A Vector Integer-Valued Moving Average Model for High Frequency Financial Count Data*

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Abstract

A vector integer-valued moving average (VINMA) model is introduced. The VINMA model allows for both positive and negative correlations between the counts. The conditional and unconditional first and second order moments are obtained. The CLS and FGLS estimators are discussed. The model is capable of capturing the covariance between and within intra-day time series of transaction frequency data due to macroeconomic news and news related to a specific stock. Empirically, it is found that the spillover effect from Ericsson B to AstraZeneca is larger than that from AstraZeneca to Ericsson B.

Key Words: Count data, Intra-day, Time series, Estimation, Reaction time, Finance.

JEL Classification: C13, C22, C25, C51, G12, G14.

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1 Introduction

This paper introduces a Vector Integer-Valued Moving Average (VINMA) model. The VINMA is developed to capture covariance in and between stock transactions time series. Each transaction refers to a trade between a buyer and a seller in a volume of stocks for a given price. A transaction is impounded with information such as volume, price and spread. The trading intensity or the number of transactions for a fixed interval of time and the durations can be seen as inversely related since the more time elapses between successive transactions the fewer trades take place. Easley and O'Hara (1992) shows that a low trading intensity implies no news. Engle (2000) models time according to the autoregressive conditional duration (ACD) model of Engle and Russell (1998) and finds that longer durations are associated with lower price volatilities. One obvious advantage of the VINMA model over extensions of the ACD model is that there is no synchronization problem due to different onsets of durations in count data time series. Hence, the spread of shocks and news is more easily studied in the current framework.

The VINMA allows for both negative and positive correlation in the count series and the integer-value property of counts is taken into account. The VINMA model arises from the integer-valued autoregressive moving average (INARMA) model, which is related to the conventional ARMA class of Box and Jenkins (1970). The INARMA model for pure time series is independently introduced by McKenzie (1986) and Al-Osh and Alzaid (1987). An important difference between the continuous variable vector MA (VMA), a special case of vector ARMA model, and the VINMA model is that the latter has parameters that are interpreted as probabilities and hence the values of the parameters are restricted to unit intervals. An introductory treatise of count data is available in, e.g., Cameron and Trivedi (1998).

Until now, the only studies based on the INMA class for intra-day transactions data appear to be Brännäs and Quoreshi (2004) and Quoreshi (2006ab). Quoreshi (2006a) develops a bivariate integer-valued moving average (BINMA) model that satisfies the natural conditions of a count data model. This model is employed to measure the reaction time for news or rumors and how new information is spread through the system. The VINMA model is more general than the BINMA model and enables the study of spillover effects of news from one stock to the other.

2 The VINMA Model

Assume that there are M intra-day series, $y_{1t}, y_{2t}, \ldots, y_{Mt}$, for the number of stock transactions in $t = 1, \ldots, T$ time intervals. Assume further that the dependence between y_{it} and $y_{jt}, i \neq j$, emerges from common underlying factor(s) such as macro-economic news, rumors, etc. Moreover, news related to the y_{jt} series may also have an impact on y_{it} and vice versa.

The covariation within and between the count data variables can be modelled by a VINMA(q) model, with $q = \max(q_1, q_2, \ldots, q_M)$, which can be written on the form

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{Mt} \end{pmatrix} = \begin{pmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{Mt} \end{pmatrix} + \begin{pmatrix} \alpha_{111} & \alpha_{121} & \cdots & \alpha_{1M1} \\ \alpha_{211} & \alpha_{221} & & \alpha_{2M1} \\ \vdots & \vdots & & \vdots \\ \alpha_{M11} & \alpha_{M21} & & \alpha_{MM1} \end{pmatrix} \circ \begin{pmatrix} u_{1t-1} \\ u_{2t-1} \\ \vdots \\ u_{Mt-1} \end{pmatrix} + \ldots$$

$$+ \begin{pmatrix} \alpha_{11q} & \alpha_{12q} & \cdots & \alpha_{1Mq} \\ \alpha_{21q} & \alpha_{22q} & & \alpha_{2Mq} \\ \vdots & \vdots & & \vdots \\ \alpha_{M1q} & \alpha_{M2q} & & \alpha_{MMq} \end{pmatrix} \circ \begin{pmatrix} u_{1t-q} \\ u_{2t-q} \\ \vdots \\ u_{Mt-q} \end{pmatrix} \tag{1a}$$

or compactly as

$$\mathbf{y}_t = \mathbf{u}_t + \mathbf{A}_1 \circ \mathbf{u}_{t-1} + \ldots + \mathbf{A}_q \circ \mathbf{u}_{t-q}. \tag{1b}$$

The integer-valued innovation sequence $\{\mathbf{u}_t\}$ is assumed independent and identically distributed (iid) with $E(\mathbf{u}_t) = \boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_M)'$ and $Cov(\mathbf{u}_t) = \boldsymbol{\Omega}$. Obviously, there is reason to expect the $\mathbf{A}_i$ matrices to become sparser as i increases.

The binomial thinning operator distinguishes the VINMA model from the VMA model. By employing the binomial thinning operator in (1a-b) we account for the integer-value property of count data. The operator can be written

$$\mathbf{A} \circ \mathbf{u} = \begin{pmatrix} \sum_{i=1}^{M} \alpha_{1i} \circ u_i \\ \vdots \\ \sum_{i=1}^{M} \alpha_{Mi} \circ u_i \end{pmatrix} \tag{2}$$

where $\alpha_{ki} \circ u_i = \sum_{j=1}^{u_i} z_{jki}$. The $\{z_{jki}\}$ is assumed to be an iid sequence of 0-1 random variables with $\Pr(z_{jki} = 1) = \alpha_{ki} = 1 - \Pr(z_{jki} = 0)$ and the z_{jki} and u_i are assumed to be independent. Since $\alpha_{ki} \in [0, 1]$, $\alpha_{ki} \circ u_i \in [0, u_i]$.

Some conditional and unconditional moment properties of $\alpha_{ki} \circ u_i$ and $\mathbf{A} \circ \mathbf{u}$ are given in the Appendix.

For the covariance matrix $\mathbf{\Omega}$ we assume

$$\mathbf{\Omega}_{ij} = Cov(u_{it}, u_{js}) = \begin{cases} \sigma_{ij} - \lambda_i \lambda_j, & \text{for } t = s \\ 0, & \text{for } t \neq s \end{cases}$$

with $\sigma_{ij} = E(u_{it}u_{jt}), i \neq j$, and $\sigma_{ij} = \sigma_i^2 = E(u_{it}^2), i = j$. The off-diagonal elements $\mathbf{\Omega}_{ij}, i \neq j$, can be positive or negative depending on the relative sizes of σ_{ij} versus $\lambda_i \lambda_j$.

Retaining the previous assumptions, the conditional (on the previous observations, $Y_{t-1} = y_{1t-1}, y_{1t-2}, ..., y_{2t-1}, y_{2t-2}, ...$) first and second order moments for the VINMA(q) model are, in analogy with Brännäs and Hall (2001) and Quoreshi (2006a),

$$E(\mathbf{y}_t|Y_{t-1}) = E_{t|t-1} = \boldsymbol{\lambda} + \sum_{i=1}^{q} \mathbf{A}_i \mathbf{u}_{t-i} \tag{3a}$$

$$\begin{aligned} \mathbf{\Gamma}_{t,t-k|t-1} &= E[(\mathbf{y}_t - E_{t|t-1})(\mathbf{y}_{t-k} - E_{t-k|t-k-1}) \mid Y_{t-1}] \\ &= \begin{cases} \mathbf{\Omega} + \sum_{i=1}^{q} \mathbf{H}_{it}, & k = 0 \\ \mathbf{0}, & \text{otherwise} \end{cases} \end{aligned} \tag{3b}$$

where diag$(\mathbf{H}_{it}) = \mathbf{B}_i \mathbf{u}_{t-i}$ with $\mathbf{B}_i$ an $M \times M$ matrix with elements $(\mathbf{B}_i)_{jk} = \alpha_{jki}(1 - \alpha_{jki})$. Since the conditional variance varies with $\mathbf{u}_{t-i}$ there is conditional heteroskedasticity. When $M = 2$ and matrices $\mathbf{A}_i$ are diagonal the VINMA(q) collapses into the BINMA($q_{1,}q_2$) model of Quoreshi (2006a).

The unconditional first and second order moments for VINMA(q) model can be written

$$E\mathbf{y}_t = \left[\mathbf{I} + \sum_{i=1}^{q} \mathbf{A}_i \right] \boldsymbol{\lambda} \tag{4a}$$

$$Cov(\mathbf{y}_t, \mathbf{y}_{t-k}) = \begin{cases} \mathbf{\Omega} + \sum_{i=1}^{q} \mathbf{A}_i \mathbf{\Omega} \mathbf{A}_i' + \sum_{i=1}^{q} \mathbf{G}_i, & \text{for } k = 0 \\ \mathbf{A}_k \mathbf{\Omega} + \sum_{i=1}^{q} \mathbf{A}_{k+i} \mathbf{\Omega} \mathbf{A}_i', & \text{for } k = 1, 2, \ldots, q \\ \mathbf{0}, & \text{for } \mid k \mid > q \end{cases} \tag{4b}$$

with diag$(\mathbf{G}_i) = \mathbf{B}_i \boldsymbol{\lambda}$.

We may wish to include explanatory variables in the VINMA(q) model setup. This is most easily done by introducing a time-varying $\boldsymbol{\lambda}_t$ (Brännäs, 1995)

$$\lambda_{jt} = \exp(\mathbf{x}_{jt} \boldsymbol{\beta}_j) \geq 0, \quad j = 1, \ldots, M. \tag{5}$$

Previous prices, etc. are included in $\mathbf{x}_{jt}$. To obtain a more flexible conditional variance specification in (3b) we may let σ_j^2 become time dependent σ_{jt}^2. Allowing σ_{jt}^2 to depend on past values of σ_{jt}^2, u_{jt}, σ_{it}^2 and u_{it}, for $i \neq j$, and explanatory variables, using an exponential form, we may specify (cf. Nelson, 1991)

$$\begin{aligned} \operatorname{diag}(\boldsymbol{\Omega}_t) &= \exp\Bigg[\boldsymbol{\phi}_0 + \sum_{i=1}^{P} \boldsymbol{\Phi}_i \operatorname{diag}(\boldsymbol{\Omega}_{t-i}) \qquad (6) \\ &\quad + \sum_{i=1}^{Q} \boldsymbol{\Theta}_i \operatorname{diag}\left(\widetilde{\mathbf{u}}_{t-i}\widetilde{\mathbf{u}}'_{t-i}\right) + \sum_{i=1}^{R} \boldsymbol{\Psi}_i \mathbf{x}'_{t-i}\Bigg] \end{aligned}$$

where $\widetilde{\mathbf{u}}_{t-i} = \mathbf{u}_{t-i} - \boldsymbol{\lambda}_{t-i}$. The $\boldsymbol{\phi}_0$ is an M vector with elements ϕ_{j0}, $\boldsymbol{\Phi}_i$, $\boldsymbol{\Theta}_i$ and $\boldsymbol{\Psi}_i$ are $M \times M$ matrices.

3 Estimation

As we specify the model with first and second order moment conditions the conditional least squares (CLS), the feasible generalized least squares (FGLS) and the generalized method of moments (GMM) estimators are first hand candidates for estimation. Here, we only consider the CLS and FGLS estimators. The CLS and FGLS have the residual

$$\mathbf{e}_{1t} = \mathbf{y}_t - E(\mathbf{y}_t|Y_{t-1}) \qquad (7)$$

in common and both the CLS and FGLS estimators of $\boldsymbol{\psi} = (\boldsymbol{\psi}'_1, \boldsymbol{\psi}'_2, \ldots, \boldsymbol{\psi}'_M)'$ with $\boldsymbol{\psi}_j$ containing the α-parameters of the jth equation minimize a criterion function of the form

$$S = \sum_{t=q+1}^{T} \mathbf{e}'_{1t} \widehat{\mathbf{W}}_t^{-1} \mathbf{e}_{1t}, \qquad (8)$$

where $\mathbf{e}_{1t} = (e_{11t}, e_{21t}, \ldots, e_{M1t})'$, with respect to the unknown parameters. For CLS, $\widehat{\mathbf{W}}_t = \mathbf{I}$, while for FGLS $\widehat{\mathbf{W}}_t = \widehat{\boldsymbol{\Gamma}}_{t,t-k|t-1}$, where $\widehat{\boldsymbol{\Gamma}}_{t,t-k|t-1}$ is a estimate of the conditional covariance matrix in (3b). To calculate the sequences $\{\mathbf{e}_{1t}\}$ we employ $\mathbf{e}_{1t} = \mathbf{u}_t - \boldsymbol{\lambda}_t$.

The conditional variance and the covariance prediction errors

$$\begin{aligned} e_{j2t} &= e_{j1t}^2 - \sigma_{jt}^2 - \sum_{i=1}^{q}\sum_{k=1}^{M} \alpha_{jki}(1-\alpha_{jki})u_{kt-i} \qquad (9) \\ e_{j3t} &= e_{i1t}e_{j1t} - \boldsymbol{\Omega}_{ij}, \quad \text{for } i \neq j \qquad (10) \end{aligned}$$

are used for FGLS estimation. $S_{j2} = \sum_{t=s}^{T} e_{j2t}^2$ and $S_{j3} = \sum_{t=s}^{T} e_{j3t}^2$, where $s = \max(q, P, Q, R)+1$, are minimized with respect to the parameters of the function σ_{jt}^2 and σ_{ij} and with the CLS estimates for the jth equation $\widehat{\boldsymbol{\psi}}_j$ and $\widehat{u}_{jt}$ kept fixed. For time invariant $\boldsymbol{\Omega}$ a simple and obvious moment estimator

$$\widehat{\boldsymbol{\Omega}} = (T-s)^{-1} \sum_{t=s}^{T} \left[(\mathbf{e}_{1t}' \mathbf{e}_{1t}) - \sum_{i=1}^{q} \mathbf{H}_{it} \right]$$

follows from (3b). The covariance matrix estimators for CLS and FGLS are

$$Cov(\widehat{\boldsymbol{\psi}}_{CLS}) = (\sum_{t=s}^{T} \frac{\partial \mathbf{e}_{1t}}{\partial \boldsymbol{\psi}'} \frac{\partial \mathbf{e}_{1t}}{\partial \boldsymbol{\psi}})^{-1}$$

$$Cov(\widehat{\boldsymbol{\psi}}_{FGLS}) = (\sum_{t=s}^{T} \frac{\partial \mathbf{e}_{1t}}{\partial \boldsymbol{\psi}'} \widetilde{\boldsymbol{\Gamma}}_{t,t-k|t-1}^{-1} \frac{\partial \mathbf{e}_{1t}}{\partial \boldsymbol{\psi}})^{-1}.$$

The $\widetilde{\boldsymbol{\Gamma}}_{t,t-k|t-1}$ is the covariance matrix for the residual series from FGLS estimation.

4 Empirical Results

Tick-by-tick data for Ericsson B and AstraZeneca are aggregated over five minute intervals of time. The covered period is November 5-December 12, 2002. Since our intention is to capture ordinary transactions we have deleted trading before 0935 (trading opens at 0930) and after 1714 (order book closes at 1720). There are altogether 2392 observations for each stock series. Both CLS and FGLS estimators are employed for the VINMA model and the AIC criterion is used to find lag lengths for the VINMA(q) model. The FGLS estimator turns out to be the better one in terms of eliminating serial correlations. The parameters for Ericsson B (α_{11i}) and AstraZeneca (α_{22i}) estimated by FGLS are presented in Figure 1 (left panel). All estimates are positive and significant at the 5 percent level. The parameters to capture spillover effects from AstraZeneca to Ericsson B (α_{12i}) and from Ericsson B to AstraZeneca (α_{21i}) are presented in Figure 1 (right panel). About 19 percent of the estimated mean transactions for AstraZeneca is due to spillover effects while about 2 percent of the estimated mean transactions for Ericsson B is due to spillover.[1] The α_{21i} are all significant until lag 16 except for lags $6-8$ and 13, while the α_{12i} are all

[1] To calculate the percent of mean transactions for y_j due to spillover effect from y_k we employ $100 \cdot (\sum_{i=0}^{q} \alpha_{jki} u_{kt-i}/E(y_{jt}|Y_{t-1}))$.

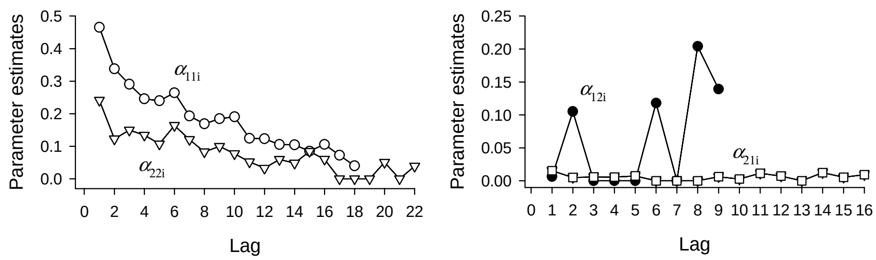

Figure 1: The circles and the triangles are the moving average parameters for Ericsson B ($\widehat{\lambda}_1 = 13.16$) and AstraZeneca ($\widehat{\lambda}_2 = 1.97$) (left figure). The solid circles capture the impact of AstraZeneca on Ericsson B while the squares capture the impact of Ericsson B on AstraZeneca (right figure).

significant until lag 9 except for lags $1, 3-5$ and 7. This implies that Ericsson B influences AstraZeneca for a longer period of time than is the case in the other direction.

The estimation results for the VINMA and BINMA (see Quoreshi, 2006a) models are summarized in Table 1. For FGLS, the VINMA model is marginally better than the BINMA model in terms of goodness of fit. For the VINMA model, the adjusted R^2 for Ericsson B increases from the BINMA model by 1.1 percent while it increases by 4.9 percent for AstraZeneca. It is found that news related to AstraZeneca Granger-causes Ericsson B and vice versa. The conditional correlations between the stock series at lag zero estimated with VINMA and BINMA models are 0.16 are 0.15, respectively. This implies that the intensity of trading for both stocks moves in the same direction, i.e. increases or decreases, due to macroeconomic news and news related to a specific stock. The corresponding estimated unconditional correlation for the VINMA model is 0.21 while the correlation between the two stock series in the sample is 0.28.

For FGLS (CLS), the α_{11i} and α_{22i} estimates give a mean reaction time (RT_m) of 26.07 (26.21) and 23.23 (17.23) minutes, respectively.[2] For FGLS (CLS), the α_{11i} and α_{22i} estimates give a median reaction (RT_{me}) time of 20 (20) and 15 (10), respectively. Hence, for the measurement of reaction time, the choice of mean or median reaction time matters.

[2] As measures of reaction times to macroeconomic news/rumors in the $\{u_{jt}\}$ sequence on y_{jt} we use the mean lag $\sum_{i=0}^{q_j} i\alpha_{jji}/w$, where $w = \sum_{i=0}^{q_j} \alpha_{jji}$ and where $\alpha_{jj0} = 1$. Alternatively, we use the median lag, which is the smallest k such that $\sum_{i=0}^{k} \alpha_{jji}/w \geq 0.5$.

Table 1: Results for VINMA and BINMA models for Ericsson B (Stock 1) and AstraZeneca (Stock 2).

	VINMA				BINMA	
	CLS		FGLS		FGLS	
	Stock 1	Stock 2	Stock 1	Stock 2	Stock 1	Stock 2
$\overline{R}^2$	0.505	0.209	0.504	0.231	0.498	0.221
RT_m	26.21	17.12	26.07	23.23	25.91	24.64
RT_{me}	20.00	10.00	20.00	15.00	20.00	15.00
LB_{30}	31.58	70.26	32.87	17.96	34.67	18.41
$\widehat{\rho}_{0\|t-1}$	0.159		0.160		0.150	

5 Concluding Remarks

This study introduces a vector integer-valued moving average (VINMA) model. The conditional and unconditional first and second order moments are obtained. The VINMA model allows for both positive and negative correlations between the counts. The model is capable of capturing the covariance between and within intra-day time series of transaction frequency data due to macroeconomic news and news related to a specific stock. In its empirical application, we found that the spillover effect from Ericsson B to AstraZeneca is larger than that from AstraZeneca to Ericsson B. The FGLS estimator performs better than the CLS estimator in terms of eliminating serial correlations.

Appendix

Conditionally on the integer-valued u_i, $\alpha_{ki} \circ u_i$ is binomially distributed with $E(\alpha_{ki} \circ u_i \mid u_i) = \alpha_{ki} u_i$, $V(\alpha_{ki} \circ u_i \mid u_i) = \alpha_{ki}(1-\alpha_{ki})u_i$ and $E[(\alpha_{ki} \circ u_i)(\alpha_{kj} \circ u_j) | \, u_i, u_j] = \alpha_{ki}\alpha_{kj}u_i u_j$, for $i \neq j$. Unconditionally it holds that $E(\alpha_{ki} \circ u_i) = \alpha_{ki}\lambda_i$, $V(\alpha_{ki} \circ u_i) = \alpha_{ki}^2\sigma_i^2 + \alpha_{ki}(1-\alpha_{ki})\lambda_i$ and $E\left[(\alpha_{ki} \circ u_i)(\alpha_{kj} \circ u_j)\right] = \alpha_{ki}\alpha_{kj}E(u_i u_j)$, for $i \neq j$, where $E(u_i) = \lambda_i$ and $V(u_i) = \sigma_i^2$.

Assuming independence between and within the thinning operations, conditionally on $M \times 1$ integer-valued vector $\mathbf{u}$, $\mathbf{A} \circ \mathbf{u}$ has

$$
\begin{aligned}
E\left(\mathbf{A} \circ \mathbf{u} \mid \mathbf{u}\right) &= \mathbf{A}\mathbf{u} \\
E\left[(\mathbf{A}_i \circ \mathbf{u}_{t-i})(\mathbf{A}_j \circ \mathbf{u}_{t-j})' \mid \mathbf{u}_{t-i}, \mathbf{u}_{t-j}\right] &= \begin{cases} \mathbf{A}_i \mathbf{u}_{t-i}\mathbf{u}_{t-i}'\mathbf{A}_i' + \mathbf{H}_{it}, & \text{for } i = j \\ \mathbf{A}_i \mathbf{u}_{t-i}\mathbf{u}_{t-j}'\mathbf{A}_j', & \text{for } i \neq j \end{cases}
\end{aligned}
$$

where the $\mathbf{A}$ is a $M \times M$ matrix with elements $\alpha_{ki} \in [0,1]$ and $\text{diag}(\mathbf{H}_{it}) = \mathbf{B}\mathbf{u}_{t-i}$. The $\mathbf{B}$ is an $M \times M$ matrix with elements $\alpha_{ki}(1-\alpha_{ki})$. The corresponding unconditional first and second order moments are

$$
\begin{aligned}
E\left(\mathbf{A} \circ \mathbf{u}\right) &= \mathbf{A}E\left(\mathbf{u}\right) = \mathbf{A}\boldsymbol{\lambda} \\
E\left[(\mathbf{A}_i \circ \mathbf{u}_{t-i})(\mathbf{A}_j \circ \mathbf{u}_{t-j})'\right] &= \begin{cases} \mathbf{A}_i E\left(\mathbf{u}_{t-i}\mathbf{u}_{t-i}'\right)\mathbf{A}_i' + \mathbf{G}, & \text{for } i = j \\ \mathbf{A}_i E\left(\mathbf{u}_{t-i}\mathbf{u}_{t-j}'\right)\mathbf{A}_j', & \text{for } i \neq j \end{cases}
\end{aligned}
$$

where $\text{diag}(\mathbf{G}) = \mathbf{B}\boldsymbol{\lambda}$.

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