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Sick Pay, Health and Work

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Abstract: The purpose of this paper is to analyze the effects of different sickness insurance regimes on the employee decision reporting sick or not. We can think of the design problem as a representative employer's decision to determine the optimal relationship between the wage and the sickness pay. The employee bases her decision to work or not on this relative price and her exogenously given health status that varies between individuals. We believe that the incentives present in the model are able to tell as about relevant aspects of the incentives involved in a state managed sickness insurance system. We calculate how the control variables depend on parameters such as the average productivity of the worker, the average productivity of the substitute, the wage of the substitute, and the search cost to find a substitute. Since we assume that the health status of the work force is heterogeneous and represented by a distribution function, we are also able to calculate the change in the work participation rate, as a function of the parameters.

Key words: Sickness insurance design, wage setting, and labour force participation.

JEL Codes: I18, J2, J28.

Introduction

The purpose of this paper is to analyze the effects of different sickness insurance regimes on the employee decision reporting sick or not. The model, borrowed from Rikner (2002), belongs to the class of optimal design models. Here we produce the optimal design of a simple sickness insurance system. We can think of the design problem as a representative employer's decision to determine the optimal relationship between the wage and the sickness pay. The employee bases her decision to work or not on this relative price and her exogenously given health status that varies between individuals. It is up to the employer to find the wage and the sick-pay that maximizes the firm's expected profit.

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The paper differs with regard to previous literature on one major issue; the treatment of asymmetric information (hidden action and hidden information). In for example Weiss (1985) Coles and Treble (1993, 1996), one distinguishes between absence without or with a good cause, which sometimes results in a moral hazard problem that can be taken care of by an incentive compatibility constraint, and/or the contract is provisional on the observation of a probability². The probability to become sick is modeled by a random draw from the binominal distribution. The firm is typically an assembly line firm that has to optimally choose the permissible rate of absenteeism, the wage per period and the number of employees. In this paper we ignore the moral hazard problem completely by introducing a non-manipulative health status through a distribution function.

Although we formally deal with an employer provided sick-pay, we believe that the incentives present in the model are able to tell as about relevant aspects of the incentives involved in a state managed sickness insurance system. The reason is that an alternative interpretation of the model is that a social planner, unable to handle the moral hazard problem, decides the optimal wage and the optimal sick-pay. We take the model in Rikner (2002) a little further by examining how the control variables depend on parameters such as the average productivity of the worker, the average productivity of the substitute, the wage of the substitute, and the search cost to find a substitute. Since we assume that the health status of the work force is heterogeneous and represented by a distribution function, we are able to calculate the change in the work participation probability and its dependence on the parameters³.

Like Rikner, we introduce a participation constraint. Since we essentially assume that there is only one representative individual, only the exogenously given health status of the individuals differ, while all individuals have the same productivity as workers, we do not need any incentive compatibility constraint.

² The underlying general theory is available in Radner (1981).

³ Our approach also differs from a recent paper by Engström and Holmlund (2005) that introduces a general equilibrium model with search unemployment and incorporates absenteeism as a distinct labor force state.

In spite of the fact that the analysis involves some rather subtle relationships our findings are quite straightforward; e.g., if the (average) productivity of the employee increases, or if the cost of hiring a substitute would increase (via a higher wage or a higher search cost) the regular employee will be able to increase her wage but the sick-pay will fall.

A change in these parameters will also increase the work participation rate. The opposite result will follow if the average productivity of the substitute worker increases. The results are to a considerable extent driven by the trade offs between the wage and the sick-pay that are generated by the participation constraint.

The model

The model is designed to fulfill the following conditions:

- (i) It includes a health variable, h , and a distribution function $F(h)$, with support on the closed interval $[0,1]$. Zero means that the individual is completely unhealthy (six feet under), and the upper bound means that the individual is completely healthy (fit as a fiddle).
- (ii) The individual makes the decision whether to go to work or not
- (iii) A firm (or a social planner) will formally provide the sickness insurance and set the wage rate, subject to the share of the work force that will show up for work.

When the employee works, she derives utility from the net income of work. When she is sick she derives utility from the sick-pay. For simplicity we assume that the utility function is the same in the two states. More formally:

$$u^w = u(w) \quad u(0) = 0 \tag{1}$$

when the individual works, and

$$u^I = u(I) \tag{2}$$

when she is sick, where w denotes the wage and I denotes the sick pay. The utility function is twice continuously differentiable, increasing in income and strictly concave. The second component of the utility function is an effort function that handles the non-pecuniary parts of work or absence. We assume that all individuals face the same effort function:

$$e = e(h) \tag{3}$$

It measures the disutility from working as a function of the health status. It is assumed to be twice continuously differentiable, decreasing in h and strictly convex. When the individual is “fit as a fiddle”, $h=1$, $e(1)=0$. Moreover, $\lim_{h \rightarrow 0} e(h) = +\infty$, i.e., the effort will be prohibitive. We also assume that if the individual does not work, the effort level is assumed to be zero independent of the health status of the individual. Moreover, the effort level is not connected to the productivity of the individual.

If we put the above pieces together it follows that the individual will choose to work as long as

$$u(w) - e(h) \geq u(I) \tag{4}$$

i.e., the individual will work as long as the net utility of working is higher than the utility of staying at home. Solving for h at which the individual is indifferent between the two states (equation 4 holds with equality), we obtain a threshold $h^*(w, I)$ which tells us what health level that is at least necessary to prefer working to staying at home. This threshold is decreasing in the wage, $h_w^* < 0$, and increasing in the compensation level⁴, $h_I^* > 0$. Hence, the probability for a randomly drawn individual to be on a sick leave is $F[h^*(w, I)]$, and she will work with probability $1 - F[h^*(w, I)]$. The analysis of the threshold also reveals that the function $h^*(w, I)$ has the following properties: $h_{Iw}^* = h_{wI}^* > 0$, $h_{ww}^* > 0$, and $h_{II}^* < 0$. The above assumptions means that, given the vector (w, I) , the expected utility for an average worker is given by:

$$E[U] = \int_{h^*(w, I)}^1 (u(w) - e(h)) f(h) dh + \int_0^{h^*(w, I)} u(I) f(h) dh \tag{5}$$

⁴ The assumptions that are made about the utility components mean that a threshold will always exist. The derivatives are obtained by totally differentiating the constraint under the equality.

The first integral on the right hand side is the utility from working, and the second integral measures the utility from reporting sick. For the employee to stay in the potential “work force”, the expected utility has to match a certain level, here referred to as the participation utility level, P , i. e., $E[U] \geq P$. One interpretation is that a worker knows the health distribution, but not her place in the distribution, and will remain in the work force provided that the expected utility exceeds a certain utility level.

Profit maximization

We assume that the employee produces a fixed output equal to Y and is paid a wage w , to be determined by the employer. Hence the firms profit from the regular employee when she works is

$$\pi^w = Y - w \quad (7)$$

Moreover, when she is sick:

$$\pi^I = -I \quad (8)$$

where the sick-pay is determined by the firm. If the regular employee is on sick leave, the firm is forced to employ a substitute. The substitute will produce Y^s and she is paid an exogenously given salary w^s . Further, hiring a substitute is connected with a recruiting (training) cost, here denoted θ . This means that the firm profit from the substitute is

$$\pi^s = Y^s - w^s - \theta \quad (9)$$

To make the problem non-trivial we assume that, even if no respect is taken to the sick-pay, the firms will prefer regular workers to substitutes, which implies that

$$Y - w > Y^s - w^s - \theta \quad (10)$$

Finally, since the probability for the employee to be - according to her preferences and health status - too sick to work is $F(h^*(w, I))$, the expected profit function for the firm can be written

$$\pi(w, I) = [1 - F(h^*(w, I))(Y - w) - F(h^*(w, I))I + F(h^*(w, I))(Y^s - w^s - \theta)] \quad (11)$$

To guarantee that there is a unique maximum for this problem, one has to assume that the expected profit function is strictly concave in the control variables w and I , which will mean certain non-trivial restrictions on the distribution function. We can also live with an assumption that there exists an interior global optimum, which does not imply global concavity.

We will work through three different nested version of the model. In the first version (Model 1) we maximize profit with respect to w and I . This is the most general version of the model.

In the second version of the model (Model 2) we use w and $z = \frac{I}{w}$ as control variables; z will be referred to as the replacement ratio. Finally, in the third version of the model (Model 3), we assume that the replacement ratio is exogenously given. This means that w will be the only control variable. All three models are optimized subject to the participation constraint.

The optimal contract with wage and sick pay as control variables-Model 1

The overall interest of the employer is to maximize expected profit, and we will use this to derive the properties of the optimal wage/sick-pay contract. Below we present the first order conditions for the optimal contract, which if we have assumed strict concavity of the objective function in the control variables they are not only necessary, but also sufficient for an optimum. Under our less specific assumption we will have to assume that there is a global optimum where that the second order conditions are fulfilled.

We derive the comparative static effects on the optimal wage/sick-pay contract of changes in the parameter vector $\alpha = [Y, Y^s, w^s, \theta, P]$. Since the decision to work is a function of the wage/sick-pay contract, we can find out how the parameters affect the work participation rate (or rather the share of the work force that show up for work), $[1 - F(h(w, I))]$.

The optimal value function is the solution to the following optimization problem

$$\Pi(Y, Y^s, w^s, \theta) = \max_{w, I} [1 - F(h^*(w, I))][Y - w] - F(h^*(w, I))I + F(h^*(w, I))[Y^s - w^s - \theta] \quad (12)$$

$$\text{s.t.} \quad E[U] = \int_{h^*(w,I)}^1 [u(w) - e(h)] f(h) dh + \int_0^{h^*(w,I)} u(I) f(h) dh \geq P \quad (13)$$

The Lagrangian of the problem can be written

$$\begin{aligned} L = & [1 - F(h^*(w, I))][Y - w] - F(h^*(w, I))I + F(h^*(w, I))[Y^s - w^s - \theta] \\ & + \mu \left(\int_{h^*(w,I)}^1 [u(w) - e(h)] f(h) dh + \int_0^{h^*(w,I)} u(I) f(h) dh - P \right) \end{aligned} \quad (14)$$

We will assume that the participation constraint is binding. Loosely speaking, this means that without the constraint an employer (or social planner) would choose a contract which gives a lower expected utility.

The first order conditions for an interior maximum can be written:

$$\begin{aligned} \text{(i)} \quad L_\mu &= u(w)[1 - F(h^*(w, I))] + u(I)[F(h^*(w, I))] - \int_{h^*(w,I)}^1 e(I) f(h) dh - P = 0 \\ \text{(ii)} \quad L_w &= [f(h^*(w, I))\{-h_w^*[A]\} - [1 - F(h^*(w, I))](1 - u_w\mu) = 0 \\ \text{(iii)} \quad L_I &= [F(h^*(w, I))](u_I\mu - 1) - f(h^*(w, I))h_I^*[A] = 0 \end{aligned} \quad (15)$$

where $A = [Y - w - (Y^s - w^s - I - \theta)] > 0$ from the assumption in equation (10)⁵. The first

equation in (15), $L_\mu = 0$, tells us that the participation constraint is binding. The second

equation, $L_w = 0$, tells us that the marginal revenue of increasing the wage equals the

marginal cost. To interpret condition (15ii) we use condition (15iii) to show that

$u_w\mu < 1$ and $u_I\mu > 1$, which, using the strict concavity of the utility function ($u_{ww} < 0$),

confirms that the optimal wage, w^* , is strictly greater than the optimal sick pay, I^* .

However, this follows also, trivially, from equation (4), since otherwise nobody would show up for work. More importantly, we know that $1 - u_w\mu > 0$ and $u_I\mu - 1 > 0$, Hence, the first term

⁵ In the derivation of the first order conditions we have used that $B = [u(w) - u(I) - e(h^*(w, I))] \equiv 0$.

in (15ii) is the expected marginal revenue from an increase in the wage, which show up because a higher wage rate decreases the threshold to work, and $-h_w^*A > 0$ is the gain from getting rid of a substitute worker at the margin, while $f(h^*(w, I))$ is the probability to be at the margin. The second term is the expected increase in the *net* marginal wage payments to the share of the workforce that show up for work at the given wage. By increasing the wage you induce easier participation, which deducts $u_w\mu$ from the marginal wage increase, making $1 - u_w\mu > 0$ the marginal net wage cost.

Finally the third, $L_I = 0$, tells us that the marginal revenue of increasing the sick pay equals the marginal cost from an increase in the sick pay. The first term is the (expected) net gain from increasing the sick pay. This is positive since the marginal revenue from easier participation, $u_I\mu$, is larger than the marginal cost of increasing the sick pay, $= 1$. The second terms the (expected) marginal cost of having to employ a substitute worker fA due to the marginal increase in the threshold increase, $h_I > 0$.

The solution vector $\mathbf{x}(\boldsymbol{\alpha}) = [\mu(\boldsymbol{\alpha}), w(\boldsymbol{\alpha}), I(\boldsymbol{\alpha})]$, will be a function of the parameter vector $\boldsymbol{\alpha}$. The aim is to determine how the $\mathbf{x}$ vector changes when the vector $\boldsymbol{\alpha}$ changes. Totally differentiating the first order conditions result in an equation system that compactly can be written:

$$\mathbf{L}_H \mathbf{dx} = \mathbf{B} d\boldsymbol{\alpha} \quad (16)$$

which has the solution $d\mathbf{x} = \mathbf{L}_H^{-1} \mathbf{B} d\boldsymbol{\alpha}$. For the readers convenience we write down the full specification of (16).

$$\begin{bmatrix} 0 & L_{\mu w} & L_{\mu I} \\ L_{w\mu} & L_{ww} & L_{wI} \\ L_{I\mu} & L_{Iw} & L_{II} \end{bmatrix} \begin{bmatrix} d\mu \\ dw \\ dI \end{bmatrix} =
\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ f(h^*(w, I))h_w^* & -f(h^*(w, I))h_w^* & f(h^*(w, I))h_w^* & f(h^*(w, I))h_w^* & 0 \\ f(h^*(w, I))h_I^* & -f(h^*(w, I))h_I^* & f(h^*(w, I))h_I^* & f(h^*(w, I))h_I^* & 0 \end{bmatrix} * \begin{bmatrix} \partial Y \\ \partial Y^s \\ \partial w^s \\ \partial \theta \\ \partial P \end{bmatrix} \quad (17)$$

By using the properties of the second order conditions for a maximum we can prove the following result:

Proposition 1: *The solution of the equation system (17) yields the following qualitative information:*

- a) $\frac{\partial w}{\partial Y} = \frac{\partial w}{\partial w^s} = \frac{\partial w}{\partial \theta} = -\frac{\partial w}{\partial Y^s} > 0$
- $\frac{\partial I}{\partial Y} = \frac{\partial I}{\partial w^s} = \frac{\partial I}{\partial \theta} = -\frac{\partial I}{\partial Y^s} < 0$
- b) $\frac{\partial \mu}{\partial P} > 0$
- c) $\frac{\partial w}{\partial P}$ and $\frac{\partial I}{\partial P}$ cannot both be negative

Proof: See Appendix

The first claim in Proposition 1 tells us that marginal increases in the productivity of the employee, the wage of the substitute, and cost of hiring have positive and identical effects on the wage of the employee. The effect of a marginal increase in the productivity of the substitute has an effect of the same absolute size, but with the opposite sign. Moreover, marginal increases in the productivity of the employee, the wage of the substitute, and the cost of hiring, have negative and identical effects on the sick-pay. The effect of a marginal

increase in the productivity of the substitute has an effect of the same absolute size but the opposite sign⁶.

The second claim can be given a more detailed interpretation by using the Envelope Theorem to show that $\frac{dL}{dP} = -\mu(P) < 0$. In other words, an increase in the minimum utility level for work participation decreases the expected profit of the firm. This means that claim **b** shows that the marginal cost to the firm of a marginal increase in the participation utility level

$-\frac{\partial^2 L}{\partial P^2} = \frac{\partial \mu}{\partial P}$ is positive. In other words, the profit function is strictly concave in P .

The third claim is almost the mirror image of the second claim. If the participation utility level increases the firm has to improve the expected utility level of the employee. This can be done by either raising the wage or the sick pay, or both.

The employment participation rate

The employment participation rate in the optimal contract is given by

$$R(\alpha) = 1 - F\{h^*[w(\alpha), I(\alpha)]\} \quad (18)$$

This means that

$$\frac{\partial R(\alpha)}{\partial \alpha_i} = -f(h^*(w, I)) \left[\frac{\partial h^*}{\partial w} \frac{\partial w}{\partial \alpha_i} + \frac{\partial h^*}{\partial I} \frac{\partial I}{\partial \alpha_i} \right], \quad i=1, \dots, 5 \quad (19)$$

By using the properties of the threshold function $h^*(w, I)$, and Proposition 1, we can prove the following claim:

⁶ If we make the parameters in equation (10) increasing and non-linear functions of the parameters means that the results in Proposition 1 are qualitatively, but not quantitatively the same.

Proposition 2: *The qualitative partial effects on the share of people working (the participation rate) from changes in the parameter vector α are the following:*

$$\frac{\partial R(\alpha)}{\partial Y} = \frac{\partial R(\alpha)}{\partial w^s} = \frac{\partial R(\alpha)}{\partial \theta} = -\frac{\partial R(\alpha)}{\partial Y^s} > 0$$

Proof: See Appendix

In other words, marginal increases in the productivity of the employee, the wage of the substitute, and the recruitment cost of hiring have positive and identical effects on the employment participation rate. Like in Proposition 1 the effect of a marginal change in the productivity of the substitute is of the same magnitude, but opposite sign.

A modification of the model – Model 2

A typical sick insurance is not paid in a lump sum fashion, but rather as a share of the wage income. In our model individual labor supply is fixed, so we model the insurance as a share of the optimal wage, that is $I = zw$. As nobody would work if $z \geq 1$, we know that the replacement ratio, z , fulfills $0 \leq z < 1$.

For this model, we can write equation (4) for the threshold that determines whether you work or not, as

$$u(w) - u(zw) = e(h) \quad (4a)$$

which can be solved for h to yield

$$\begin{aligned} h^*(w, zw) &= e^{-1}[u(w) - u(zw)] \\ \frac{\partial h^*}{\partial w} &= \frac{1}{e_h(\cdot)}[u_w(w) - zu_w(zw)] = h_w^*(w, zw) \\ \frac{\partial h^*}{\partial z} &= \frac{-wu_w(w)}{e_h(\cdot)} = h_z^*(w, zw) > 0 \end{aligned} \quad (20)$$

The derivative h_z^* can be signed, but the effect of the wage on the threshold is a priori not possible to sign. We should, however, expect that it is negative at the optimal wage contract.

Otherwise, an increase in the wage would give perverse incentives to work; under a higher wage you would need to be healthier to go to work. We will return to this complication below.

The Lagrangian of the maximization problem can now be written

$$\begin{aligned} \tilde{L} = & [1 - F(h^*(w, zw))][Y - w] - F(h^*(w, zw)zw) + F(h^*(w, zw))[Y^s - w^s - \theta] \\ & + \mu \left(u(w)[1 - F(h^*(w, zw))] + u(zw)[F(h^*(w, zw))] - \int_{h^*(w, zw)}^1 e(h)f(h)dh - P \right) \end{aligned} \quad (22)$$

The first order conditions are

$$\begin{aligned} \text{(i)} \quad \frac{\partial \tilde{L}}{\partial \mu} = & u(w)[1 - F(h^*(w, zw))] + u(zw)[F(h^*(w, zw))] - \int_{h^*(w, zw)}^1 e(h)f(h)dh - P = 0 \\ \text{(ii)} \quad \frac{\partial \tilde{L}}{\partial w} = & [1 - F(h^*(w, zw))](\mu u_w(w) - 1) + zF(h^*(w, zw))(\mu u_w(zw) - 1) - f(h^*(w, zw)) \frac{dh^*}{dw} (\tilde{A} + \mu \tilde{B}) = 0 \\ \text{(iii)} \quad \frac{\partial \tilde{L}}{\partial z} = & F(h^*(w, zw))(\mu u_z(zw) - 1)w - f(h^*(w, zw))h_z^*(\tilde{A} + \mu \tilde{B})w = 0 \end{aligned} \quad (23)$$

The first condition in (23) is the participation constraint, which will be fulfilled with equality in an interior solution. Here $\tilde{A} = Y - w - (Y^s - w^s - zw - \theta)$, and

$\tilde{B} = [u(w) - u(zw) - e(h^*(w, I))] \equiv 0$ is the threshold equation. Moreover, $\frac{dh^*}{dw}$ is the total

derivative of the threshold h^* with respect to w and consists of several partial derivatives. If we put $zw = I$, we get

$$\frac{dh^*}{dw} = \frac{\partial h^*}{\partial w} + \frac{\partial h^*}{\partial I} \frac{\partial I}{\partial w} = \frac{\partial h^*}{\partial w} + \frac{\partial h^*}{\partial I} z \quad (24)$$

where the first term on the right hand side is negative and the second is positive. Hence the sign of this derivative is not possible to determine, not even at the optimum. This means, as we will see, that the insurance system can have some unwanted effects. Remember that the sign of the corresponding derivative in Model 1 has a negative sign, i.e., an increase in the wage, *ceteris paribus*, will diminish the health status necessary to show up for work. Hence, a positive sign would mean that an increase in the wage means that you have to be healthier to work after a wage increase.

The reason behind the ambiguity is straightforward. We have assumed that the utility functions are the same under both regimes. The utility function is strictly concave and a sick-pay lower than the wage means that the marginal utility from a higher sick-pay is larger than the marginal utility of an increase in the wage. Since the following difference between the marginal utilities determines the sign of the critical derivative, i.e.,

$$\text{sign} \frac{\partial h^*}{\partial w} = -\text{sign}[u_w(w) - zu_I(zw)] \quad (25)$$

The derivative will be ambiguous, since the term within parentheses on the right hand has an ambiguous sign.

Comparative statics

We are now ready to derive some of the comparative static results that are buried in Model 2. To this end we differentiate the first order conditions in equation (23) with respect to the parameters. The details are available in the appendix.

Given that the sign of the critical derivative is negative at the optimum, we have the following result

Proposition 3 *In Model 2, under the assumption that $\frac{dh^*}{dw} < 0$ at optimum, it holds that*

$$a) \quad \frac{\partial w}{\partial Y} = \frac{\partial w}{\partial w^s} = \frac{\partial w}{\partial \theta} = -\frac{\partial w}{\partial Y^s} > 0$$

$$b) \frac{\partial z}{\partial Y} = \frac{\partial z}{\partial w^s} = \frac{\partial z}{\partial \theta} = -\frac{\partial z}{\partial Y^s} < 0$$

Proof: See Appendix

Under the assumptions of the Proposition, the qualitative results are the same as in Model 1.

However, if $\frac{dh^*}{dw} > 0$ at optimum, none of the derivatives under a) and b) can be signed.

It is also worth noting that the sick-pay at optimum consists of a product, i. e. and the derivatives $\frac{\partial I}{\partial \alpha_i}$ are ambiguous, and this holds independently of the critical derivative in equation (25).

The participation rate will be “well behaved” under the assumption in Proposition 3.

Proposition 4: If $\frac{dh^*}{dw} < 0$, the qualitative partial effects on the share of people working (the participation rate) from a change in the parameter vector α are the following:

$$\frac{\partial R(\alpha)}{\partial Y} = \frac{\partial R(\alpha)}{\partial w^s} = \frac{\partial R(\alpha)}{\partial \theta} = -\frac{\partial R(\alpha)}{\partial Y^s} > 0$$

Proof: See Appendix

Clearly, if the critical derivative in equation (25) is positive, the effect on the participation rate is ambiguous.

A sub-optimal insurance -Model 3

In Model 2 the general design of the sick insurance is very similar to many, not to say most, existing insurance system. Nevertheless, the analysis is not able to give a meaningful answer to one of the most frequently asked questions: How does a change in the replacement ratio z affect the participation rate? The reason is that both the wage, and the replacement rate are

endogenous, so a meaningful answer has to be conditioned on which parameter that shifts the replacement ratio in the first place.

A less ambitious, and perhaps also more realistic, task is to assume that the replacement ratio is a parameter that is chosen by the policy maker. This will give a simpler model with only two variables, w and μ . This means that the objective function will look as before, but now z is a parameter. The first order conditions will be reduced by one equation, since the derivative of the objective function with respect to z vanishes. For the readers convenience we reproduce the first order conditions below, which correspond to the two first equations in equation (23).

$$(i) \frac{\partial \tilde{L}}{\partial \mu} = u(w)[1 - F(h^*(w, zw))] + u(zw)[F(h^*(w, zw))] - \int_{h^*(w, zw)}^1 e(h) f(h) dh - P = 0 \quad (23a)$$

(ii)

$$\frac{\partial \tilde{L}}{\partial w} = [1 - F(h^*(w, zw))](\mu u_w(w) - 1) + zF(h^*(w, zw))(\mu u_w(zw) - 1) - f(h^*(w, zw)) \frac{dh^*}{dw} (\tilde{A} + \mu \tilde{B}) = 0$$

The parameter vector has been augmented by one parameter, and this changes the previous comparative statics considerably, and it also produces a well defined and reasonably signed effect on the optimal wage rate from a change in the replacement ratio.

Proposition 5: *Under the conditions of Model 3, we obtain the following comparative static effects:*

$$a) \frac{\partial w}{\partial Y} = \frac{\partial w}{\partial w^s} = \frac{\partial w}{\partial \theta} = -\frac{\partial w}{\partial Y^s} = 0$$

$$b) \frac{\partial w}{\partial z} < 0$$

Proof: *See appendix*

The intuition behind the results in the **a** part of Proposition 5 is that the participation constraint is not directly affected by the parameter vector, only the first order condition for w , and, even if an increase in the productivity is good for the firm, there is no incentive to adjust

the wage to compensate the employee. It is more profitable to cash in the whole productivity gain. In the same spirit, an increase in the productivity would tempt the employer to decrease the wage of the employee, but this would violate the participation constraint. Hence, the wage is kept constant. In Model 2 it is possible to fine tune the effects from a change in the two productivity variables by changing both the wage and the sick-pay (the replacement ratio times the wage) in opposite directions.

The effect on the wage from the increase in the replacement ratio on the optimal wage (b in Proposition 5) is negative. The intuition here is that an exogenous increase in the replacement ratio at a constant wage would increase the level of the threshold $h^*(w, zw)$, and also violate the participation constraint. Hence, to satisfy the participation constraint the wage has to be decreased. Put differently, the firm uses the exogenous improvement in the sick pay to increase its profit by lowering the salary of its employees.

What remains to be shown is how a change in the replacement ratio affects the participation rate. For simplicity, we write the participation rate as a function of the parameter z , since a marginal change in all other parameters will leave the participation rate unchanged (line a in Proposition 5). Hence we have

$$R = R(h^*(w(z), I(z))) \quad (26)$$

Differentiating with respect to z yields

$$\begin{aligned} \frac{dR}{dz} &= \frac{\partial R}{\partial h} \left[\frac{\partial h^*}{\partial w} \frac{\partial w}{\partial z} + \frac{\partial h^*}{\partial I} \frac{\partial I}{\partial z} \right] = \frac{\partial R}{\partial h} \left[\frac{\partial h^*}{\partial w} \frac{\partial w}{\partial z} + \frac{\partial h^*}{\partial I} \left(\frac{\partial I}{\partial w} \frac{\partial w}{\partial z} + \frac{\partial I}{\partial z} \right) \right] = \\ &= \frac{\partial R}{\partial h} \left[\frac{\partial h^*}{\partial w} \frac{\partial w}{\partial z} + \frac{\partial h^*}{\partial I} \left(z \frac{\partial w}{\partial z} + w \right) \right] = \frac{\partial R}{\partial h} \left[\frac{dh^*}{dw} \frac{\partial w}{\partial z} + w \frac{\partial h^*}{\partial I} \right] \end{aligned} \quad (27)$$

which cannot be signed. Again the ambiguously depends on the term $\frac{dh^*}{dw}$, which again cannot be signed. However, since $\frac{\partial R}{\partial h} < 0$, we can prove the following result

Proposition 6: *In the suboptimal model an increase in the replacement ratio will decrease the participation rate if $\frac{dh^*}{dw} \leq 0$.*

The proof is a direct consequence of the fact that under the condition in Proposition 6, the expression in the last hatched parenthesis in equation (27) is positive.

Concluding comments

In this paper we model the optimal design of a sick-pay system in a partial equilibrium setting. One interpretation of our modeling attempt is that the firm controls both the wage and the sick-pay, but an equally reasonable interpretation is that we are modeling the optimal decisions of a social planner. She maximizes the expected profits of the firm under a participation constraint that the expected utility of a randomly drawn individual is higher than a certain exogenously given utility level. When the design is set, the health status of the worker determines whether she will show up for work or not. If not, the firm has to employ a substitute with a lower net productivity.

Three versions of the model are introduced and the most general one is Model 1, where the social planner determines both the wage and sick pay of the employee. The level of the sick-pay is not tied to the wage by an endogenously determined replacement ratio, but determined independently of the wage. This model gives very intuitive and clear cut effects from changes in the parameters on the control variables, and on the participation rate. For example, an increase in the productivity of the employee increases the wage, decreases the sick-pay and increases the participation rate.

In Model 2 we tie the sick-pay to the wage by making the sick pay an optimal determined share of the wage, the replacement ratio. The idea is to mimic the design of many existing sick-pay systems. The new design means that it is no longer possible to show that an increase in the salary, at a given replacement ratio, increases the participation rate. The reason is that the increase in the wage indirectly increases the sick-pay, and that the marginal utility of the sick-pay is higher than the marginal utility of the wage. The two marginal utilities work in opposite directions. This means that the income effect from wage increases can generate a lower participation rate; work is an “inferior good”.

The third model is a simplified version of Model 2, where we treat the replacement ratio as a parameter that can be changed by a policy maker. This means that we can study the effect of a change in the replacement ratio on the endogenously set wage and the participation rate. From the participation constraint it follows that the wage is decreased, but the effect on the participation ratio is again ambiguous.

A little thought reveals that Model 1 gives the highest value social value, since Model 2 is restricted to a linear relationship between the wage and the replacement ratio with a slope equal to the wage. In Model 1 there is no such restriction, and everything you can accomplish in Model 2 can be accomplished in Model 1, but not the other way around (Model 3 is obviously inferior to the other two). Moreover, it also produces sound results on the participation rate from changes in the exogenous parameters. Model 2 has the built in possibility to make work an inferior good, and reveals that the design has some unwanted properties from a social standpoint.

The model structure is a bare bone model in many respects, and this is intentional. We have tried to pinpoint the key determinants of a sick-pay system in the absence of asymmetric information. Given perfect knowledge of workers productivity and willingness to work, how should an insurance system be designed, and how does it answer to changes in economic parameters.

What about the introduction of asymmetric information? Would this fundamentally change the previous analysis? Say that there is a very simple health distribution with only two types; good health people and bad health people. This is, in terms of health status, a more simple set up than the above, so let us add the reasonable complication that bad health people are less productive than good health people. The bad health people would on average have a larger natural absenteeism than the good health type, and they would care relatively more about the sick pay than the wage compared to the more productive type. Given that the firm (social planner) knows that there are two types and the productivity of the respective types, she would design one contract for the high productivity people with a wage higher than the corresponding wage for the low productivity type, and a sick-pay that is possibly lower than the corresponding sick-pay in the contract aimed at the low productivity type. Such a scheme would sort them, and modulo some boundary problem, within each productivity/health type

the design should be similar to the one we have studied in order model world. This remains, however, to be shown.

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Appendix: Proof of Propositions

To prove Proposition 1, we note that the system determinant of the equation system in equation (16) is positive, i.e. , $\det \mathbf{L}_H = H > 0$. This follows from the second order conditions for optimum Moreover the elements in the matrix are

$$L_{\mu\mu} = 0$$

$$L_{\mu w} = [1 - F(h^*(w, I))] \mu_w = L_{w\mu} > 0$$

$$L_{\mu I} = [F(h^*(w, I))] \mu_I = L_{I\mu} > 0$$

$$L_{ww} = [1 - F(h^*(w, I))] \mu_{ww} \mu + f(h^*(w, I)) h_w^* [2 - \mu(2u_w + e_{h^*} h_w^*)] - (f_{h^*} h_w^* + f(h^*(w, I)) h_{ww}^*) [A] < 0$$

$$L_{wI} = L_{Iw} = f(h^*(w, I)) (h_I^* [1 - \mu u_w] - h_w^* [1 - \mu u_I] + \mu e_{h^*} h_I^* h_w^*) - (f_{h^*} h_w^* + f(h^*(w, I)) h_{wI}^*) [A] = ?$$

$$L_{II} = [F(h^*(w, I))] \mu_{II} \mu + f(h^*(w, I)) h_I^* [2 - \mu(2u_I + e_{h^*} h_I^*)] - (f_{h^*} h_I^* + f(h^*(w, I)) h_{II}^*) [A] < 0$$

$$L_{II} = [F(h^*(w, I))] \mu_{II} \mu + f(h^*(w, I)) h_I^* [2 - \mu(2u_I + e_{h^*} h_I^*)] - (f_{h^*} h_I^* + f(h^*(w, I)) h_{II}^*) [A] < 0$$

The right hand side of the equation system (16) is given in equation (17)

Moreover, we define H_{ij} as the co-factor of the i:th row and j:th column of H.

The cofactors of the system determinants are

$$H_{11} = \begin{vmatrix} L_{ww} & L_{wI} \\ L_{Iw} & L_{II} \end{vmatrix} > 0 \quad \text{Positive}$$

$$H_{12} = H_{21} = \begin{vmatrix} L_{w\mu} & L_{wI} \\ L_{I\mu} & L_{II} \end{vmatrix} * (-1)^3 = ? \quad \text{Not signed}$$

$$H_{13} = H_{31} = \begin{vmatrix} L_{w\mu} & L_{ww} \\ L_{I\mu} & L_{wI} \end{vmatrix} * (-1)^4 = ? \quad \text{Not signed}$$

$$H_{22} = \begin{vmatrix} 0 & L_{\mu I} \\ L_{I\mu} & L_{II} \end{vmatrix} < 0 \quad \text{Negative}$$

$$H_{23} = H_{32} = \begin{vmatrix} 0 & L_{\mu w} \\ L_{I\mu} & L_{II} \end{vmatrix} * (-1)^5 > 0 \quad \text{Positive}$$

$$H_{33} = \begin{vmatrix} 0 & L_{\mu w} \\ L_{w\mu} & L_{ww} \end{vmatrix} < 0 \quad \text{Negative}$$

Remark: From second order conditions for optimum $H_{ii} < 0$ for $i=2,3$.

The results in claim **a** and **b** of **Proposition 1** now follow from solving the system by using Cramer's rule to obtain

a)

$$\frac{\partial w}{\partial Y} = \frac{\partial w}{\partial w^s} = \frac{\partial w}{\partial \theta} = \frac{H_{22} f(h^*(w, I)) h_w^* + H_{23} f(h^*(w, I)) h_I^*}{H} = -\frac{\partial w}{\partial Y^s} > 0$$

$$\frac{\partial I}{\partial Y} = \frac{\partial I}{\partial w^s} = \frac{\partial I}{\partial \theta} = \frac{H_{32} f(h^*(w, I)) h_w^* + H_{33} f(h^*(w, I)) h_I^*}{H} = -\frac{\partial I}{\partial Y^s} < 0$$

b)

$$\frac{\partial \mu}{\partial P} = \frac{H_{11}}{H} > 0$$

The proof of claim **c** is given in the main text.

Proof of Proposition 2

The employment participation rate in the optimal contract is given by

$$R(\alpha) = 1 - F\{h^*[w(\alpha), I(\alpha)]\}$$

And

$$\frac{\partial R(\alpha)}{\partial \alpha} = -f(h^*(w, I)) \left[\frac{\partial h^*}{\partial w} \frac{\partial w}{\partial \alpha} + \frac{\partial h^*}{\partial I} \frac{\partial I}{\partial \alpha} \right]$$

By using that $h_w^* < 0$, and $h_I^* > 0$, and the results in Proposition 1, we are able to state that

$$\frac{\partial R(\alpha)}{\partial Y} = \frac{\partial R(\alpha)}{\partial w^s} = \frac{\partial R(\alpha)}{\partial \theta} = -\frac{\partial R(\alpha)}{\partial Y^s} > 0$$

Proofs of Proposition 3

The equation system of the totally differentiated first order conditions has the following explicit form

$$\begin{bmatrix} 0 & \tilde{L}_{\mu w} & \tilde{L}_{\mu z} \\ \tilde{L}_{w\mu} & \tilde{L}_{ww} & \tilde{L}_{wz} \\ \tilde{L}_{z\mu} & \tilde{L}_{zw} & \tilde{L}_{zz} \end{bmatrix} * \begin{bmatrix} d\mu \\ dw \\ dz \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ f(h^*(w, zw)) \frac{dh^*}{dw} & -f(h^*(w, zw)) \frac{dh^*}{dw} & f(h^*(w, zw)) \frac{dh^*}{dw} & f(h^*(w, zw)) \frac{dh^*}{dw} & 0 \\ f(h^*(w, zw)) h_z^* & -f(h^*(w, zw)) h_z^* & f(h^*(w, zw)) h_z^* & f(h^*(w, zw)) h_z^* & 0 \end{bmatrix} * \begin{bmatrix} dY \\ dY^s \\ dw^s \\ d\theta \\ d\tilde{P} \end{bmatrix}$$

Once again, we define the determinant of the matrix $\tilde{\mathbf{L}}_{\tilde{H}}$ as $\det \tilde{\mathbf{L}}_{\tilde{H}} = \tilde{H}$. Again, we define

$\tilde{H}_{ij}$ as the co-factor of the i:th row and j:th column of $\tilde{H}$ and expand to get

$$\tilde{H}_{11} = \begin{vmatrix} \tilde{L}_{ww} & \tilde{L}_{wz} \\ \tilde{L}_{zw} & \tilde{L}_{zz} \end{vmatrix} > 0 \quad \text{Positive}$$

$$\tilde{H}_{12} = \tilde{H}_{21} = \begin{vmatrix} \tilde{L}_{w\mu} & \tilde{L}_{wz} \\ \tilde{L}_{z\mu} & \tilde{L}_{zz} \end{vmatrix} * (-1)^3 = ? \quad \text{Not signed}$$

$$\tilde{H}_{13} = \tilde{H}_{31} = \begin{vmatrix} \tilde{L}_{w\mu} & \tilde{L}_{ww} \\ \tilde{L}_{z\mu} & \tilde{L}_{wz} \end{vmatrix} * (-1)^4 = ? \quad \text{Not signed}$$

$$\tilde{H}_{22} = \begin{vmatrix} 0 & \tilde{L}_{\mu z} \\ \tilde{L}_{z\mu} & \tilde{L}_{zz} \end{vmatrix} < 0 \quad \text{Negative}$$

$$\tilde{H}_{23} = \tilde{H}_{32} = \begin{vmatrix} 0 & \tilde{L}_{\mu w} \\ \tilde{L}_{z\mu} & \tilde{L}_{zz} \end{vmatrix} * (-1)^5 > 0 \quad \text{Positive}$$

$$\tilde{H}_{33} = \begin{vmatrix} 0 & \tilde{L}_{\mu w} \\ \tilde{L}_{w\mu} & \tilde{L}_{ww} \end{vmatrix} < 0 \quad \text{Negative}$$

Remark: From the second order conditions for optimum $\tilde{H}_{ii} < 0$, for $i=2,3$.

Using Cramer's rule to solve the system and the assumption $\frac{dh^*}{dw} < 0$ at optimum proves the claims in Proposition 3.

Proof of Proposition 4

For the proof of Proposition 4, see the proof of Proposition 2.

Proof of Proposition 5

The equation system for a model with z exogenous is

$$\begin{bmatrix} 0 & \tilde{L}_{\mu w} \\ \tilde{L}_{w\mu} & \tilde{L}_{ww} \end{bmatrix} * \begin{bmatrix} d\mu \\ dw \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -\tilde{L}_{\mu z} \\ f(h^*(w, zw))\frac{dh^*}{dw} & -f(h^*(w, zw))\frac{dh^*}{dw} & f(h^*(w, zw))\frac{dh^*}{dw} & f(h^*(w, zw))\frac{dh^*}{dw} & 0 & -\tilde{L}_{wz} \end{bmatrix} * \begin{bmatrix} dY \\ dY^s \\ dw^s \\ d\theta \\ dP \\ dz \end{bmatrix}$$

Using Cramer's rule yields

$$\frac{\partial w}{\partial Y} = \frac{\partial w}{\partial w^s} = \frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial Y^s} = \frac{0}{H} = 0$$

$$\frac{\partial w}{\partial z} = \frac{\tilde{L}_{w\mu} \tilde{L}_{\mu z}}{H} < 0,$$

which prove the claims in proposition 5.

