}_t\}$. Also asymmetric effects are characterized through the matrices \mathbf{F}_i^+ (\mathbf{F}_i^-) and \mathbf{G}_i^+ (\mathbf{G}_i^-). Empirically, it is important to realize that the estimation of (4) may become infeasible with too generously parameterized specifications. Reducing lag lengths and introducing sparse matrix specifications are two ways of reducing the number of parameters; see Section 3 for a data-driven model specification procedure.

Various moment properties, and distributional results for univariate ARasMA models have been reported by Brännäs and De Gooijer (1994) and Brännäs and Ohlsson (1999), and for univariate ARasMA-quadratic GARCH models by Brännäs and De Gooijer (2004). Since $V(\mathbf{y}_t) = \mathbf{A}_0^{-1} E_{\mathcal{F}_{t-1}}(\mathbf{H}_t)(\mathbf{A}_0^{-1})' + V_{\mathcal{F}_{t-1}}[E(\mathbf{y}_t|\mathcal{F}_{t-1})]$, obtaining an explicit expression for the unconditional variance of $\{\mathbf{y}_t\}$ is a far from trivial problem.

2.2 Identification

We say that the system of simultaneous vector equations is identified when the parameters of the model can be uniquely estimated. Since estimation of the structural vector ARasMAasQGARCH model will be in terms of its reduced form it is obvious that parameter matrices \mathbf{A}_0 and \mathbf{D}_0 play important roles. For instance, if \mathbf{A}_0 can be determined from lagged \mathbf{y}_{t-i} parameters, all other parameters can be obtained uniquely. The situation is analogous for \mathbf{D}_0 . The imposition of some sort of normalization restriction is necessary but not sufficient to achieve identification. A "traditional" solution is to impose long-run restrictions and/or sign restrictions on the parameters. However, within the context of our empirical analysis, we feel that these restrictions are difficult to defend. Instead we rely on a methodology proposed by Rigobon (2003) and Rigobon and Sack (2003) who showed that identification can be achieved if there is conditional heteroskedasticity in the data. The key idea is based on the movement of structural innovations $\{\mathbf{u}_t\}$ and the movement of the conditional covariances between them. The heteroskedasticity adds equations to the system, but also some unknowns. So, it is essential to impose some restrictions on the covariances to be able to use the variation in the second moments to solve the problem of identification. Rigobon (2003) derives necessary conditions for identification in case there are discrete regimes in the variances of the structural shocks. In our structural vector model, the variances of the shocks are allowed to evolve in a continuous manner. Thus giving rise to a continuum of regimes for identifying the system.

3. Estimation and Model Use

Given a multivariate normality assumption on $\{\varepsilon_t\}$ the prediction error

$$\mathbf{y}_t - E(\mathbf{y}_t | \mathcal{F}_{t-1}) = \mathbf{A}_0^{-1} \mathbf{u}_t = \mathbf{A}_0^{-1} \mathbf{H}_t^* \boldsymbol{\varepsilon}_t \equiv \mathbf{v}_t$$
(5)

is i.i.d. $N(\mathbf{0}, \mathbf{\Gamma}_t)$ distributed with $\mathbf{\Gamma}_t = \mathbf{A}_0^{-1} \mathbf{H}_t(\mathbf{A}_0^{-1})'$; recall (3). Here, \mathbf{H}_t is the conditional variance expression in reduced form, containing among other things the \mathbf{D}_0 matrix. Given

observations up till time T, the log-likelihood function takes the form

$$\ln L \propto -\frac{1}{2} \sum_{t=s}^{T} \ln |\mathbf{\Gamma}_{t}| - \frac{1}{2} \sum_{t=s}^{T} \mathbf{v}_{t}' \mathbf{\Gamma}_{t}^{-1} \mathbf{v}_{t}$$
$$\propto (T-s) \ln |\mathbf{A}_{0}| - \frac{1}{2} \sum_{t=s}^{T} \left(\ln |\mathbf{H}_{t}| + \mathbf{u}_{t}' \mathbf{H}_{t}^{-1} \mathbf{u}_{t} \right), \tag{6}$$

where $s = \max(p, q, r) + 1$. For practical quasi maximum likelihood estimation we use the RATS 6.0 package and employ robust standard errors.

To obtain the final model specification we advocate the following stepwise procedure.

- 1. Univariate ARasMA-asQGARCH models containing specifications for both mean returns and conditional variance are first estimated. Select models that minimize AIC or some other appropriate model selection criterion. Thus, we implicitly assume that there are no interactions between the series.
- 2. Using results from step 1 introduce simultaneity in the structural form, i.e. add \mathbf{A}_0 . Consider thereafter the expansion to non-diagonal matrices in the returns expression. Choose the specification that minimizes AIC. The \mathbf{A}_0 is the final parameter matrix to be reduced. For this step the volatility functions obtained in step 1 are taken as given, but $\{\hat{\mathbf{u}}_t\}$ changes in the iterative steps.
- 3. Using results from steps 1 and 2 introduce simultaneity in the volatility function, i.e. add \mathbf{D}_0 . Consider thereafter the expansion to non-diagonal matrices in the volatility expression. Choose the specification that minimizes AIC. The \mathbf{D}_0 is the final parameter matrix to be reduced and the $\{\hat{\mathbf{u}}_t\}$ -sequence are taken as given from step 2.
- 4. In a final step all parameters are estimated jointly.

Given the estimated model, it is of interest to test hypotheses about simultaneity, asymmetry, and the Moscow effect. Given the likelihood framework and our specification procedure, Wald and likelihood ratio (LR) test statistics are relatively easy to implement.

We first consider tests of simultaneity and do so in terms of the \mathbf{A}_0 matrix. The reasoning with respect to \mathbf{D}_0 is analogous. We say that there is a simultaneous effect between markets i and j if $(\mathbf{A}_0)_{ij} \neq 0$ and $(\mathbf{A}_0)_{ji} \neq 0$. When $(\mathbf{A}_0)_{ij} \neq 0$ but $(\mathbf{A}_0)_{ji} = 0$ there is a recursive structure and causation is unidirectional from market j to market i. When $(\mathbf{A}_0)_{ij} = (\mathbf{A}_0)_{ji} = 0$ there is no causation between returns. When all off-diagonal elements equal zero $\mathbf{A}_0 = \mathbf{I}$ and the structural and reduced forms are identical.

Next we consider testing against asymmetric effects and do so in terms of the \mathbf{B}_i^+ and $\mathbf{B}_i^$ matrices. We may form $\mathbf{B}_i^{\bigtriangledown} = \mathbf{B}_i^+ - \mathbf{B}_i^-$ (i = 1, ..., q), and test whether this matrix is equal to zero or whether it is nonzero. We then make no distinction between the case of both matrices having nonzero parameters $(\mathbf{B}_i^+)_{ij}$ and $(\mathbf{B}_i^-)_{ij}$ in all places and the case where, say, $(\mathbf{B}_i^-)_{ij} = 0$. Testing against asymmetric effects of Moscow is in terms of the parameter matrices \mathbf{C}_i^+ and \mathbf{C}_i^- (i = 1, ..., r). For asymmetric effects in volatility the parameter matrices \mathbf{F}_i^+ and \mathbf{F}_i^- as well as \mathbf{G}_i^+ and \mathbf{G}_i^- are focused.

For no effects of Moscow on returns all matrices \mathbf{C}_i^+ and \mathbf{C}_i^- must be identical to a zero matrix, while for volatility all \mathbf{G}_i^+ and \mathbf{G}_i^- must be zero.

When we wish to use or, as here, evaluate the model in financially interesting and meaningful ways, portfolio allocation and VaR measures are of obvious interest. Two problems both stemming from the use of index series arise; how to get back to the index and what price related to the index should we consider.

First, the index is determined from the inverse of the change variable $y_{it} = 100 \ln(I_{it}/I_{it-1})$, i.e. as $I_{it} = I_{it-1} \exp(y_{it}/100)$ for stock market *i*. We get $E(I_{it}|\mathcal{F}_{t-1}) = I_{it-1}E(\exp(y_{it}/100)|\mathcal{F}_{t-1})$ $\approx I_{it-1}(1 + E(y_{it}|\mathcal{F}_{t-1})/100)$ where the first order approximation of the exponential function is reasonable for the small values of $y_{it}/100$. Using the same first order approximation we get $V(\mathbf{I}_t|\mathcal{F}_{t-1}) = \mathbf{I}_{t-1}^{\circ}V(\mathbf{y}_t|\mathcal{F}_{t-1})\mathbf{I}_{t-1}^{\circ}/100^2$, where \mathbf{I}_t° is a matrix with elements I_{it} on the diagonal and zeroes elsewhere. These expressions are useful if we wish to forecast the index and to give its forecast variance. Second, trading is not directly in terms of the index. The presence of index funds and standard options tied to the index are reasonable justifications for using the index as a price. The chosen approach is to use the return series as is and then emphasize the return as an indicator of market risk (e.g., McNeil and Frey, 2000).

For portfolio allocation we adopt the tangency portfolio (e.g., Campbell et al., 1997, ch 5). At time T + 1 we have

$$\mathbf{a}_{T+1} = V^{-1}(\mathbf{y}_{T+1}|\mathcal{F}_T) \cdot \left[E(\mathbf{y}_{T+1}|\mathcal{F}_T) - R_f \mathbf{1}\right]/A,\tag{7}$$

where $A = \mathbf{1}' V^{-1}(\mathbf{y}_{T+1}|\mathcal{F}_T) \cdot [E(\mathbf{y}_{T+1}|\mathcal{F}_T) - R_f \mathbf{1}]$, R_f is the risk free rate, and $\mathbf{1}$ is a column vector of ones. Hence, $\mathbf{1}' \mathbf{a}_{T+1} = 1$. For the VaR-measure under normality, a time invariant allocation vector \mathbf{a} , and a probability α , Gourieroux and Jasiak (2001, ch 16) give:

$$R_{T+1} = -\mathbf{a}' E(\mathbf{y}_{T+1} | \mathcal{F}_T) + \Phi^{-1}(1-\alpha) \left[\mathbf{a}' V(\mathbf{y}_{T+1} | \mathcal{F}_T) \mathbf{a} \right]^{1/2}.$$
(8)

This VaR measure is in terms of returns; one in terms of indices can also be devised by simply replacing \mathbf{y}_{T+1} by \mathbf{I}_{T+1} and using the expressions given above. Using shock scenarios in terms of the \mathbf{u}_t vector or in terms of $\mathbf{x}_t^{+/-}$ and $\mathbf{z}_t^{+/-}$, the \mathbf{a}_{T+1} and R_{T+1} can be calculated and then evaluated and subjected to comparisons. To cast light on effects of simultaneity, the univariate models can be compared to the simultaneous model system in terms of the portfolio or VaR metrics either as above or over some historical period. Note, that both measures are subject to sampling variation in estimated mean return and risk functions. Britten-Jones (1999) and others have discussed the variation in allocation weights, while Christoffersen and Gonçalves (2005) among others have discussed the issue for VaR measures.

4. Data

The data used in this paper are capitalization weighted daily stock price indices of the Estonian (Tallinn, TALSE), Latvian (Riga, RIGSE), Lithuanian (Vilnius, VILSE) and Russian (Moscow,



Figure 1: Indices of the Baltic stock exchanges (December 31, 1999 = 100).

RTS) stock markets. All prices are transformed into Euros from local currencies, except for Estonia where stock market trading is in Euro. The data-set covers January 3, 2000 to August 16, 2006, for a total of T = 1729 observations, cf. Figure 1 for the three Baltic indices. Both indices and exchange rates are collected from DataStream. The irregularity in the summer of 2001 in the Riga index (RIGSE) is due to a power struggle in its largest company (Latvijas Gaze). Instead of elaborating on modelling to contain this irregular period, the Riga series is adjusted in the following simplistic way: For a speculation period from July 25 to September 3, 2001, observations are replaced by interpolated values.

Due to some differences in holidays for the involved countries the series have different shares of days for which index stock price are not observable. Linear interpolation was used to fill the gaps for all series. The resulting series are then throughout for a common trading week. All returns are calculated as $y_t = 100 \cdot \ln(I_t/I_{t-1})$, where I_t is the daily price index. Table 1 reports descriptive statistics for the daily returns. The Ljung-Box statistics for 10 lags (LB₁₀) indicate significant serial correlations. The large kurtoses for Riga, Tallinn and Vilnius indicate leptokurtic densities. Table 2 presents cross correlations for the Baltic return series and for a squared returns. Table 3 gives lagged cross correlations. For instance, the table indicates that Tallinn is positively affected by Vilnius both within the day and with up to three lags. There appears to be no impact from Riga.

Figure 2 gives scatterplots for pairs of returns series with a nonparametric regression line (LOWESS default settings in RATS 6.0). Visual inspection indicates that there is weak dependence between Riga and Tallinn for the majority of observations, while for the other plots there appear to be positive relationships.

Table 1: Descriptive statistics for return series.

Exchange	Mean	Variance	$\operatorname{Min}/\operatorname{Max}$	Skewness	Kurtosis	LB_{10}
Riga	0.10	1.77	-9.27/10.29	0.18	10.72	45.93
Tallinn	0.10	1.05	-5.87/12.02	1.09	15.94	51.43
Vilnius	0.09	1.05	-12.12/5.32	-0.91	13.82	46.87
Moscow	0.12	4.93	-11.92/10.23	-0.47	3.27	16.37

Note: LB_{10} is the Ljung-Box statistic evaluated at 10 lags.

Table 2: Cross correlations for Baltic stock markets returns and squared returns.

		Return	s		Squared Returns							
	Riga	Tallinn	Vilnius	_	Riga	Tallinn	Vilnius					
Riga	1				1							
Tallinn	0.134	1			0.161	1						
Vilnius	0.141	0.208	1		0.023	0.032	1					



Figure 2: Cross plots for Baltic returns series. One negative outlier for Vilnius is outside the figure and three positive ones for Tallinn.

Table 3: Cross correlations for Baltic stock markets returns (in the order Riga, Tallinn and Vilnius). Significant entries are indicated by signs and subindex indicates lag.

$$\begin{pmatrix} 1 & + & + \\ \cdot & 1 & + \\ \cdot & + & 1 \end{pmatrix}_{0}, \begin{pmatrix} - & \cdot & \cdot \\ \cdot & \cdot & + \\ \cdot & + & + \end{pmatrix}_{1}, \begin{pmatrix} \cdot & \cdot & + \\ \cdot & \cdot & + \\ \cdot & \cdot & + \end{pmatrix}_{2}, \begin{pmatrix} + & \cdot & \cdot \\ \cdot & \cdot & + \\ \cdot & \cdot & + \end{pmatrix}_{3}, \begin{pmatrix} - & \cdot & \cdot \\ \cdot & \cdot & - \\ \cdot & \cdot & + \end{pmatrix}_{4}$$

5. Results

The empirical results are presented first in terms of the return function and later in terms of the volatility function. Table A contains estimated univariate models. The empirical specifications are obtained by the steps outlined in Section 3.

For the return function of $\{\mathbf{y}_t\}$, cf. eq (1), when returns are in the order Riga, Tallinn and Vilnius, the estimated function is

With respect to simultaneity, the $\hat{\mathbf{A}}_0$ matrix indicates a recursive structure; the returns of the Riga index depends within the day positively on both the index returns of Tallinn and Vilnius, while returns in Tallinn are positively influenced by those of Vilnius. Riga returns have no impact on the returns of neither Tallinn nor Vilnius, and Tallinn returns have no influence on those of

Vilnius. The only lagged influence arises for Vilnius at lag two, cf. the \hat{A}_2 matrix. For Riga returns Moscow has a quite symmetric and positive effect within the day. For Tallinn we instead find asymmetric and negative effects spread over lags 0-2, and for Vilnius negative shocks out of Moscow appear to have larger impact than positive shocks. For shocks arising in the three Baltic stock exchanges we find that a positive shock in Riga at lag one has a negative impact on current returns, and in addition negative lag two shocks of Tallinn and Vilnius have negative effects. Positive shocks in Tallinn have stronger effects than equally sized negative shocks, and there are negative shocks of both Riga and Vilnius at lag 2. The off-diagonal elements of lagged shocks suggests that there are some shock-spillovers; Riga returns are negatively influenced by Tallinn and Vilnius shocks at lag two, while Tallinn is impacted by Riga and Vilnius shocks at lag one.

The estimated volatility function has the form

Only two elements in $\hat{\mathbf{D}}_0$ are significant, the volatility of Vilnius depends negatively but weakly on that of Tallinn in the same time period, while Riga depends positively on Tallinn. As expected

Hypothesis	Wald	df	Measure	Riga	Tallinn	Vilnius
Simultaneity-Returns	27.0	3	LB_{10}	10.08	5.82	22.75
Simultaneity-Risk	7.81	2	LB_{10}^2	11.77	1.63	1.14
Asymmetry-Return-Moscow	160.9	6	Skewness	0.47	0.54	-0.30
Asymmetry-Return-Innovation	74.4	8	Kurtosis	4.33	6.31	6.06
Asymmetry-Risk-Moscow	92.8	6	$_{\mathrm{JB}}$	1403.7	2936.8	2659.2
Asymmetry-Risk-Innovation	6033	7	R^2	0.05	0.18	0.06

Table 4: Simultaneity and asymmetry tests together with model evaluation measures.

volatilities are quite persistent, cf. the $\hat{\mathbf{D}}_1$ -matrix estimates. In the very short term (within the day) a higher than average Moscow risk marginally reduces risk in Riga, while the effect is an enhancing one for Vilnius. Already after one day there appears to remain little impact of Moscow risk for Vilnius. This is also true for negative shocks in all three stock markets.

The conditional covariances are very small and insignificantly estimated as $\mathbf{H}_{t,1,2} = 0.003$ (s.e. = 0.023), $\mathbf{H}_{t,1,3} = 0.000$ (0.033) and $\mathbf{H}_{t,2,3} = 0.000$ (0.025).

The model evaluation phase considers formal tests against simultaneity in returns and in risk as well as tests against asymmetric effects arising from Moscow or from the innovations of the model system. As a first but informal test supporting the joint models rests on the likelihoods under the univariate models and the joint model; the likelihood ratio statistic is then LR = 181.8. Table 4 summarizes the formal test results and also gives the serial correlation properties and the goodness-of-fit for the model. The Wald tests are all significant with *p*-values less than 0.02. There is then evidence of simultaneity as well as of asymmetric effects. When it comes to serial correlation properties in standardized and squared standardized residuals there appears to be remaining serial correlation in only one series, the standardized residuals of Vilnius. The standardized residuals are nonnormal and leptokurtic.

Next, we consider the estimated volatility functions in some more detail in Figures 3-4. Figure 3 shows the estimated $\mathbf{H}_{t,i,i}$ functions for the final part of the series. It is quite clear from this figure that the volatilities of Riga and Vilnius are larger than those of Tallinn. This pattern reenforces the sample variance ordering of Table 1. The estimated volatility functions are positively correlated, cf. Figure 4. Since covariance estimates $\mathbf{H}_{t,i,j}$ between the innovations of stock exchanges are very small the resulting time-varying conditional correlations are also very small and always smaller than 0.05. The implied estimated conditional correlations between $\{\mathbf{y}_t\}$ variables are much larger and also positive throughout, cf. Figure 5. Average conditional correlations are relatively close to the sample correlations of Table 2.

Portfolio allocations and VaR measures one-step-ahead are depicted in Table 5. These measures are based on forecast equations



Figure 3: Estimated volatility functions for the final part of the sample period.



Figure 4: Plots of estimated volatilities (some outlying volatilities fall outside the graphs).



Figure 5: Estimated conditional correlations between the returns of the stock markets for the final part of the sample period.

Tab	ble 5 :	Portf	olio a	and '	VaR	effects	of	shocks	in	innovati	ons	and I	Moscov	w (J	oint),	togetl	her	with
a ui	nivari	iate n	nodel	(Sin	igle)	case.	The	e VaR	is ł	based on	pro	babili	ity 0.0	l25 ε	and a	portfo	olio '	with
weig	ghts ().333 :	for ea	ach in	ndex	(VaR-	A)	and w	ith	the weig	tts o	obtaiı	ned in	the	Base	case (VaR	-B).

]	Portfolio	Allocat		VaR						
		Joint			Single	;		А		В		
	Riga	Tallinn	Vilnius	Riga	Tallinn	Vilnius	Joint	Single	Joint	Single		
Base case	0.24	0.66	0.10	0.32	0.50	0.18	1.23	0.91	1.66	0.83		
Shock-Riga	0.27	0.64	0.09	0.19	0.60	0.21	1.19	1.15	1.65	0.98		
-Tallinn	0.30	0.54	0.16	0.35	0.45	0.19	1.23	0.99	1.64	1.06		
-Vilnius	0.26	0.72	0.02	0.37	0.58	0.05	1.42	0.99	2.02	0.81		
-Moscow (x)	0.23	0.67	0.10	0.31	0.51	0.18	1.25	0.90	1.67	0.82		
-Moscow (z)	0.24	0.62	0.14	0.27	0.50	0.23	1.36	1.07	1.77	1.03		

$$E(\mathbf{y}_{T+1}|\mathcal{F}_{T}) = \hat{\mathbf{A}}_{0}^{-1} \left[\hat{\mathbf{A}}_{2} \mathbf{y}_{T-1} + \sum_{i=1}^{2} \left(\hat{\mathbf{B}}_{i}^{+} \hat{\mathbf{u}}_{T+1-i}^{+} + \hat{\mathbf{B}}_{i}^{-} \hat{\mathbf{u}}_{T+1-i}^{-} \right) \right. \\ \left. + \hat{\mathbf{c}}_{0} + \sum_{i=0}^{2} \left(\hat{\mathbf{C}}_{i}^{+} \mathbf{x}_{T-i}^{+} + \hat{\mathbf{C}}_{i}^{-} \mathbf{x}_{T-i}^{-} \right) \right] \\ V(\mathbf{y}_{T+1}|\mathcal{F}_{T}) = \hat{\mathbf{A}}_{0}^{-1} \hat{\mathbf{H}}_{T+1} (\hat{\mathbf{A}}_{0}^{-1})'$$

and depend on the histories of \mathbf{y}_t , $\hat{\mathbf{u}}_t$, and \mathbf{x}_t for the conditional return and additionally on the histories of \mathbf{H}_t and \mathbf{z}_t for the conditional volatility. Since the impact of Moscow is in the same period we set future values $(x_{T+1} \text{ and } z_{T+1})$ for Moscow close to their values at the end of the series, i.e. as $x_{T+1}^+ = 0.1$ and $z_{T+1}^- = -4$. This is the Base case design. For the portfolio allocation exercise the risk free rate is set at 1.07, which is the level of the Euro market government bond yield by the end of the sample period.

The allocation for the Tallinn stock exchange is 0.66, while 0.24 of the portfolio should be placed in Riga and 0.10 in Vilnius. Using the same setup but using instead the univariate models (Single) of Table A, gives a much lower allocation for Tallinn and higher ones for both Riga and Vilnius.¹ The two model forms differ in simultaneity but also with respect to other features of the dynamic model. Therefore, we cannot infer with certainty that the differences are due solely to simultaneous effects. The VaR measures for probability 0.025 are for the simultaneous model with equal weights 1.23 and for the univariate models 0.91. For the weights obtained with the weights of the Base case we get 1.66 and 0.82, respectively.

To study the sensitivity of the Base case results we next shock the individual elements of $\hat{\mathbf{u}}_T$ (the final residuals are individually multiplied by a factor 3). For shocks in the Tallinn and Vilnius stock markets the allocations for these markets are reduced. Figure 6 illustrates this for an increasingly negative shock in Tallinn. With a decrease in the Tallinn weight comes relatively more weight for Riga than for Vilnius. The allocations obtained using the univariate models differ from those based on the joint model, mainly such that the weights for Riga and Vilnius are larger and those for Tallinn are smaller.

We also consider shocks arising in Moscow returns (x_{T+1}^+) is set to 1). This appears to have only minor impact. For Moscow risk we change from $z_{T+1}^- = -4$ to $z_{T+1}^+ = 4$ and note an increase for Vilnius and a reduction for Tallinn allocations.

The VaR measure changes little for shocks in Tallinn but responds more to shocks in Vilnius and in Moscow risk. The VaR:s based on the univariate models are smaller than the corresponding measures for the joint model. When the weights of the Base case are used the VaR:s increase markedly throughout. Figure 6 studies the impacts on VaR of Moscow shocks in more detail. Changes in risk have rather small effects, while Moscow return changes have a more sizeable and asymmetric effect.

¹In shocking the stock markets, note that the residuals of the joint and univariate models differ both in sizes and signs. The underlying sizes of residuals in the univariate models have not been changed but shocks are throughout in the direction of the joint model.



Figure 6: Allocations after shocking the final negative residual for Tallinn (left exhibit, a value on the x-scale larger than +1 means a larger negative shock). VaR effects of shocks to Moscow return and risk (right exhibit).

6. Conclusion

The paper has introduced simultaneity into a multivariate and nonlinear time series model framework to study jointly the indices of the Baltic states stock exchanges. Unlike previous studies (e.g., Rigobon and Sack, 2003, De Wet, 2006, Lee, 2006), we allow for simultaneity in returns and volatility separately. The model allows us to capture "within a day" information transmission between the stock markets under study. Since information transmission between markets is virtually instantaneous (e.g., Engle and Russell, 1998) a study based on daily sampling frequency should take into account simultaneous reactions to movements in other relevant assets or markets. Moreover, the model is able to capture asymmetric impacts of lagged positive and negative shocks on returns and volatility processes. We argue that measuring simultaneous and asymmetric spillovers is important for a number of reasons, including optimal portfolio allocation and risk management.

Empirically, we illustrate the importance of simultaneity with respect to Baltic stock markets. In these closely related markets simultaneity is likely to arise due to geographic proximity, common institutional setup as well as common large traders, among other things. We found strong evidence of simultaneous effects/interaction in both returns and volatility. In returns, Riga is dependent on the indices of Tallinn and Vilnius, Tallinn is dependent on Vilnius, while Vilnius is not influenced by the other two markets. For volatility, we find within a day spillovers from Tallinn to both Riga and Vilnius. In addition, we found asymmetric effects of Moscow returns on the index returns in the Baltic exchanges, and asymmetric effects of Moscow risk on volatilities.

To illustrate the importance of simultaneous interaction between markets we obtain the portfolio allocations and value at risk measures for the multivariate and univariate models. Portfolio allocation results indicate that optimal portfolio weights are more sensitive to shocks when simultaneity is not accounted for. VaR measures indicate that the variability in losses that may occur due to shocks to the market is larger when simultaneity is not accounted for.

The simultaneous and dynamic econometric model generalizes previous univariate models by allowing for simultaneity but also for cross-effects of innovations. As in any simultaneous model we can therefore talk about direct, indirect and total effects in the return and volatility functions. The direct effects can be seen in the estimation results, while the portfolio and value at risk results build on total effects. To estimate the model we employ full information maximum likelihood. The suggested stepwise specification procedure resulted in a model with important deviations from corresponding univariate models. Estimation of the final model does not result in numerical problems despite the fact that the model is quite richly parametrized.

Acknowledgements

The financial support from the Wallander-Hedelius foundation to the first three authors and from the Nordea foundation to Albina Soultanaeva are gratefully acknowledged. A previous version of this paper was presented at the Nordic Econometric Meeting 2007.

		Ri	ga			Tall	inn		Vilnius				
Variables	Retu	ırn	Ri	sk	Retu	ırn	Ris	sk	Ret	urn	Ris	sk	
y_{t-2}									0.057	0.021			
u_{t-1}^+	-0.146	0.048	-0.072	0.018	0.252	0.041			0.162	0.045	0.273	0.030	
u_{t-2}^{+}					0.117	0.037	0.021	0.022					
u_{t-3}^+													
u_{t-1}^-			0.394	0.071	0.119	0.046	-0.190	0.181			-0.279	0.023	
u_{t-2}^-			-0.283	0.064									
h_{t-1}			0.944	0.005			0.917	0.009			0.829	0.024	
u_{t-1}^2			0.389	0.034			0.113	0.034			0.093	0.031	
u_{t-2}^2			-0.322	0.031			-0.135	0.031			-0.113	0.026	
x, z_t^+	0.050	0.021	-0.001	0.001							0.034	0.005	
x, z_{t-1}^+									0.046	0.0167	-0.032	0.005	
x, z_t^-	0.105	0.021	0.121	0.0167	0.120	0.011	0.046	0.012	0.126	0.015	0.007	0.004	
x, z_{t-1}^-			-0.114	0.0167	0.046	0.012	-0.050	0.012					
x, z_{t-2}^-					0.029	0.013							
Constant	0.177	0.033	0.079	0.012	0.114	0.027	-0.035	0.004	0.141	0.027	0.004	0.020	
AIC	2086.9				1164.5				1446.8				
$\ln L, R^2$	-1029.5		0.03		-566.87			0.16	-709.41			0.06	
LB_{10}	10.84		8.83		7.01		1.53		21.57		1.53		
Skew, Kurt, JB	0.43	5.60	2303.5		0.439	6.86	3446.6		-0.23	6.48	3030.7		

Table A: Estimation results for univariate models.

Notes:

References

- Brännäs K, De Gooijer JG. 1994. Autoregressive-asymmetric moving average models for business cycle data. *Journal of Forecasting* **13**, 529-544.
- Brännäs K, De Gooijer JG. 2004. Asymmetries in conditional mean and variance: modelling stock returns by asMA-asQGARCH. *Journal of Forecasting* **23**, 155-171.
- Brännäs K, Ohlsson H. 1999. Asymmetric time series and temporal aggregation. Review of Economics and Statistics 81, 341-344.
- Brännäs K, Soultanaeva A. 2006. Influence of news in Moscow and New York on returns and risks on Baltic state stock indices. Umeå Economic Studies 696, Umeå University.
- Britten-Jones M. 1999. The sampling error in estimates of mean-variance efficient portfolio weights. *Journal of Finance* LIV, 655-671.
- Campbell JY, Lo AW, MacKinlay AC. 1997. *The Econometrics of Financial Markets.* 2nd edition, Princeton University Press, Princeton.
- Christoffersen P, Gonçalves S. 2005. Estimation in financial risk management. *Journal of Risk* **7**, 1-28.
- De Goeij P, Marquering W. 2005. The generalized asymmetric dynamic covariance model. *Finance Research Letters* **2**, 67-74.
- De Wet WA. 2006. A structural GARCH model: An application on South African data. *Economic Modelling* 23, 775-791.
- Engle RF, Kroner KF. 1995. Multivariate simultaneous generalized ARCH. *Econometric Theory* **11**, 122-150.
- Engle RF, Russell JR. 1998. Autoregressive conditional duration: A new model for irregularly spaced transaction data. *Econometrica* **66**, 1127-1162.
- Gannon GL. 2004. Simultaneous volatility transmission and spillover effects. School Working Paper Series 2004/10, Deakin University.
- Gannon GL. 2005. Simultaneous volatility transmissions and spillover effects: U.S. and Hong Kong stock and futures markets. *International Review of Financial Analysis* 14, 326-336.
- Gourieroux C, Jasiak J. 2001. Financial Econometrics. Princeton University Press, Princeton.
- Kroner KF, Ng VK. 1998. Modeling asymmetric comovements of asset returns. The Review of Financial Studies 11, 817-844.
- Lee KY 2006. The contemporaneous interactions between the U.S., Japan and Hong Kong stock markets. *Economics Letters* **90**, 21-27.
- McNeil AJ, Frey R. 2000. Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance* 7, 271-300.
- Rigobon R. 2003. Identification through heteroskedasticity. The Review of Economics and Statistics LXXXV, 777-792.
- Rigobon R, Sack B. 2003. Spillovers across U.S. financial markets. NBER Working Paper 9640.