International Cooperation over Green Taxes: On the Impossibility of Achieving a Probability-One Gain*

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Abstract

This paper concerns international coordination of environmental taxation. The main purpose is to study the global welfare effects that will arise, if there is an agreement between countries to slightly increase their emission taxes. We show that even if each individual country has chosen its prereform emission tax to be 'too low' in comparison with the marginal cost of the environmental damage caused by that particular country's emissions, implementation of the agreement will not necessarily increase the welfare level.

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1 Introduction

It is a conventional wisdom that the overall - or global - welfare level will increase, if countries cooperate instead of forming their environmental policies in isolation. The reason is that the polluting behavior of agents in one country affects the wellbeing of consumers in other countries. In the absence of cooperation, part of the external effects of environmental damage will remain uninternalized, even if all other policies are designed in an optimal way from society's point of view.

In practice, however, 'cooperation' is not likely to mean the implementation of a first best cooperative equilibrium. It is more realistic to assume that the countries agree upon smaller projects[1], the purposes of which are to (slightly) improve the resource allocation in comparison with the initial equilibrium. This paper analyzes the welfare consequences of an agreement between countries to increase their emission taxes. Contrary to most previous studies on 'international environmental policy'[2], the alternative to cooperation will not necessarily be the outcome of a noncooperative Nash-game between the countries. Such Nash-games typically imply that the agents in each country behave as if the resource allocation is decided upon by a 'national social planner', who treats the outcome of choices made by other countries as exogenous. As a consequence, any externalities that would otherwise arise from the release of emissions by domestic producers will become fully internalized, whereas the externalities caused by the interaction between countries remain uninternalized. Our analysis treats the Nash equilibrium as one out of many possible prereform equilibria.

[1] The Rio (1992) and Kyoto (1997) agreements to reduce the release of CO_2 emissions are examples of such cooperative projects. The CO_2 tax constitutes an important instrument for implementation of these agreements.

[2] See e.g. Mäler (1989), Barrett (1990, 1994), Carraro and Siniscalco (1993), Cesar (1994), Tahvonen (1994, 1995) and Mäler and de Zeeuw (1995).

We shall not address the conditions under which coalitions are likely to form. The main purpose is, instead, to study the global welfare consequences that will arise, if the countries agree to slightly increase their emission taxes. The analysis will be conducted in a dynamic general equilibrium model, in which the 'global welfare level' is measured by a Benthamite welfare function defined over the countries involved.

We also compare the results with those from two of our previous studies; Aronsson and Löfgren (1999a, 1999b). The first of these earlier studies addresses, among other things, the welfare effect of higher emission taxation in the context of a 'one-country' economy, whereas the second analyzes the welfare effect of an agreement between countries to increase their emission taxes when the prereform equilibrium is represented by the outcome of a noncooperative Nash-game between individual countries. The comparison between a one-country economy and a multi-country economy is particularly interesting from the point of view of environmental taxation. In the context of a one-country economy, and if the prereform emission tax is 'too low' in comparison with the marginal social cost of the environmental damage caused by emissions, one can show that an increase in the emission tax increases the welfare level. As will be made clear below, this result does not in general carry over to a multi-country economy: even if the prereform emission tax in each individual country falls short of the marginal social cost of the environmental damage caused by that particular country's emissions, implementation of the agreement to increase the emission taxes does not necessarily improve the global welfare level.

The outline of the paper is as follows. In Section 2, we will present the model as well as derive the noncooperative Nash equilibrium in open loop form and the cooperative equilibrium. Even if these two equilibrium concepts are not the basis for the analysis to follow, they constitute important reference cases by which to compare the market equilibrium. Section 3 introduces the market equilibrium and contains the main results. Section 4 concludes the paper.

2 A Reference Model

We begin this section with a presentation of the model, which is based on Aronsson and Löfgren (1999b). We will then briefly characterize the equilibria resulting from a noncooperative Nash-game in open loop form between the countries and from full cooperation, respectively. Since the number of countries is not important (as long as there is more than one country), we simplify the analysis by studying a two-country economy. The population in each country is constant and normalized to one.

2.1 The Model

The instantaneous utility function facing the consumer in country i, $i = 1, 2$, takes the form

$$u_i(t) = u_i(c_i(t), z_i(t))$$

where $c_i(t)$ is consumption and $z_i(t)$ an indicator of environmental quality at time t. The instantaneous utility function is increasing, twice continuously differentiable and strictly concave in its arguments. If we denote the part of the stock of pollution generated by production in country i by x_i, the indicator of environmental quality in country i is defined by the concave function

$$z_i(t) = z_i(x_1(t), x_2(t))$$

where $\partial z_i / \partial x_1 < 0$ and $\partial z_i / \partial x_2 < 0$ for all x_1, x_2.

Output is produced by labor (normalized to one), physical capital and emissions (through the use of energy input). Net output is determined by the production function

$$y_i(t) = f_i(k_i(t), g_i(t))$$

where k_i is the capital stock per unit of labor and g_i is energy per unit of labor. We assume that the function $f_i(\cdot)$ is nondecreasing in g_i, twice continuously differentiable and strictly concave. Since y_i measures net output, depreciation of physical capital can make the marginal product of capital negative, provided the physical capital stock is large enough. The stock of physical capital accumulates according to

$$\dot{k}_i(t) = f_i(k_i(t), g_i(t)) - c_i(t) \tag{1}$$

The stock of pollution accumulates through the release of emissions. In the model, these originate from the production of energy. To simplify the analysis, we will disregard the process of producing energy and assume that emissions in country i at t are equal to $g_i(t)$, which means that the differential equation for $x_i(t)$ is written

$$\dot{x}_i(t) = g_i(t) - \gamma x_i(t) \tag{2}$$

where γ is the rate of depreciation.

2.2 The Nash Noncooperative Open Loop Solution

It is well known that differential games are very difficult to solve analytically, and that an equilibrium solution may not exist[3]. However, given that a solution does exist, it turns out that envelope properties of the value function

[3] Explicit solutions usually require a set of simplifying assumptions; see e.g. Lancaster (1973), Hoel (1978), Clark (1980), Levhari and Mirman (1980), Dockner et al. (1985) and Tahvonen (1994). In a more general setting, however, very few insights emerge (even in terms of qualitative statements). One of the most comprehensive statements of the theory has been provided by Basar and Olsder (1982).

enable us to derive a set of specific results relevant for cost benefit analysis[4]. To be able to carry out the welfare analysis, suppose to begin with that the resource allocation in each country is decided upon by a planner, who takes the path for the part of the stock of pollution created by the other country as exogenous.

For country i, the planner chooses $c_i(t)$ and $g_i(t)$ to maximize

$$U_i(0) = \int_0^\infty u_i(c_i(t), z_i(t))e^{-\theta t}dt$$

subject to the equations of motion for k_i and x_i, initial conditions $k_i(0) = k_{i0}$ and $x_i(0) = x_{i0}$ and terminal conditions $\lim_{t\to\infty} k_i(t) \geq 0$ and $\lim_{t\to\infty} x_i(t) \geq 0$. The parameter θ represents the rate of time preference, which is assumed to be the same in both countries.

The present value Hamiltonian is written

$$H_i(t) = u_i(c_i(t), z_i(t))e^{-\theta t} + \lambda_i(t)\dot{k}_i(t) + \mu_i(t)\dot{x}_i(t) \tag{3}$$

where λ_i and μ_i are present value shadow prices in terms of utility. In addition to equations (1) and (2), as well as to the initial and terminal conditions, the necessary conditions are (neglecting the time indicator)[5]

$$\frac{\partial u_i(c_i, z_i)e^{-\theta t}}{\partial c_i} - \lambda_i = 0 \tag{4}$$

$$\lambda_i \frac{\partial f_i(k_i, g_i)}{\partial g_i} + \mu_i = 0 \tag{5}$$

[4] See also Löfgren (1999), who conducts cost benefit analyses in the context of Nash and Stackelberg differential 'fish games' under open loop and feedback loop.

[5] The transversality conditions are necessary provided that certain growth conditions are fulfilled. These growth conditions serve as upper bounds on the influence of the state variables on the functions involved. For further details, the reader is referred to Seierstad and Sydsaeter (1987, Theorem 16, Chapter 3).

$$\dot{\lambda}_i = -\lambda_i \frac{\partial f_i(k_i, g_i)}{\partial k_i} \tag{6}$$

$$\dot{\mu}_i = -\frac{\partial u_i(c_i, z_i)e^{-\theta t}}{\partial z_i} \frac{\partial z_i}{\partial x_i} + \mu_i \gamma \tag{7}$$

$$\lim_{t \to \infty} \lambda_i \geq 0 \ (= 0 \text{ if } \lim_{t \to \infty} k_i > 0) \tag{8}$$

$$\lim_{t \to \infty} \mu_i \geq 0 \ (= 0 \text{ if } \lim_{t \to \infty} x_i > 0) \tag{9}$$

Now, let

$$\Lambda_i^n(t) = (c_i^n(t), g_i^n(t)), \ \forall \ t$$

solve planner i's optimization problem. We define $(\Lambda_1^n(t), \Lambda_2^n(t))$ for $t \in [0, \infty)$ to be a Nash equilibrium, if
(i) $\{\Lambda_1^n(t)\}_0^\infty$ solves the decision problem of country 1 conditional on $\Lambda_2(t) = \Lambda_2^n(t)$ for all t, and
(ii) $\{\Lambda_2^n(t)\}_0^\infty$ solves the decision problem of country 2 conditional on $\Lambda_1(t) = \Lambda_1^n(t)$ for all t.
The superindex "n" will be used throughout the paper to denote the noncooperative Nash equilibrium in open loop form.

Note that the Nash equilibrium concept only internalizes part of the welfare effect caused by emissions. Equation (7) implies that the shadow price of pollution relevant for country i only reflects the utility effects of pollution facing the consumer in country i, whereas the utility effect relevant for the consumer in the other country is neglected. According to equation (5), the marginal product of emissions in the noncooperative equilibrium, $\partial f_i(k_i^n, g_i^n)/\partial g_i$, is equal to $-\mu_i^n/\lambda_i^n$, which is the shadow price of x_i in real terms. In the context of a market economy, $-\mu_i^n/\lambda_i^n$ is also interpretable as an emission tax designed to make the market economy reproduce the noncooperative equilibrium.

2.3 The Cooperative Solution

To derive the cooperative solution, where the external effects are fully internalized at the global level, suppose to begin with that a global planner maximizes the sum[6] of the countries' utility functions, $U_1(0)$ and $U_2(0)$, subject to equations of motion for the state variables (k_1, k_2, x_1 and x_2) as well as to the initial and terminal conditions defined above.

Among the necessary conditions, we find[7]

$$\frac{\partial u_i(c_i, z_i)e^{-\theta t}}{\partial c_i} - \lambda_i = 0 \tag{10}$$

$$\lambda_i \frac{\partial f_i(k_i, g_i)}{\partial g_i} + \mu_i = 0 \tag{11}$$

$$\dot{\lambda}_i = -\lambda_i \frac{\partial f_i(k_i, g_i)}{\partial k_i} \tag{12}$$

$$\dot{\mu}_i = -\frac{\partial u_i(c_i, z_i)e^{-\theta t}}{\partial z_i}\frac{\partial z_i}{\partial x_i} - \frac{\partial u_j(c_j, z_j)e^{-\theta t}}{\partial z_j}\frac{\partial z_j}{\partial x_i} + \mu_i \gamma \tag{13}$$

$$\lim_{t\to\infty} \lambda_i \geq 0 \ (= 0 \text{ if } \lim_{t\to\infty} k_i > 0) \tag{14}$$

$$\lim_{t\to\infty} \mu_i \geq 0 \ (= 0 \text{ if } \lim_{t\to\infty} x_i > 0) \tag{15}$$

for $i = 1, 2$ and $i \neq j$. Let

$$\Lambda_i^*(t) = (c_i^*(t), g_i^*(t)), \ \forall \ t$$

for $i = 1, 2$ solve the planner's optimization problem, where the superindex "*" is used to denote the cooperative equilibrium.

[6]This assumption is made to preserve simplicity, since we shall be concerned with efficiency aspects of green tax reforms. Maximizing a more general welfare function would not change anything essential.

[7]See footnote 5.

By comparing these necessary conditions with those of the previous subsection, it is clear that the only formal difference refers to the equation of motion for the shadow price of pollution. In the cooperative equilibrium, there are no remaining external effects, since the shadow price of pollution facing country i will reflect all utility effects caused by that country's polluting behavior. In the context of a market economy, we would interpret $-\mu_i^*/\lambda_i^*$ as an emission tax designed to make the market economy replicate the cooperative equilibrium.

3 The Imperfect Market Economy

It is convenient to begin with a brief description of the outcome in the controlled market economy. Suppose an emission tax, $\tau_i^0(t)$, is imposed on the firm in each country, and that the tax revenues, $\tau_i^0(t)g_i(t)$, are given to the consumer in the form of a lump-sum transfer. To take a short cut to the decentralized market equilibrium, note that the main difference between a social planner problem and a market economy is that x_i is not treated as a state variable by the agents in a decentralized setting. It is, instead, a side effect of the firms' production decisions, and its path is exogenous to the consumer. The optimization problems facing the representative consumer and firm in each country are presented in the Appendix. We shall here concentrate on the equilibrium outcome of their behavior.

3.1 The Conditional Equilibrium

In addition to the initial condition for the physical capital stock and the transversality condition for the shadow price of physical capital, the necessary conditions are;

$$\frac{\partial u_i(c_i^0, z_i^0)e^{-\theta t}}{\partial c_i} - \lambda_i^0 = 0 \tag{16}$$

$$\frac{\partial f_i(k_i^0, g_i^0)}{\partial g_i} - \tau_i^0 = 0 \tag{17}$$

$$\dot{\lambda}_i^0 = -\lambda_i^0 \frac{\partial f_i(k_i^0, g_i^0)}{\partial k_i} \tag{18}$$

$$\dot{k}_i^0 = f(k_i^0, g_i^0) - c_i^0 \tag{19}$$

for $i = 1, 2$, and $t \in [0, \infty)$, where the time indicator has been dropped for notational convenience, and the superindex "0" is used to denote the market equilibrium. Note also that these conditions are general equilibrium conditions, i.e. they are derived by combining the necessary conditions for the consumer and the firm. We shall refer to equations (16)-(19) as the conditional equilibrium, since they originate from the assumption that the private agents in each country optimize conditional on the emission tax path $\{\tau_i^0(t)\}_0^\infty$.

By comparing the conditional equilibrium - which is the outcome of a market economy - with the two equilibrium concepts discussed in Section 2, it is easy to see that the latter two equilibria are special cases of the more general conditional equilibrium. Formally[8]

Observation 1: *(i) If $\tau_i^0(t) = \tau_i^n(t) = -\mu_i^n(t)/\lambda_i^n(t)$, $\forall$ t and $i = 1, 2$, the decentralized economies will reproduce the noncooperative Nash-equilibrium in open loop form. (ii) If $\tau_i^0(t) = \tau_i^*(t) = -\mu_i^*(t)/\lambda_i^*(t)$, $\forall$ t and $i = 1, 2$, the decentralized economies will replicate the cooperative equilibrium.*

We shall refer to $\tau_i^n(t)$ as the "noncooperative" Pigouvian tax and $\tau_i^*(t)$ as the "full" Pigouvian tax for country i. To emphasize the difference between the two Pigouvian related emission tax paths even further, let us solve equa-

[8] See Aronsson and Löfgren (1999b) and van der Ploeg and de Zeeuw (1992).

tions (7) and (13), respectively, subject to the transversality condition. This enables us to derive[9]

$$\tau_i^n(t) = -\{\int_t^\infty \frac{\partial u_i^n(s)e^{-\theta s}}{\partial z_i}\frac{\partial z_i^n(s)}{\partial x_i}e^{-\gamma(s-t)}ds\} \,/\, \lambda_i^n(t)$$

$$\tau_i^*(t) = -\{\int_t^\infty \left[\frac{\partial u_i^*(s)}{\partial z_i}\frac{\partial z_i^*(s)}{\partial x_i} + \frac{\partial u_j^*(s)}{\partial z_j}\frac{\partial z_j^*(s)}{\partial x_i}\right] e^{-\theta s}e^{-\gamma(s-t)}ds\} \,/\, \lambda_i^*(t)$$

Clearly, $\tau_i^n(t)$ only takes into account that pollution in country i affects the utility of the consumer in country i (a consequence of the noncooperative solution concept). The latter problem is absent at the global level in the cooperative solution, because $\tau_i^*(t)$, reflects all utility effects of pollution caused by country i.

3.2 Tax Reforms in the Conditional Equilibrium

In general, there is no reason to believe that the conditional equilibrium will coincide with a noncooperative Nash equilibrium if the countries do not coordinate their environmental policies. This is so because the implementation of such an equilibrium would require enormous amounts of information: the policy makers must be able to solve 'national' command optimum problems in order to design noncooperative Pigouvian taxes. In this subsection, we use the conditional equilibrium represented by equations (16)-(19) as a starting point for policy analysis. The purpose is to study what factors determine the welfare consequences that arise, if the countries agree to slightly increase their emission taxes.

The initial tax structure is given by the paths $\{\tau_i^0(t)\}_0^\infty$ for $i = 1, 2$. We want to measure the welfare effects of increasing these emission taxes to $\tau_1^0(t)+\alpha$ and $\tau_2^0(t)+\beta$, respectively, for all t, where α and β are small positive

[9] We assume that the terminal conditions on the stocks of pollution are not binding, in which case we can use $\lim_{t\to\infty}\mu_i^n(t) = 0$ and $\lim_{t\to\infty}\mu_i^*(t) = 0$.

constants. To focus on efficiency aspects of the tax reform, the additional tax revenues in each country are given to the consumer in the form of a lump-sum subsidy.

The value function is written

$$W^0(0;\xi) = \sum_{i=1}^{2} V_i^0(0;\xi) = \int_0^\infty \left[\sum_{i=1}^{2} u(c_i^0(t;\xi), z_i^0(t;\xi))\right] e^{-\theta t} dt \tag{20}$$

where ξ is a parameter vector with α and β as two of its elements, while $V_i^0(0;\xi)$ is the value (or maximized utility) function of country i. The cost benefit rules we are looking for can be obtained by differentiating the value function with respect to α and β, respectively, and evaluating the resulting derivatives at the points where $\alpha = 0$ and $\beta = 0$. Our concern will then be to evaluate the welfare effect, $\partial W^0(0;\xi)/\partial\alpha + \partial W^0(0;\xi)/\partial\beta$, following the implementation of the agreement.

It is straight forward to derive these measures by using the dynamic envelope theorem[10]. We show in the Appendix that the cost benefit rule for α can be written as

$$\begin{aligned} \frac{\partial W^0(0;\xi)}{\partial\alpha} &= \int_0^\infty [\lambda_1^0\tau_1^0 \frac{\partial g_1^0}{\partial\alpha} + \frac{\partial u_1^0 e^{-\theta t}}{\partial z_1}\frac{\partial z_1^0}{\partial x_1}\frac{\partial x_1^0}{\partial\alpha} + \frac{\partial u_1^0 e^{-\theta t}}{\partial z_1}\frac{\partial z_1^0}{\partial x_2}\frac{\partial x_2^0}{\partial\alpha} \\ &\quad + \lambda_2^0\tau_2^0 \frac{\partial g_2^0}{\partial\alpha} + \frac{\partial u_2^0 e^{-\theta t}}{\partial z_2}\frac{\partial z_2^0}{\partial x_1}\frac{\partial x_1^0}{\partial\alpha} + \frac{\partial u_2^0 e^{-\theta t}}{\partial z_2}\frac{\partial z_2^0}{\partial x_2}\frac{\partial x_2^0}{\partial\alpha}] dt \end{aligned} \tag{21}$$

where the parameter vector ξ and the time indicator have been suppressed in order to avoid unnecessary notations.

In general, what causes a small change in taxation to affect the welfare level is that the tax was not optimally chosen prior to the reform. The only preexisting distortions here refer to the possibly suboptimal use of emissions

[10]See Seierstad (1981) and Seierstad and Sydsaeter (1987). A theorem on the differentiability of the value function in infinite horizon optimal control problems, which has been derived by Atle Seierstad, is presented by Aronsson et al. (1997, Appendix to Chapter 4).

in both countries: the emission tax paths $\{\tau_i^0(t)\}_0^\infty$, $i = 1, 2$, do not necessarily reflect the disutility of pollution. Therefore, the global welfare effect of increasing country 1's emission tax arises via responses in g_1^0, g_2^0, x_1^0 and x_2^0, whereas all other behavioral responses will vanish from the welfare measure as a consequence of optimization. Note that the welfare effect of increasing country 2's emission tax, $\partial W^0(0;\xi)/\partial\beta$, is analogous to equation (21): it can be written in terms of the changes in g_1^0, g_2^0, x_1^0 and x_2^0 arising from higher emission taxation in country 2;

$$\begin{aligned}\frac{\partial W^0(0;\xi)}{\partial \beta} &= \int_0^\infty [\lambda_1^0\tau_1^0\frac{\partial g_1^0}{\partial \beta} + \frac{\partial u_1^0 e^{-\theta t}}{\partial z_1}\frac{\partial z_1^0}{\partial x_1}\frac{\partial x_1^0}{\partial \beta} + \frac{\partial u_1^0 e^{-\theta t}}{\partial z_1}\frac{\partial z_1^0}{\partial x_2}\frac{\partial x_2^0}{\partial \beta} \\ &\quad + \lambda_2^0\tau_2^0\frac{\partial g_2^0}{\partial \beta} + \frac{\partial u_2^0 e^{-\theta t}}{\partial z_2}\frac{\partial z_2^0}{\partial x_1}\frac{\partial x_1^0}{\partial \beta} + \frac{\partial u_2^0 e^{-\theta t}}{\partial z_2}\frac{\partial z_2^0}{\partial x_2}\frac{\partial x_2^0}{\partial \beta}]dt\end{aligned} \tag{22}$$

To go further, it will be convenient to write each country's emission tax as if it is a 'biased estimate' of the marginal value of the environmental damage caused by that particular country's emissions. Formally, the emission taxes in the conditional equilibrium will be written as

$$\tau_i^0(t) = -\int_t^\infty [\frac{\partial u_i^0(s)}{\partial z_i}\frac{\partial z_i^0(s)}{\partial x_i} + \frac{\partial u_j^0(s)}{\partial z_j}\frac{\partial z_j^0(s)}{\partial x_i} + b_i(s)]e^{-\theta s}e^{-\gamma(s-t)}ds \;/\; \lambda_i^0(t) \tag{23}$$

for all t, where $i = 1, 2$, and $j \neq i$. The term $b_i(t)$ is the bias relevant for country i at time t, i.e. the magnitude by which the actual emission tax deviates from the correct valuation of the damages to the environment caused by country i.

We can now derive a useful result;

Theorem: *If the emission taxes in the conditional equilibrium take the form of equation (23), the cost benefit rules (21) and (22) can be written as*

$$\begin{aligned}\frac{\partial W^0(0;\xi)}{\partial \alpha} &= -\sum_{i=1}^{2}\int_0^{\infty} b_i(t)\frac{\partial x_i^0(t)}{\partial \alpha}e^{-\theta t}dt \\ \frac{\partial W^0(0;\xi)}{\partial \beta} &= -\sum_{i=1}^{2}\int_0^{\infty} b_i(t)\frac{\partial x_i^0(t)}{\partial \beta}e^{-\theta t}dt\end{aligned}$$

Proof: see the Appendix.

To evaluate the welfare effect of the agreement to increase the emission taxes permanently at time 0, $\partial W^0(0;\xi)/\partial\alpha + \partial W^0(0;\xi)/\partial\beta$, we would require knowledge of b_1 and b_2 for all t, as well as how α and β affect x_1^0 and x_2^0. Clearly, by increasing the emission tax in one of the countries, we would expect the firms in that particular country to reduce their release of emissions, meaning that $\partial g_1^0(t)/\partial\alpha < 0$ and $\partial g_2^0(t)/\partial\beta < 0$, all t. In fact, if it was not possible to reduce emissions via the tax system, the whole discussion about green taxes would be meaningless from the point of view of environmental concern. Therefore, since $x_i^0(t) = x_i(0)e^{-\gamma t} + \int_0^t g_i^0(s)e^{\gamma(s-t)}ds$, we have $\partial x_1^0(t)/\partial\alpha < 0$ and $\partial x_2^0(t)/\partial\beta < 0$, all t. However, it remains unclear how increased emission taxation in country 1 (2) affects x_2^0 (x_1^0), so the signs of the derivatives $\partial x_1^0/\partial\beta$ and $\partial x_2^0/\partial\alpha$ cannot be determined without additional information. As a consequence, even if the emission tax chosen by each country underestimates the marginal damage to the environment caused by that particular country's emissions (i.e. even if $b_1 > 0$ and $b_2 > 0$ for all t), it is not in general possible to sign the welfare effect following implementation of the agreement[11].

To be able to relate the welfare effect to the biases b_1 and b_2, one would need to impose restrictions regarding how the environmental policy under-

[11] It is important to make a distinction between a small cooperative project - which is the concern here - and implementation of the full cooperative equilibrium. If the prereform emission taxes fall short of their full Pigouvian counterparts, a discrete tax increase towards the 'Pigouvian levels' will, of course, increase the welfare level.

taken by each country affects the production decision of the other country. This idea is used in Proposition 1 below[12];

Proposition 1 *Suppose that* $u_i(c_i, z_i) = \phi_i(c_i) + \kappa_i z_i$ *and* $z_i = \rho_i^i x_i + \rho_i^j x_j$ *for* $i = 1, 2$ *and* $j \neq i$*, where* κ_i*,* ρ_i^i *and* ρ_i^j *are constants. Then, if* $b_1(t) > 0$ *and* $b_2(t) > 0$ *for all* t*, we have* $\partial W^0(0;\xi)/\partial\alpha > 0$ *and* $\partial W^0(0;\xi)/\partial\beta > 0$.

The proof rests on the fact that, in the case of additive separability between goods, a change in α (β) will not affect the marginal utility of consumption in country 2 (1). Linearity will then imply that the emission tax in country 1 (2) does not depend on x_2 (x_1). It follows that the "ambiguous parts" of the welfare change measure will vanish.

Although special, the case of additive separability and linearity is interesting, because it provides an intuitive explanation as to when implementation of the agreement will improve the welfare level. Given the form of the utility function suggested by the proposition, a sufficient condition would be that the prereform emission tax paths systematically underestimate the marginal disutility of pollution. However, if we were to relax the assumptions that the instantaneous utilities are additive and linear along the lines of Proposition 1, this result will no longer necessarily apply. We cannot, in general, rule out the possibility that higher emission taxation in, say, country 1 affects x_1^0 and x_2^0 in opposite directions (the same argument applies to higher emission taxation in country 2).

[12] The assumption about functional form of the instantaneous utility function in Proposition 1 may seem overly restrictive. In fact, it is sufficient to assume that $u_1(\cdot)$ is additive and linear in x_2 and vice versa. At the same time, it would be somewhat peculiar to assume that $u_1(\cdot)$ is nonlinear in x_1 and linear in x_2, which motivates the functional form assumption underlying the proposition.

3.3 Some Other Special Cases

The theorem in the previous subsection also provides a generalization of the results derived in previous studies. Most earlier studies on environmental policy have (at least implicitly) assumed that environmental damage is a 'national' problem by using *representative agent models*. In the context of the framework set out above, this assumption would imply $\partial z_i/\partial x_j = 0$ for $i = 1, 2$, and $j \neq i$. As a consequence, $\partial W^0(0;\xi)/\partial\alpha = \partial V_1^0(0;\xi)/\partial\alpha$ and $\partial W^0(0;\xi)/\partial\beta = \partial V_2^0(0;\xi)/\partial\beta$. We can then use the theorem to establish the following special case;

Corollary 1: *If* $u_i(c_i, z_i(x_1, x_2)) = \psi_i(c_i, x_i)$ *for* $i = 1, 2$*, the cost benefit rules for* α *and* β *reduce to read*

$$\begin{aligned}\frac{\partial W^0(0;\xi)}{\partial\alpha} &= \frac{\partial V_1^0(0;\xi)}{\partial\alpha} = -\int_0^\infty b_1(t)\frac{\partial x_1^0(t)}{\partial\alpha}e^{-\theta t}dt \\ \frac{\partial W^0(0;\xi)}{\partial\beta} &= \frac{\partial V_2^0(0;\xi)}{\partial\beta} = -\int_0^\infty b_2(t)\frac{\partial x_2^0(t)}{\partial\beta}e^{-\theta t}dt\end{aligned}$$

This special case was originally derived by Aronsson and Löfgren (1999a) in the context of a one-country economy. It has (at least) two interesting implications; (i) if $b_i(t) > 0$ for all t, it is always welfare improving to increase the emission tax of country i, and (ii) if $0 \leq b_i(t) < -\partial\psi_i(c_i^0(t), x_i^0(t))/\partial x_i(t)$ for all t, then the conditional equilibrium (with suboptimal emission taxes from society's point of view) is welfare superior to the uncontrolled market economy (without emission taxation). The upper bound of the interval for the bias term is simply to assure that $\tau_i^0(t)$ is a tax and not a subsidy.

We mentioned in the introduction that previous studies on international environmental policies are often based on the assumption that the alternative to cooperation is the outcome of a noncooperative Nash-game between 'national social planners'. Aronsson and Löfgren (1999b) analyze the welfare effect of an agreement among countries to increase their emission taxes, when the prereform equilibrium is the outcome of a *noncooperative Nash-game in open loop form*. By comparing equations (7) and (23), it is apparent that such a Nash-game will imply $b_i = -[\partial u_j(\cdot)/\partial z_j][\partial z_j(\cdot)/\partial x_i]$ for $i = 1, 2$, and $j \neq i$ at the equilibrium. The theorem will then imply;

$$\frac{\partial W^n(0;\xi)}{\partial \alpha} = \int_0^\infty [\frac{\partial u_2^n(t)}{\partial z_2}\frac{\partial z_2^n(t)}{\partial x_1}\frac{\partial x_1^n(t)}{\partial \alpha} + \frac{\partial u_1^n(t)}{\partial z_1}\frac{\partial z_1^n(t)}{\partial x_2}\frac{\partial x_2^n(t)}{\partial \alpha}]e^{-\theta t}dt \tag{24}$$

$$\frac{\partial W^n(0;\xi)}{\partial \beta} = \int_0^\infty [\frac{\partial u_1^n(t)}{\partial z_1}\frac{\partial z_1^n(t)}{\partial x_2}\frac{\partial x_2^n(t)}{\partial \beta} + \frac{\partial u_2^n(t)}{\partial z_2}\frac{\partial z_2^n(t)}{\partial x_1}\frac{\partial x_1^n(t)}{\partial \beta}]e^{-\theta t}dt \tag{25}$$

Recall that the noncooperative Nash equilibrium in Section 2 means that any externalities that would otherwise arise from the release of emissions by domestic producers become internalized, whereas the externalities which are due to the interaction among countries remain uninternalized. As a consequence, one would expect the cost benefit rule to reflect interaction effects across countries from changes in their emission taxes. According to equations (24) and (25), this is precisely what happens, since α and β affect $u_1(\cdot)$ via x_2 and $u_2(\cdot)$ via x_1. All other behavioral effects of higher emission taxation vanish from the welfare change measure as a consequence of optimization on a national basis. Nevertheless, even if the prereform equilibrium is the outcome of a noncooperative Nash-game, the welfare effect of higher emission taxation can go in either direction[13].

[13] A similar qualitative result will emerge, if the open loop assumption were to be replaced by a feedback loop assumption. However, as indicated by Jensen and Lockwood (1998), it may be somewhat restrictive to assume differentiability of the value function in a feedback loop Nash equilibrium. They show, within a class of linear-quadratic differential games, that the value function may be discontinuous even if the game itself has a very simple structure. They also provide sufficient conditions for differentiability, which turn out to be related to the conditions for a unique equilibrium.

However, under the conditions of Proposition 1 - with the instantaneous utility function being additive in arguments as well as linear in x_1 and x_2 - equations (24) and (25) will of course imply $\partial W^n(0;\xi)/\partial\alpha > 0$ and $\partial W^n(0;\xi)/\partial\beta > 0$. Another implication will be that the national welfare effect of each reform vanishes, i.e. $\partial V_1^n(0;\xi)/\partial\alpha = 0$ and $\partial V_2^n(0;\xi)/\partial\beta = 0$. The positive global welfare effect originates from the influence of α on country 2 (via the decrease in x_1) and of β on country 1 (via the decrease in x_2). *In other words, if the policy maker in one country does not believe that the other country will stick to the agreement, he/she will have no incentive to increase the emission tax.*

4 Discussion

It is sometimes argued that cooperation between countries with regard to environmental policy is preferable to the case where each individual country forms its policy in isolation. Indeed, if cooperation means implementation of a full cooperative equilibrium, this argument is correct. In practice, however, cooperation is not likely to mean that countries pool their resources in order to implement such an equilibrium concept. It is more realistic to assume that countries agree upon smaller projects (such as e.g. the Rio and/or Kyoto agreements), the purpose of which are to improve the resource allocation in comparison with the initial equilibrium. We have, in this paper, analyzed the welfare consequences of such a project; namely, the welfare effects of an agreement among countries to slightly increase their emission taxes. Independently of what the prereform equilibrium looks like, we found that the implementation of this project need not increase the overall - or global - welfare level.

In comparison with the representative agent models (or one-country economies) which have often been used in previous studies on the effects of green taxes, the multi-country framework provides much more complicated interactions that need to be taken into account. In the context of a one-country economy - and with no distortions other than environmental damage arising from production - one would normally find that welfare increases monotonically with the emission tax as long as this tax falls short of the value of the marginal externality. One of the main insights here is that this result does not in general carry over to a multi-country economy, where the environmental policies undertaken by one country are likely to affect the production decisions in other countries.

This means that, even if all preexisting emission taxes fall short of the value of the externalities created by emissions in each country, and although the cooperative equilibrium is welfare superior to any other equilibrium, a small step towards the cooperative equilibrium does not necessarily increase the global welfare level. Therefore, global externalities are likely to complicate environmental policy considerably and in ways not recognized by previous research.

5 Appendix

The private agents' optimization problems

The consumer in country i chooses consumption, $c_i(t)$, at each instant such as to maximize

$$U_i(0) = \int_0^\infty u_i(c_i(t), z_i(t))e^{-\theta t} dt \tag{A1}$$

subject to

$$\dot{k}_i(t) = \pi_i(t) + r_i(t)k_i(t) + w_i(t) + T_i(t) - c_i(t) \tag{A2}$$

$$k_i(0) = k_{i0} > 0 \tag{A3}$$

$$\lim_{t\to\infty} k_i(t)\exp(-\int_0^t r_i(s)ds) \geq 0 \tag{A4}$$

where w_i is labor income, r_i the interest rate, π_i profit income (the consumer owns the firms) and T_i a lump-sum transfer. The consumer treats z_i, w_i, r_i, π_i and T_i as exogenous during optimization. Equation (A4) is a so called 'No Ponzi Game' condition.

The representative firm treats w_i and r_i as exogenous and behaves as if it chooses $k_i(t)$ and $g_i(t)$ to maximize

$$\pi_i(t) = f_i(k_i(t), g_i(t)) - r_i(t)k_i(t) - \tau_i(t)g_i(t) - w_i(t) \tag{A5}$$

where τ_i is an emission tax.

Finally, the tax revenues from the emission tax are returned lump-sum to the consumer, which means that

$$\tau_i(t)g_i(t) = T_i(t) \tag{A6}$$

The cost benefit rule given by equation (21)

By differentiating equation (20) with respect to α and evaluating the resulting derivative at the point where $\alpha = 0$, we obtain

$$\begin{aligned}\frac{\partial W^0(0;\xi)}{\partial\alpha} = & \int_0^\infty [\frac{\partial u_1^0}{\partial c_1}\frac{\partial c_1^0}{\partial\alpha} + \frac{\partial u_1^0}{\partial z_1}\frac{\partial z_1^0}{\partial x_1}\frac{\partial x_1^0}{\partial\alpha} + \frac{\partial u_1^0}{\partial z_1}\frac{\partial z_1^0}{\partial x_2}\frac{\partial x_2^0}{\partial\alpha} \\ & + \frac{\partial u_2^0}{\partial c_2}\frac{\partial c_2^0}{\partial\alpha} + \frac{\partial u_2^0}{\partial z_2}\frac{\partial z_2^0}{\partial x_1}\frac{\partial x_1^0}{\partial\alpha} + \frac{\partial u_2^0}{\partial z_2}\frac{\partial z_2^0}{\partial x_2}\frac{\partial x_2^0}{\partial\alpha}]e^{-\theta t}dt\end{aligned} \tag{A7}$$

where the time indicator and the vector ξ have been suppressed at the right hand side of equation (A7). To be able to rewrite equation (A7) into the form of equation (21), note first from equation (1) that

$$\frac{\partial^2 k_i^0(t)}{\partial t\partial\alpha} = \frac{\partial f_i^0(t)}{\partial k_i}\frac{\partial k_i^0(t)}{\partial\alpha} + \frac{\partial f_i^0(t)}{\partial g_i}\frac{\partial g_i^0(t)}{\partial\alpha} - \frac{\partial c_i^0(t)}{\partial\alpha} = \frac{\partial^2 k_i^0(t)}{\partial\alpha\partial t} \tag{A8}$$

with $i = 1, 2$, where $f_i^0(\cdot) = f_i(k_i^0, g_i^0)$ and the last equality comes from Young's theorem. Note also that

$$\begin{aligned} \int_0^\infty \lambda_i^0(t) \frac{\partial^2 k_i^0(t)}{\partial t \partial \alpha} dt &= \lambda_i^0(t) \frac{\partial k_i^0(t)}{\partial \alpha} |_0^\infty - \int_0^\infty \dot{\lambda}_i^0(t) \frac{\partial k_i^0(t)}{\partial \alpha} dt \qquad \text{(A9)} \\ &= -\int_0^\infty \dot{\lambda}_i^0(t) \frac{\partial k_i^0(t)}{\partial \alpha} dt \end{aligned}$$

since $k_i(0)$ is fixed and $\lim_{t \to \infty} \lambda_i^0(t) = 0$ is the transversality condition corresponding to the market equilibrium. By solving equation (A8) for $\partial c_i^0 / \partial \alpha$, substituting into equation (A7), and then using equations (18) and (A9), gives equation (21).

Proof of the theorem

With equation (21) at our disposal, let us begin by differentiating equation (2) with respect to α;

$$\frac{\partial^2 x_i^0(t)}{\partial t \partial \alpha} = \frac{\partial g_i^0(t)}{\partial \alpha} - \gamma \frac{\partial x_i^0(t)}{\partial \alpha} = \frac{\partial^2 x_i^0(t)}{\partial \alpha \partial t} \qquad \text{(A10)}$$

By assuming that the emission tax function, $\tau_i^0(t)$, is differentiable with respect to time, and that $\tau_i^0(t)$ approaches a finite number when t goes to infinity, we shall make use of the result;

$$\begin{aligned} \int_0^\infty \lambda_i^0(t) \tau_i^0(t) \frac{\partial^2 x_i^0(t)}{\partial \alpha \partial t} dt &= \lambda_i^0(t) \tau_i^0(t) \frac{\partial x_i^0(t)}{\partial \alpha} |_0^\infty \qquad \text{(A11)} \\ &\quad - \int_0^\infty [\dot{\lambda}_i^0(t) \tau_i^0(t) + \lambda_i^0(t) \dot{\tau}_i^0(t)] \frac{\partial x_i^0(t)}{\partial \alpha} dt \\ &= -\int_0^\infty [\dot{\lambda}_i^0(t) \tau_i^0(t) + \lambda_i^0(t) \dot{\tau}_i^0(t)] \frac{\partial x_i^0(t)}{\partial \alpha} dt \end{aligned}$$

since $x_i(0)$ is fixed. By solving equation (A10) for $\partial g_i^0 / \partial \alpha$, substituting into equation (21), and then using equations (18) and (A11), gives

$$\begin{aligned}
\frac{\partial W^0(0;\xi)}{\partial \alpha} &= \int_0^\infty [\{\lambda_1^0 \tau_1^0 (\frac{\partial f_1^0}{\partial k_1} + \gamma) - \lambda_1^0 \dot{\tau}_1^0 \\
&+ \frac{\partial u_1^0 e^{-\theta t}}{\partial z_1} \frac{\partial z_1^0}{\partial x_1} + \frac{\partial u_2^0 e^{-\theta t}}{\partial z_2} \frac{\partial z_2^0}{\partial x_1}\} \frac{\partial x_1^0}{\partial \alpha} \\
&+ \{\lambda_2^0 \tau_2^0 (\frac{\partial f_2^0}{\partial k_2} + \gamma) - \lambda_2^0 \dot{\tau}_2^0 \\
&+ \frac{\partial u_1^0 e^{-\theta t}}{\partial z_1} \frac{\partial z_1^0}{\partial x_2} + \frac{\partial u_2^0 e^{-\theta t}}{\partial z_2} \frac{\partial z_2^0}{\partial x_2}\} \frac{\partial x_2^0}{\partial \alpha}] dt
\end{aligned} \tag{A12}$$

where the time indicator has been suppressed for notational convenience. Finally, note that equation (23) implies;

$$\begin{aligned}
\dot{\tau}_i^0 &= \{(\partial u_i^0/\partial z_i)(\partial z_i^0/\partial x_i) + (\partial u_j^0/\partial z_j)(\partial z_j^0/\partial x_i) + b_i\} e^{-\theta t}/\lambda_i^0 \\
&+ \tau_i^0 [\frac{\partial f_i^0}{\partial k_i} + \gamma]
\end{aligned} \tag{A13}$$

for $i = 1, 2$ and $j \neq i$, and the time indicator has been suppressed once again. Substituting equation (A13) into equation (A12) gives the cost benefit rule for α in the theorem. The cost benefit rule for β can be derived in a similar way.

6 References

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