

# Environmental Policy and Product Specialization\*

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## **Abstract**

This paper characterizes income and commodity taxation as the outcome of a noncooperative Nash game in a two-country economy where one of the countries produces an environmentally clean good, while the other produces a dirty good. Among the results, it is shown that the commodity tax on the dirty good implemented by each country does not contain any term that directly serves to correct for the external effect. Instead, the country producing the dirty good internalizes part of the domestic external effect by choosing a relatively high marginal income tax rate.

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# 1 Introduction

In the literature on transboundary environmental problems<sup>1</sup>, it is recognized that some kind of cooperation is generally required in order to reach a globally optimal resource allocation. The reason is that, in the absence of cooperation, part of the external effects of environmental damage will remain uninternalized, even if all other policies are optimally chosen from the point of view of society, since domestic objectives typically govern individual countries. At the same time, and despite the presence of certain international agreements, there is still considerable freedom for individual countries to choose their own policy on a national basis. As a consequence, it is important to understand the incentives on which individual countries base their policy decisions. This paper analyzes environmental policy as part of the outcome of a noncooperative Nash game between countries, where each country solves a mixed tax problem conditional on the policies chosen by other countries. The purpose is to introduce some additional elements into this framework; namely, production specialization and policy incentives associated with endogenous world market producer prices.

The study of environmental policy in the context of optimal taxation is typically based on 'one-country' model economies, where environmental damage is generated by the aggregate demand of a certain commodity; often referred to as a 'dirty' good. A seminal contribution is Sandmo (1975) dealing with environmental policy in a second best economy, where commodity taxes are used to raise a given public revenue. His main result is that the social value of the marginal external effect enters the tax formula for the

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<sup>1</sup>Transboundary environmental problems have received a lot of attention in the literature; see e.g. Mäler (1989), Barrett (1990, 1994), Tahvonen (1994, 1995) and Aronsson and Löfgren (2000).

commodity that gives rise to environmental damage, whereas it has no direct effect on the tax formulas for other commodities. A similar result is derived by Pirttilä and Tuomala (1997), although their study is based on a mixed tax problem, where the linear commodity taxes are supplemented by a non-linear income tax<sup>2</sup>. Aronsson and Blomquist (2004) extend the framework used by Pirttilä and Tuomala into a two-country economy, where the interactions between the countries refer to labor mobility and spillover effects of environmental damage; they do not consider specialization in the production. Their contribution is to compare the tax policy supporting a noncooperative Nash equilibrium with that of a cooperative equilibrium. A basic conclusion in most earlier studies is that environmental damage typically provides an incentive to modify the commodity tax structure<sup>3</sup>; let be that the corrective terms entering the commodity tax formulas differ depending on whether each country chooses its environmental policy in isolation or cooperates with other countries.

The studies mentioned above are either based on one-country model economies or, in case a global economy is being considered, a production structure without specialization. An important question is whether this assumption is important for our understanding of environmental policy. The production possibilities facing real world market economies give rise to specialization (at least to some extent), implying that different countries concentrate on producing different types of commodities. As a consequence, some countries become exporters of dirty goods (e.g. the oil producing countries),

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<sup>2</sup>See also the related work by e.g. Cremer and Gahvari (2001) and Cremer et al. (2001).

<sup>3</sup>See also the literature dealing with environmental tax reforms in the presence of other tax distortions, e.g. Bovenberg and de Mooij (1994), Bovenberg and Goulder (1996), Schneider (1997), Bovenberg and van der Ploeg (1998), Aronsson (1999), Koskela and Schöb (1999) and Parry et al. (1999).

whereas others become importers. In this paper, we take the specialization argument to its extreme point by considering full specialization in the production. Our analysis is based on a stylized model with a world economy comprising two countries and two commodities; one of the countries produces an environmentally clean good and the other a dirty good. The two commodities are consumed by the residents in both countries, and the aggregate (worldwide) consumption of the dirty good gives rise to environmental damage. This means that the resulting equilibrium involves international trade as well as transboundary spillover effects of environmental damage. Furthermore, our framework implies that the national governments recognize how their policies influence the world market producer prices and incorporate this information into their optimization problems; one of the driving forces behind the results. Each country is characterized by two ability-types, implying that environmental policy is studied simultaneously with redistribution; a starting point which appears reasonable to us. The government in each country faces a mixed tax problem, where the set of national tax instruments includes a nonlinear income tax and linear commodity taxes.

Our results show that, if the resource allocation is a noncooperative Nash equilibrium, where each country chooses its policy in isolation and treats the policy variables of the other country as exogenous, the national governments do not use commodity taxation for the purpose of internalizing the domestic external effects associated with environmental damage. The reason is that, for a given level of production, each national government acts as if a reduction of the domestic demand for the dirty good leads to a corresponding increase in consumption elsewhere. Therefore, each country perceives that the aggregate worldwide demand for the dirty good and, as a consequence, the environmental damage will be unaffected by its own commodity tax on

the dirty good.

If policies aimed at the demand side of the economy will be ineffective to combat the environmental problem, national governments can only reduce emissions by targeting the supply side. Export/import tariffs may seem to be an obvious set of instrument to accomplish this task. However, one can easily argue that such instruments are not fully available in a world economy with free trade agreements<sup>4</sup>. Taking also this argument to its extreme point, we disregard trade policy in what follows; instead, our focus will be on how to use income taxation to influence the supply side. Our results show that the effective marginal tax rate facing the residents in the country that produces the dirty good depends explicitly on the domestic marginal value of reduced environmental damage. As such, this effect works to increase the effective marginal tax rates which, in turn, reduces the labor supply and the output of the dirty good.

The outline of the paper is as follows. Section 2 contains a description of the model and a characterization of the outcome of private optimization. In section 3, we address the income and commodity tax structure implicit in a noncooperative Nash equilibrium and present our results. Section 4 contains a summary and discussion of the results.

## 2 The Model

Consider a two-country economy, where the firms in country  $C$  produce an environmentally clean good, while the firms in country  $D$  produce an environmentally dirty good. In all other important respects, the countries are

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<sup>4</sup>This assumption may be justified by the GATT agreements, which restrict the use of trade barriers.

identical. We neglect migration in what follows by assuming that the populations are immobile. There are two types of consumers in each country; a low-ability type (denoted by superindex 1) and a high-ability type (denoted by superindex 2). This distinction refers to productivity, meaning that the high-ability type faces a higher before tax wage than the low-ability type. In each country, we normalize the number of consumers of each ability-type to one.

The consumers have identical preferences defined by the utility function  $U(\mathbf{X}^{h,i}, Z^{h,i}, E)$ , where  $\mathbf{X}^{h,i}$  is a vector of private goods consumed by ability-type  $i$  ( $i = 1, 2$ ) in country  $h$  ( $h = C, D$ ),  $Z^{h,i}$  is leisure consumed by the same individual, while  $E$  denotes the environmental damage. Leisure is, in turn, defined as  $Z^{h,i} = H - L^{h,i}$ , where  $H$  is a time endowment and  $L^{h,i}$  the hours of work. In what follows, we assume that the individual consumers treat  $E$  as exogenous. The vector  $\mathbf{X}^{h,i}$  contains a clean good,  $X_c^{h,i}$ , and a dirty good  $X_d^{h,i}$ . The function  $U(\cdot)$  is increasing in  $X_c^{h,i}$ ,  $X_d^{h,i}$  and  $Z^{h,i}$ , decreasing in  $E$  and strictly quasiconcave. We also assume that  $X_c^{h,i}$  and  $X_d^{h,i}$  are normal goods. The producer price of the clean good is normalized to one, and the clean good is untaxed. The consumer price of the dirty good in country  $h$  is given by  $Q_d^h = P_d + t_d^h$ , where  $P_d$  is the producer price and  $t_d^h$  the commodity tax.

Individual  $i$  in country  $h$  chooses  $X_c^{h,i}$ ,  $X_d^{h,i}$  and  $L^{h,i}$  to maximize utility subject to the budget constraint. Following Christiansen (1984), it is convenient to solve the optimization problem in two stages. First, we solve the utility maximization problem conditional on the hours of work. This problem is written

$$\max_{X_c^{h,i}, X_d^{h,i}} U(\mathbf{X}^{h,i}, Z^{h,i}, E) \quad (1)$$

subject to

$$B^{h,i} = \mathbf{Q}^h \mathbf{X}^{h,i} \quad (2)$$

where  $\mathbf{Q}^h = (1, Q_d^h)$  is the vector of consumer prices and  $B^{h,i}$  the after-tax income. The solution to this problem gives the conditional indirect utility function,  $V^{h,i} = V(Q_d^h, B^{h,i}, Z^{h,i}, E)$ . In the second stage, the hours of work are chosen to maximize the conditional indirect utility function subject to the budget constraint  $B^{h,i} = I^{h,i} - T^h(I^{h,i})$ , where  $I^{h,i} = w^{h,i} L^{h,i}$  and  $T^h(\cdot)$  is the income tax function. The first order condition is written

$$T_I^{h,i} = 1 - \frac{V_Z^{h,i}}{w^{h,i} V_B^{h,i}} \quad (3)$$

in which  $T_I^{h,i}$  is the marginal income tax rate.

Let us continue with the production side. In each country, the goods market is competitive, and the production technology is characterized by constant returns to scale. Given these characteristics, the number of firms in each country is not important and will be normalized to one. The production function is written  $Y^h = F^h(\mathbf{L}^h)$  for  $h = C, D$ , where  $\mathbf{L}^h = (L^{h,1}, L^{h,2})$ . By defining  $n^h = L^{h,2}/L^{h,1}$  and  $f^h(n^h) = F^h(\mathbf{L}^h)/L^{h,1}$  for  $h = C, D$ , the first order conditions become

$$w^{C,1} = [f^C(n^C) - n^C f'_n(n^C)], \quad w^{C,2} = f'_n(n^C) \quad (4)$$

$$w^{D,1} = P_d [f^D(n^D) - n^D f'_n(n^D)], \quad w^{D,2} = P_d f'_n(n^D) \quad (5)$$

Therefore, in each country, the wage ratio,  $w^{h,1}/w^{h,2}$ , will be a function of the hours of work ratio,  $n^h$ , i.e.

$$\frac{w^{h,1}}{w^{h,2}} = \phi^h(n^h) \quad (6)$$

with  $\partial\phi^h(n^h)/\partial n^h = \phi_n^h(n^h) > 0$ .

The environmental damage is determined by the aggregate worldwide consumption of the dirty good, i.e.

$$E = \sum_h \sum_i X_d(P_d^h + t_d^h, B^{h,i}, L^{h,i}, E) \quad (7)$$

implying that there is a transboundary environmental problem. The world market equilibrium for the dirty good is written

$$F^D(\mathbf{L}^D) \equiv \sum_h \sum_i X_d(Q_d^h, B^{h,i}, L^{h,i}, E) \quad (8)$$

This equation implicitly defines the producer price as a function of private income, work hours, commodity taxes and the environmental damage, i.e.

$$P_d = P_d(\mathbf{B}^C, \mathbf{B}^D, \mathbf{L}^C, \mathbf{L}^D, t_d^C, t_d^D, E) \quad (9)$$

where  $\mathbf{B}^h = (B^{h,1}, B^{h,2})$  for  $h = C, D$ . Finally, observe that as long as equation (8) is fulfilled, Walras' law implies that also the world market for the clean good is in equilibrium.

### 3 A Noncooperative Nash Equilibrium

We assume that each country faces a utilitarian social welfare function. This means that the social welfare function of country  $h$  can be written as

$$W^h = V^{h,1} + V^{h,2} \quad (10)$$

Ability is private information. In accordance with the majority of previous studies based on the self-selection approach to optimal taxation, we assume

that the aim of the redistributive policy is to redistribute from high income earners to low income earners. As a consequence, we would need to prevent the high-ability type in each country from pretending to be a low-ability type. The self-selection constraint that may bind in country  $h$  can then be written as

$$V^{h,2} = V(Q_d^h, B^{h,2}, H - L^{h,2}, E) \geq V(Q_d^h, B^{h,1}, H - \hat{L}^{h,2}, E) = \hat{V}^{h,2} \quad (11)$$

for  $h = C, D$ , where  $\hat{L}^{h,2} = \phi^h(n^h)L^{h,1}$  while  $\hat{V}^{h,2}$  is the utility of the mimicker.

The national tax instruments are the income tax facing each ability-type and the commodity tax on the dirty good. Since we are primarily concerned with tax policy in this paper, we disregard public provision of public and private goods. The budget constraint of the government in country  $h$  becomes

$$T^h(I^{h,1}) + t_d^h X_d^{h,1} + T^h(I^{h,2}) + t_d^h X_d^{h,2} = 0 \quad (12)$$

Since  $T^h(\cdot)$  is a general income tax, it can be used to implement any desired combination of work hours and private income for both ability-types. It is, therefore, convenient to use  $(B^{h,1}, L^{h,1}, B^{h,2}, L^{h,2})$ , instead of the parameters of  $T^h(\cdot)$ , as direct decision variables in the optimal tax problem. As a consequence, we rewrite the budget constraint of the government in order to eliminate its direct dependence on  $T^h(\cdot)$ . By using equation (12) together with the private budget constraint, the first order conditions for the firm and  $f^h(n^h) = F^h(\mathbf{L}^h)/L^{h,1}$ , we can rewrite the government's budget constraint to read

$$0 = F^C(\mathbf{L}^C) - \sum_i [P_d X_d(Q_d^C, B^{C,i}, L^{C,i}, E) + X_c(Q_d^C, B^{C,i}, L^{C,i}, E)] \quad (13)$$

$$0 = P_d F^D(\mathbf{L}^D) - \sum_i [P_d X_d(Q_d^D, B^{D,i}, L^{D,i}, E) + X_c(Q_d^D, B^{D,i}, L^{D,i}, E)] \quad (14)$$

The policy variables of the government in country  $h$  are  $B^{h,1}$ ,  $L^{h,1}$ ,  $B^{h,2}$ ,  $L^{h,2}$  and  $t_d^h$ . In addition, as will be explained below, we use equation (7) as an explicit constraint in the optimization problem, implying that  $E$  is treated as an additional decision variable. By analogy to the analyses carried out in previous studies, the latter enables us to derive explicit expressions for the national shadow prices associated with environmental damage.

The countries are assumed to play a Nash game in the sense that each country treats the other country's decision variables as exogenous. Although conventional, this assumption is important for the results to be derived below. It means that the government in country  $D$  treats  $Y^C = F(\mathbf{L}^C)$ ,  $\mathbf{B}^C$  and  $t_d^C$  as exogenous, while the government in country  $C$  treats  $Y^D = F(\mathbf{L}^D)$ ,  $\mathbf{B}^D$  and  $t_d^D$  as exogenous. We also assume that the national governments recognize that their public policies influence the world market producer price via equation (9). Since each country treats the decision variables of the other country as exogenous, it behaves as if the producer price of the dirty good is a function of its own decision variables, conditional on the decision variables facing the other country. Let  $P_d^C$  and  $P_d^D$  denote these perceptions about  $P_d$  held by the governments in countries  $C$  and  $D$ , respectively. Equation (9) then implies

$$P_d^C = P_d^C (\mathbf{B}^C, \mathbf{L}^C, t_d^C, E^C; \bar{\mathbf{B}}^D, \bar{\mathbf{L}}^D, \bar{t}_d^D) \quad (15)$$

$$P_d^D = P_d^D (\mathbf{B}^D, \mathbf{L}^D, t_d^D, E^D; \bar{\mathbf{B}}^C, \bar{\mathbf{L}}^C, \bar{t}_d^C) \quad (16)$$

where the bar “-” indicates that the variable is treated as exogenous. By analogy,  $E^C$  and  $E^D$  are the perceptions about  $E$  held by countries  $C$  and  $D$ , respectively. The variables  $E^C$  and  $E^D$  are implicitly defined by

$$\begin{aligned} E^C &= \sum_i X_d (P_d^C + \bar{t}_d^D, \bar{B}^{D,i}, \bar{L}^{D,i}, E^C) \\ &\quad + \sum_i X_d (P_d^C + t_d^C, B^{C,i}, L^{C,i}, E^C) \end{aligned} \quad (17)$$

$$\begin{aligned} E^D &= \sum_i X_d (P_d^D + \bar{t}_d^C, \bar{B}^{C,i}, \bar{L}^{C,i}, E^D) \\ &\quad + \sum_i X_d (P_d^D + t_d^D, B^{D,i}, L^{D,i}, E^D) \end{aligned} \quad (18)$$

where  $P_d^C$  and  $P_d^D$  are given by equations (15) and (16), respectively. Note that  $P_d^C = P_d^D = P_d$  and  $E^C = E^D = E$  in the Nash equilibrium.

The Lagrangean corresponding to each national policy problem can be written as

$$\begin{aligned} \mathfrak{L}^C &= V^{C,1} + V^{C,2} + \lambda^C (V^{C,2} - \hat{V}^{C,2}) + \mu^C \left( E^C - \sum_h \sum_i X_d^{h,i} \right) \\ &\quad + \gamma^C \left[ Y^C - \sum_i (X_c^{C,i} + P_d^C X_d^{C,i}) \right] \end{aligned} \quad (19)$$

$$\begin{aligned} \mathfrak{L}^D &= V^{D,1} + V^{D,2} + \lambda^D (V^{D,2} - \hat{V}^{D,2}) + \mu^D \left( E^D - \sum_h \sum_i X_d^{h,i} \right) \\ &\quad + \gamma^D \left[ P_d^D Y^D - \sum_i (X_c^{D,i} + P_d^D X_d^{D,i}) \right] \end{aligned} \quad (20)$$

and the first order conditions are given in the Appendix.

Equations (15)-(18) are important for the results to be derived below. Let us begin by briefly discussing some of their implications for environmental

policy. To exemplify, consider how  $t_d^D$  affects  $E^D$  in case the policy variables of country  $C$  are treated as fixed. Start by differentiating equation (8) with respect to  $P_d^D$  and  $t_d^D$

$$\frac{\partial P_d^D}{\partial t_d^D} = -\frac{\sum_i \partial X_d^{D,i} / \partial Q_d^D}{\sum_i \partial X_d^{C,i} / \partial Q_d^C + \sum_i \partial X_d^{D,i} / \partial Q_d^D} \quad (21)$$

Since the demand curves are downward sloping, equation (21) implies  $-1 < \partial P_d^D / \partial t_d^D < 0$ . Then, differentiating equation (18) with respect to  $t_d^D$  and using equation (21), it is straightforward to show that  $\partial E^D / \partial t_d^D = 0$ . Therefore, from the perspective of country  $D$ , commodity taxation is useless as an instrument for influencing the aggregate demand for the dirty good. The intuition is that  $\mathbf{L}^D$  and, therefore, the output of the dirty good are held constant. This means that the decrease in the domestic demand for the dirty good leads to increased exports which, in turn, generates downward pressure on the world market producer price and, as a consequence, a subsequent increase in both the foreign and domestic demand for the dirty good. Equilibrium is restored when  $P_d^D$  has decreased so much as to equalize the aggregate demand with the fixed supply. In a similar way, one can show that  $\partial E^C / \partial t_d^C = 0$ . Furthermore, these arguments can be generalized to apply to the other decision variables as well; with a fixed supply of the dirty good, it is straightforward to show that the aggregate consumption of the dirty good is unaffected by changes in  $\mathbf{B}^C$ ,  $\mathbf{B}^D$ ,  $\mathbf{L}^C$  and  $\mathbf{L}^D$ , if the national policies are chosen in isolation.

On the other hand, since  $\mathbf{L}^D$  affects the supply of the dirty good, it follows that

$$\frac{\partial E^D}{\partial L^{D,i}} = \frac{\partial F^D}{\partial L^{D,i}} > 0 \quad (22)$$

for  $i = 1, 2$ . As a consequence, policies aimed at the supply of the dirty good are perceived to influence the environmental damage. With these preliminaries at our disposal, we are now ready to analyze the optimal tax structure.

### 3.1 The Shadow Price of Environmental Damage

Previous studies on environmental policy in the context of mixed tax problems, such as Pirttilä and Tuomala (1997) and Aronsson and Blomquist (2003), show that the shadow price of environmental damage over the shadow price of the public budget constraint,  $\mu^h/\gamma^h$ , plays an important role for the optimal tax structure. We may interpret  $\mu^h/\gamma^h$  as the marginal value to country  $h$  of reduced environmental damage measured in terms of tax revenues. Let  $MWP_{EB}^{h,i} = -V_E^{h,i}/V_B^{h,i}$  denote the marginal willingness to pay by ability-type  $i$  in country  $h$  to avoid environmental damage. In addition, let us use the following short notations;

$$\Psi_d^C = - \sum_i X_d^{C,i} \quad (23)$$

$$\Psi_d^D = F^D(\mathbf{L}^D) - \sum_i X_d^{D,i} \quad (24)$$

$$\begin{aligned} \Delta^h = & -V_B^{h,1} X_d^{h,1} - (1 + \lambda^h) V_B^{h,2} X_d^{h,2} + \lambda^h \hat{V}_B^{h,2} \hat{X}_d^{h,2} \\ & - \sum_i \gamma^h \left( \frac{\partial X_c^{h,i}}{\partial Q_d^h} + P_d \frac{\partial X_d^{h,i}}{\partial Q_d^h} \right) \end{aligned} \quad (25)$$

measuring the import (export) of the dirty good to (from) the country producing the clean (dirty) good as well as the welfare effect associated with an increase in the consumer price of the dirty good in country  $h$ , respectively.

Next, consider Proposition 1;

**Proposition 1** *If the resource allocation is a noncooperative Nash equilibrium, then the shadow price of environmental damage over the shadow price of the public budget constraint is given by*

$$\begin{aligned}\frac{\mu^h}{\gamma^h} = & \sum_i MWP_{EB}^{h,i} - \lambda^{h,*} \left[ \widehat{MWP}_{EB}^{h,2} - MWP_{EB}^{h,1} \right] - \sum_i t_d^h \frac{\partial \tilde{X}_d^{h,i}}{\partial E^h} \\ & - \left( \Psi_d^h + \frac{\Delta^h}{\gamma^h} \right) \left[ \frac{\partial P_d^h}{\partial E^h} + \sum_i MWP_{EB}^{h,i} \frac{\partial P_d^h}{\partial B^{h,i}} \right]\end{aligned}$$

for  $h = C, D$ , where  $\lambda^{h,*} = \lambda^h \hat{V}_B^{h,2} / \gamma$  and  $\tilde{X}_d^{h,i}$  is the compensated domestic demand for the dirty good by agent-type  $i$  in country  $h$ .

Since the formula in Proposition 1 is calculated in the same general way as the corresponding formulas in previous studies, the proof is omitted. The terms in the first row of the formula are equivalent to, and have the same interpretations as, their counterparts in previous studies. The first term in the first row is the sum of the marginal willingness to pay for reduced environmental damage. The second term reflects the self-selection constraint; as such, it provides an incentive for the government to increase (decrease) the marginal value it attaches to reduced environmental damage, if the low-ability type is willing to pay more (less) than the mimicker for a marginal reduction of the environmental damage. Finally, the third term is a tax base effect, which arises because the environmental damage affects the demand for the dirty good. These effects are well known from previous studies and need not be further discussed here. Note, however, that in contrast to previous studies, the '*environmental feedback effect*' is equal to one (although the utility function is not separable). This reflects the fact that, from the point of view of the government in country  $h$ , the aggregate worldwide consumption

of the dirty good is unaffected by a change in  $E^h$  (see the discussion at the end of the previous subsection).

The second row of the formula in Proposition 1 is novel; it is due to producer price effects associated with  $\mathbf{B}^h$  and  $E^h$ . To understand this part of the formula, one should bear in mind that  $\mu^h/\gamma^h$  is calculated by using the first order conditions for  $E^h$  and  $\mathbf{B}^h$ . As a consequence, a change in  $E^h$  will here be accompanied by a simultaneous change in  $B^{h,i}$ , such as to balance the government's budget. This is, in turn, interpretable to mean that  $B^{h,i}$  changes in such a way, that the utility remains unchanged. We may, therefore, rewrite the second row of the formula to read

$$-\left(\Psi_d^h + \frac{\Delta^h}{\gamma^h}\right) \left[ \frac{\partial P_d^h}{\partial E^h} + \sum_i \frac{\partial P_d^h}{\partial B^{h,i}} \frac{\partial B^{h,i}}{\partial E^h} \Big|_{V=\bar{V}} \right] \quad (26)$$

where we have used that  $MWP_{EB}^{h,i} = -V_E^{h,i}/V_B^{h,i}$  is the slope of an indifference curve for ability-type  $i$  in  $(E^h, B^{h,i})$  space. We show in the Appendix that  $\Psi_d^D + \Delta^D/\gamma^D > 0$  and  $\Psi_d^C + \Delta^C/\gamma^C < 0$ . Using equation (26), it is easy to interpret the second row of the formula in the proposition. The term  $\partial P_d^h/\partial E^h$  within the square bracket appears because a change in the environmental damage affects the producer price of the dirty good. However, to maintain budget balance, the government needs to adjust  $B^{h,i}$  (captured by  $\partial B^{h,i}/\partial E^h$ ) which, in turn, affects the producer price ( $\partial P_d^h/\partial B^{h,i}$ ). If the terms within the square bracket sum to a positive number, they will contribute to increased welfare in the country producing the dirty good and reduced in welfare in the country producing the clean good. Therefore, if the sum within the square bracket in equation (26) is positive (negative), then it contributes to decrease (increase) the shadow price in country  $D$  and to increase (decrease) the shadow price in country  $C$ .

### 3.2 Commodity Taxation

Let us now turn to the optimal commodity tax structure. In the Appendix, we derive the following result;

**Proposition 2** *If the resource allocation is a noncooperative Nash equilibrium, the commodity tax on the dirty good is characterized by*

$$t_d^h = \frac{\lambda^h \hat{V}_B^{h,2}}{\gamma^h \Omega^h} \left( X_d^{h,1} - \hat{X}_d^{h,2} \right) - \frac{\Psi_d^h}{\Omega^h (1 + \partial P_d^h / \partial t_d^h)} \left[ \frac{\partial P_d^h}{\partial t_d^h} + \sum_i \frac{\partial P_d^h}{\partial B^{h,i}} X_d^{h,i} \right]$$

for  $h = C, D$ , where  $\Omega^h = \sum_i \partial \tilde{X}_d^{h,i} / \partial Q_d^h$ , while  $\Psi_d^h$  is defined by equations (23) and (24).

The most important consequence of Proposition 2 is that  $\mu^h / \gamma^h$  does not appear as a direct argument in the formula for the commodity tax. The intuition behind this result was presented above; the government in country  $h$  perceives that the commodity tax cannot be used as an instrument to affect the aggregate demand for the dirty good.

The first term on the right hand side reflects a standard result in the optimal tax literature and implies that the commodity tax may be used as an instrument to discourage mimicking. If  $\hat{X}_d^{h,2} - X_d^{h,1} > 0$  ( $< 0$ ), there is an incentive to increase (decrease) the commodity tax in order to make mimicking less attractive.

The second term on the right hand side of the tax formula in Proposition 2 captures a direct effect of  $t_d^h$  via the producer price. Since the demand curves are downward sloping (meaning that  $\Omega^h < 0$ ), and since we were able to show that  $-1 < \partial P_d^h / \partial t_d^h < 0$ , it follows that  $\Psi_d^D / [\Omega^D (1 + \partial P_d^h / \partial t_d^D)] > 0$  and  $\Psi_d^C / [\Omega^C (1 + \partial P_d^h / \partial t_d^C)] < 0$ . As such, the sign of the second term in the tax formula ultimately depends on the sign of the expression within the

square bracket. This expression reflects, in turn, how a change in  $t_d^h$  affects the producer price, if  $\mathbf{B}^h$  is simultaneously adjusted to balance the government's budget. If the direct effect ( $\partial P_d / \partial t_d < 0$ ) dominates the indirect effects ( $\sum_i X_d^{h,i} \partial P_d / \partial B^{h,i}$ ), then the second term on the right hand side contributes to decrease the commodity tax in country  $D$  and increase the commodity tax in country  $C$ . From the perspective of country  $D$ , a lower commodity tax contributes to increase the world market price in this case. The latter implies an increase in the national income and, therefore, higher welfare in country  $D$ . The opposite argument applies to country  $C$ .

### 3.3 Effective Marginal Tax Rates

In the previous subsection, we saw that the commodity tax on the dirty good does not contain any term, which explicitly serves the purpose of correcting for the external effect of environmental damage. However, such a term will influence the income tax structure in the country that produces the dirty good. To see this, we consider the effective marginal tax rates. Define the total tax paid by ability-type  $i$  in country  $h$

$$\tau^h(I^{h,i}) = T^h(I^{h,i}) + \sum_j t_j^h X_j^{h,i} \left( Q_d^h, I^{h,i} - T^{h,i}(I^{h,i}), \frac{I^{h,i}}{w^{h,i}}, E \right) \quad (27)$$

Differentiating with respect to  $I^{h,i}$  gives the effective marginal tax rate

$$\tau_I^{h,i} = T_I^{h,i} + \sum_j t_j^h \left[ \left( 1 - T_I^{h,i} \right) \frac{\partial X_j^{h,i}}{\partial B^{h,i}} + \frac{1}{w^{h,i}} \frac{\partial X_j^{h,i}}{\partial L^{h,i}} \right] \quad (28)$$

Our purpose is to relate the effective marginal tax rates to the marginal value of reduced environmental damage facing each government, as well as to producer price effects. To shorten the notations, let us define

$$\begin{aligned}
\alpha^h &= \lambda^{h,*} \left[ \frac{V_Z^{h,1}}{w^{h,1} V_B^{h,1}} - \frac{\hat{V}_Z^{h,2}}{w^{h,2} \hat{V}_B^{h,2}} (1 - \varepsilon^h) \right] > 0 \\
\beta^h &= -\lambda^{h,*} \frac{\hat{V}_Z^{h,2}}{w^{h,2} \hat{V}_B^{h,2}} \phi_n^h < 0 \\
\eta^{h,i} &= \left( \frac{V_Z^{h,i}}{w^{h,i} V_B^{h,i}} \frac{\partial P_d^h}{\partial B^{h,i}} + \frac{1}{w^{h,i}} \frac{\partial P_d^h}{\partial L^{h,i}} \right) \left( \Psi_d^h + \frac{\Delta^h}{\gamma^h} \right)
\end{aligned}$$

for  $h = C, D$ , where  $\varepsilon^h = n^h \phi_n^h / \phi^h > 0$ . Consider Proposition 3;

**Proposition 3** *If the resource allocation is a noncooperative Nash equilibrium, the effective marginal tax rates are characterized by*

$$\begin{aligned}
\tau_I^{C,1} &= \alpha^C - \eta^{C,1} \\
\tau_I^{C,2} &= \beta^C - \eta^{C,2}
\end{aligned}$$

$$\begin{aligned}
\tau_I^{D,1} &= \alpha^D + \frac{\mu^D}{\gamma^D} \frac{1}{P_d^D} - \eta^{D,1} \\
\tau_I^{D,2} &= \beta^D + \frac{\mu^D}{\gamma^D} \frac{1}{P_d^D} - \eta^{D,2}
\end{aligned}$$

In Proposition 3,  $\alpha^h$  reflects two influences of the self-selection constraint; (i) the mimicker has a flatter indifference curve in  $(B^h, I^h)$  space than the low-ability type, and (ii) the decision variables implicit in the effective marginal tax rate facing the low-ability type,  $(B^{h,1}, L^{h,1})$ , affect the self-selection constraint via the wage ratio. Similarly,  $\beta^h$  captures that the decision variables implicit in the effective marginal tax rate facing the high-ability type,  $(B^{h,2}, L^{h,2})$ , affect the self-selection constraint via the wage ratio. Therefore, in the absence of environmental damage, and if the incentives associated with production specialization were absent, this means that the low-ability type

faces a positive effective marginal tax rate, while the high-ability type faces a negative effective marginal tax rate. This is analogous to the results derived by Stiglitz (1982).

In addition to the conventional effects associated with the self-selection constraint, we would like to emphasize three other aspects of the tax formulas in the Proposition. First, and in contrast to many previous studies, note that none of the formulas reflect demand induced changes of the environmental damage. The intuition is that, although  $\mathbf{B}^h$  and  $\mathbf{L}^h$  affect the domestic demand for the dirty good, the government in country  $h$  perceives that a change in the domestic demand has no effect on the aggregate worldwide demand and, therefore, no effect on the environmental damage.

Second, the marginal income tax rate in the country that produces the dirty good constitutes an instrument by which the government can reduce the environmental damage. This is captured by  $\mu^D / (\gamma^D P_d^D)$  in the expressions for the effective marginal tax rates facing the residents in country  $D$ . If  $\mu^D / \gamma^D > 0$ , which appears to be a reasonable assumption, there is an incentive to increase the effective marginal tax rate. This reduces the hours of work and, therefore, the output of the dirty good. Country  $C$ , on the other hand, behaves as if it has no instrument available by which to influence the environmental damage.

Third, the producer price effects also show up in the context of the effective marginal tax rates, where  $\eta^{h,i}$  captures how  $B^{h,i}$  and  $L^{h,i}$  affect the world market producer price. These terms are analogous to the producer price effects discussed in the context of the commodity tax formulas in Proposition 2.

## 4 Summary and Discussion

In this paper, we characterize the optimal income and commodity tax structure in a two-country economy, where one of the countries produces a clean good, while the other produces a dirty good. The dirty good is assumed to be consumed in both countries, and the aggregate demand for the dirty good causes environmental damage. As a consequence, the economy is characterized by a transboundary environmental problem.

The results show that, if each country chooses its policy in isolation, the commodity tax on the dirty good does not contain any term that directly serves to correct for the external effect associated with environmental damage. For a given level of production, each national government acts as if a reduction in the domestic demand for the dirty good leads to a corresponding increase in consumption elsewhere. Therefore each country behaves as if its commodity tax structure does not affect the aggregate worldwide demand for the dirty good. In fact, the country that produces dirty good may have incentives to choose a relatively low commodity tax; this reduces the producer price of the dirty good which, in turn, increases exports and domestic welfare. The results also imply that the country producing the dirty good may internalize part of the domestic external effect by choosing a higher marginal income tax rate than it would otherwise have done. The intuition is that a higher marginal income tax rate reduces the labor supply and, therefore, the production of the dirty good.

In order to highlight important mechanisms characterizing open economies with production specialization, we have taken the produciton specialization argument to its extreme point, in the sense that each country only produces one good. A more general approach would be to assume that both coun-

tries produce, say, the clean good. This extension would imply that the production of the country that produces the dirty good consists of two sectors, where the output in each sector is a function of the producer price. Within such a framework, a commodity tax on the dirty good may indirectly affect the environmental externality, since the change in the producer price caused by an increase in the commodity tax influences the distribution of resources between the two production sectors. Another possible extension would be to allow for migration between countries. Introducing migration implies that the country not producing the dirty good may, nevertheless, be able to influence the output of the dirty good via the effects of tax policy on migration.

Although our model is highly stylized, the results discussed above have important implications for public policy. One such implication is that countries not producing goods that give rise to environmental damage may have no obvious instrument by which to directly affect the environmental damage. Such countries are likely to benefit from supranational environmental agreements. Another is that countries producing goods that give rise to environmental damage have incentives to internalize, at least in part, the corresponding domestic welfare effects. As such, they are less likely to gain from supranational agreements than countries not producing the dirty goods.

## 5 Appendix

The first order conditions for government  $D$  are written (where the superindex  $D$  has been suppressed)

$$\begin{aligned}
0 &= -V_Z^1 + \lambda \hat{V}_Z^2 \left( \phi + L^1 \frac{\partial \phi}{\partial L^1} \right) + \gamma \left( w^1 - \frac{\partial X_c^1}{\partial L^1} - P_d \frac{\partial X_d^1}{\partial L^1} \right) \\
&\quad + \gamma \left( \Psi_d + \frac{\Delta}{\gamma} \right) \frac{\partial P_d}{\partial L^1} - \mu \frac{\partial F}{\partial L^1} \tag{A1}
\end{aligned}$$

$$0 = V_B^1 - \lambda \hat{V}_B^2 - \gamma \left( \frac{\partial X_c^1}{\partial B^1} + P_d \frac{\partial X_d^1}{\partial B^1} \right) + \gamma \left( \Psi_d + \frac{\Delta}{\gamma} \right) \frac{\partial P_d}{\partial B^1} \tag{A2}$$

$$\begin{aligned}
0 &= -(1 + \lambda) V_Z^2 + \lambda \hat{V}_Z^2 L^1 \frac{\partial \phi}{\partial L^2} + \gamma \left( w^2 - \frac{\partial X_c^2}{\partial L^2} - P_d \frac{\partial X_d^2}{\partial L^2} \right) \\
&\quad + \gamma \left( \Psi_d + \frac{\Delta}{\gamma} \right) \frac{\partial P_d}{\partial L^2} - \mu \frac{\partial F}{\partial L^2} \tag{A3}
\end{aligned}$$

$$0 = (1 + \lambda) V_B^2 - \gamma \left( \frac{\partial X_c^2}{\partial B^2} + P_d \frac{\partial X_d^2}{\partial B^2} \right) + \gamma \left( \Psi_d + \frac{\Delta}{\gamma} \right) \frac{\partial P_d}{\partial B^2} \tag{A4}$$

$$0 = \Delta \left( 1 + \frac{\partial P_d}{\partial t_d} \right) + \gamma \Psi_d \frac{\partial P_d}{\partial t_d} \tag{A5}$$

$$\begin{aligned}
0 &= V_E^1 + (1 + \lambda) V_E^2 - \lambda \hat{V}_E^2 - \gamma \sum_i \left( \frac{\partial X_c^i}{\partial E} + P_d \frac{\partial X_d^i}{\partial E} \right) \\
&\quad + \gamma \left( \Psi_d + \frac{\Delta}{\gamma} \right) \frac{\partial P_d}{\partial E} + \mu \tag{A6}
\end{aligned}$$

The first order conditions for government  $C$  are analogous, except that the terms  $\mu(\partial F/\partial L^1)$  and  $\mu(\partial F/\partial L^2)$  in equations (A1) and (A3) disappear. Note also that equation (A.5) implies  $\Psi_d^D + \Delta^D/\gamma^D > 0$  and  $\Psi_d^C + \Delta^C/\gamma^C < 0$ .

*Proof of Proposition 2;*

Start by differentiating the consumer's budget constraint in equation (2) with respect to  $Q_d$ ;

$$\frac{\partial X_c^i}{\partial Q_d} + P_d \frac{\partial X_d^i}{\partial Q_d} = -t_d \frac{\partial X_d^i}{\partial Q_d} - X_d^i \tag{A7}$$

Next, using equations (25), (A2), (A4) and (A7), we have

$$V_B^1 = \frac{A_1}{(1 - X_d^1 \frac{\partial P_d}{\partial B^1})} + (1 + \lambda) V_B^2 \frac{X_d^2 \frac{\partial P_d}{\partial B^1}}{(1 - X_d^1 \frac{\partial P_d}{\partial B^1})} \quad (\text{A8})$$

$$(1 + \lambda) V_B^2 = \frac{A_2}{(1 - X_d^2 \frac{\partial P_d}{\partial B^2})} + V_B^1 \frac{X_d^1 \frac{\partial P_d}{\partial B^2}}{(1 - X_d^2 \frac{\partial P_d}{\partial B^2})} \quad (\text{A9})$$

where

$$\begin{aligned} A_1 &= \lambda \hat{V}_B^2 \left( 1 - \hat{X}_d^2 \frac{\partial P_d}{\partial B^1} \right) + \gamma \left( \frac{\partial X_c^1}{\partial B^1} + P_d \frac{\partial X_d^1}{\partial B^1} \right) \\ &\quad - \gamma \Psi_d \frac{\partial P_d}{\partial B^1} - \sum_i \gamma \left( t_d \frac{\partial X_d^i}{\partial Q_d} + X_d^i \right) \frac{\partial P_d}{\partial B^1} \\ A_2 &= -\lambda \hat{V}_B^2 \hat{X}_d^2 \frac{\partial P_d}{\partial B^2} + \gamma \left( \frac{\partial X_c^2}{\partial B^2} + P_d \frac{\partial X_d^2}{\partial B^2} \right) \\ &\quad - \gamma \Psi_d \frac{\partial P_d}{\partial B^2} - \sum_i \gamma \left( t_d \frac{\partial X_d^i}{\partial Q_d} + X_d^i \right) \frac{\partial P_d}{\partial B^2} \end{aligned}$$

Note that the consumer's budget constraint implies

$$\frac{\partial X_c^i}{\partial B^i} + P_d \frac{\partial X_d^i}{\partial B^i} = 1 - t_d \frac{\partial X_d^i}{\partial B^i} \quad (\text{A10})$$

Substituting equations (A8) and (A9) into equation (A5) and then using equations (A7) and (A10), we can derive

$$\begin{aligned} 0 &= t_d \sum_i \frac{\partial \tilde{X}_d^i}{\partial Q_d} \left( 1 + \frac{\partial P_d}{\partial t_d} \right) + \Psi_d \left( \frac{\partial P_d}{\partial t_d} + \frac{\partial P_d}{\partial B^1} X_d^1 + \frac{\partial P_d}{\partial B^2} X_d^2 \right) \\ &\quad + \frac{\lambda \hat{V}_B^2}{\gamma} \left[ \hat{X}_d^2 - X_d^1 \right] \left( 1 + \frac{\partial P_d}{\partial t_d} \right) \end{aligned} \quad (\text{A11})$$

Solving for  $t_d$  gives the tax formula in Proposition 2. ■

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