Income Distribution and the Optimal Provision of Local Public Goods*

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Abstract

This paper analyzes how the distribution of gross income among residents in a locality affects the local provision of public goods. The analysis is based on a two-type model, where the local residents can be either high income earners or low income earners. Local provision is assumed to reflect the actions of a benthamite planner giving equal weight to the utilities of all inhabitants. In this framework, three factors turn out to be particularly important for how the distribution of private income across local residents affects the local provision of public goods: (i) the local means of redistribution and taxation, (ii) the particular form of the local residents' utility functions and (iii) migration across localities.

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1 Introduction

There is a large literature on the demand for local public goods. Most previous studies have treated this demand as if it is generated by the preferences and income of a single individual, e.g. the median voter or a representative member of the locality[1]. An implication is that a change in the distribution of income among local residents will affect the demand for local public goods, only if it changes the budget constraint of the 'decisive resident'[2]. In particular, as long as the income and tax share of the 'decisive resident' is fixed, changes in the distribution of income among residents does not affect the allocation of local public goods. A similar conclusion emerges from Warr (1983), who analyzed private contributions to a public good in a model where the individuals act as players in a Nash game. Warr found that the distribution of income does not matter for the allocation of the public good, which will be referred to as the 'neutrality property'. This paper serves to question this 'neutrality property' from a theoretical point of view: it extends the analysis of what factors determine the provision of local public goods by focusing on the role of the distribution of income in the locality.

Why is it important to study the distribution of income in the context of local public goods? In 'real world' economies, households are heterogenous, and their characteristics are partly unobserved, which makes it difficult to identify the characteristics of a 'decisive resident'. Local governments also fulfil distributional policy objectives via e.g. redistributive policies or by supplying services which are (at least in part) private, such as social care, part of the educational program and health care services. Therefore, if the local public policies are designed both to meet equality objectives and to provide local public goods, one may expect that changes in the distribution of income affect the allocation of public goods, even if the mean income

[1]See Wildasin (1986) for a review of popular demand models.

[2]This tradition has also served as a basis for empirical analysis. The most influencial empirical study is Bergstrom and Goodman (1973), which is based on a median voter model. See also e.g. Barr and Davis (1966), Borchering and Deacon (1972) and Deacon (1978).

(e.g. the average tax base) remains unchanged. In addition, if changes in the distribution of income causes migration into or out of the locality, it may also influence the allocation of local public goods.

The purpose of this paper is to study how a mean preserving increase in the spread of income in the locality influences the provision of local public goods. The analysis is based on a two-type model, where the local residents can be either high income earners or low income earners[3]. We assume that the local government acts in accordance with a 'benthamite planner' by giving equal weight to the utilities of all local residents. Given this particular form of the social welfare function, at least three factors turn out to be extremely important for how the allocation of local public goods depends on the distribution of income; (i) the local means of redistribution, (ii) the local residents' preferences and (iii) migration across localities.

The outline of the paper is as follows. In Section 2, we begin by formulating a 'benchmark' model, which disregards the possibility of migration into or out of the locality, and where the decision maker have the means to reach a 'perfect' redistribution of private consumption. In this setting, we are able to derive a result, which is analogous to the 'neutrality property' referred to above: a mean preserving increase in the spread of income does not affect the optimal allocation of the public good. However, by relaxing the assumption that the local public sector is able to achieve a perfect redistribution of private consumption among the local residents, the 'neutrality property' will no longer apply. We provide two examples, which imply that the possibility to redistribute private consumption is restricted: one in which the only revenue source is a uniform lump-sum tax, and the other where a proportional income tax is the only source of funds for the local government. In each of these cases, a mean preserving increase in the spread of income changes the optimal provision of the public good, and the direction of this change depends on the degree of risk aversion with respect to private consumption. In Section 3, we extend the model to allow for migration in the sense that the high income earners are (imperfectly) mobile across localities, and the general conclusion here will be that migration offsets the 'neutrality property'. If the population is mobile, a mean

[3]Similar models have been used by e.g. Wildasin (1991) and Boadway et al. (1998) to analyze fiscal externalities among localities in a federation.

preserving increase in the spread of income will affect the optimal provision of the public good, even if the local government has the technical means of achieving a desired redistribution of private consumption. Section 4 concludes the paper.

2 The Benchmark Model - Perfect Redistribution

We shall begin by formulating a benchmark model, which is based on the assumptions that (i) the local public sector can redistribute private consumption perfectly, and (ii) there is no migration across localities. These assumptions will be relaxed later. There are two types of individuals in the locality, l and h, with exogenously given (gross) incomes w^l and w^h such that $w^l < w^h$. The utility function of individual i is given by

$$u^i = u\left(w^i + z^i\right) + v\left(g\right),\ i = l,h$$

where $u(\cdot)$ and $v(\cdot)$ are increasing and strictly concave functions of private consumption, $w^i + z^i$, and a locally provided pure public good, g, respectively. The term z^i is a net transfer payment (positive or negative) from the locality to individual i. Local welfare is given by the utility sum, i.e.

$$W = n^l u^l + n^h u^h$$

where n^l and n^h are, respectively, the number of low income earners and high income earners. The local public budget constraint is written

$$-n^l z^l - n^h z^h - pg = 0$$

and p denotes the unit price of the public good.

Let us assume that the resource allocation is decided upon by a benthamite planner. The benthamite planner chooses z^l, z^h and g to maximize the welfare function subject to the budget constraint. If we denote by λ the Lagrange multiplier associated with the public budget constraint, the optimization problem becomes

$$\max_{\lambda,z^l,z^h,g} \Omega = n^l u^l + n^h u^h - \lambda\left[n^l z^l + n^h z^h + pg\right] \tag{1}$$

The first-order conditions are given by

$$\lambda; \quad -n^l z^l - n^h z^h - pg = 0 \tag{2}$$

$$z^l; \; n^l u_y^l - \lambda n^l = 0 \tag{3}$$

$$z^h; \; n^h u_y^h - \lambda n^h = 0 \tag{4}$$

$$g; \; n^l u_g^l + n^h u_g^h - \lambda p = 0 \tag{5}$$

where a subindex denotes a partial derivative, e.g. $u_y^h = \partial u^h / \partial y^h$, unless we say otherwise. By using the short notations $y^l = w^l + z^l$ and $y^h = w^h + z^h$, we see that equations (3) and (4) together imply $y^l = y^h$, which means that the optimal solution to problem (1) is characterized by perfect redistribution of private consumption. We will return to the implications of perfect redistribution below.

We begin by analyzing how w^l and w^h affect the endogenous variables. The comparative statics with respect to w^l and w^h are given by

$$\partial g / \partial w^i > 0, \, \partial z^i / \partial w^i < 0 \text{ and } \partial z^i / \partial w^k > 0, \text{ for } i, k = l,h \text{ and } i \neq k$$

The intuition is that the higher the income of type i, the more he/she is willing to pay for public goods, and the less transfer payments to type i is needed to meet the equality objectives. Similarly, the higher the income of type i, the higher will be the net transfer payment to type k in order to reach perfect redistribution.

Let us now turn to the main issue of this section: how changes in the distribution of gross income will affect the allocation of the public good. To do this, we analyze the implications of a *mean preserving increase in the spread* of income. The effect on the optimal provision of the public good, following from a mean preserving increase in the spread of income in the locality, can be written in general terms as

$$\frac{\partial g}{\partial w^h}_{\overline{I}=const} = \frac{\partial g}{\partial w^h} + \frac{\partial g}{\partial w^l} \cdot \frac{\partial w^l}{\partial w^h}_{\overline{I}=const}$$

where $\overline{I} = (n^l w^l + n^h w^h)/(n^l + n^h)$. We summarize the main result of the benchmark model as follows:

Proposition 1 *The optimal solution to problem (1) implies that a mean preserving increase in the spread of income in the locality does not affect the provision of the local public good.*

The proof of Proposition 1 follows directly from a more general result in Section 3 and is, therefore, omitted. Proposition 1 gives a result analogous to the 'neutrality property' derived when the local residents act as players in a Nash game and contribute to a public good, which we referred to in the introduction. The intuition here is that, since the tax system contains a sufficient amount of instruments to obtain an equal distribution of private consumption and to achieve an efficient allocation of the public good, a mean preserving increase in the spread of income only causes the benthamite planner to change z^h and z^l to maintain the perfect redistribution. The initial allocation of the public good will then remain optimal.

What if the tax system does not redistribute income perfectly? With imperfect means of redistribution, the distribution of private consumption will be affected by a mean preserving increase in the spread of income, and one would generally expect the local government to respond by changing the provision of the public good. To see this more clearly, let us analyze two special cases of imperfect redistribution, where the only source of funds for the public good is: (i) a uniform lump-sum tax, $z^h = z^l = z$, in which case the possibility to redistribute income is absent, and (ii) a proportional income tax[4], where the net income (or private consumption) is given by $y^i = w^i(1-t)$ for $i = l, h$, and t is the local income tax rate. Let us begin with imperfect redistribution caused by lump-sum taxation:

Proposition 2 *Under a lump-sum tax scheme with imperfect redistribution in the sense $y^l < y^h$, a mean preserving increase in the spread of income increases (decreases) the optimal provision of the public good if*

$$u^l_{yy} - u^h_{yy}$$

is positive (negative). It follows that, if the degree of absolute risk aversion is nonincreasing in private consumption, a mean preserving increase in the spread of income

[4]Proportional income taxes are the main sources of funds for the local governments in the Nordic countries.

decreases the optimal provision of the public good.

Proof: see the Appendix.

Proposition 2 emphasizes the role of redistribution of income: since a uniform lump-sum tax by definition is such that $y^l < y^h$, a mean preserving increase in the spread of income will, in general, affect the provision of local public goods. Note that g may change in either direction. However, by imposing the assumption of nonincreasing absolute risk aversion, it follows that $\left|u^l_{yy}\right| > \left|u^h_{yy}\right|$, in which case it becomes optimal to decrease the provision of the public good. The reason is that, if we reduce the lump-sum tax as a response to additional spread of income, then nonincreasing absolute risk aversion will imply that the increase in the marginal utility of private consumption facing the low income earners is greater than that facing the high income earners, which is desirable given the form of the local social welfare function set out above. Note finally that Proposition 2 applies to any tax/transfer system of lump sum type as long as $y^l < y^h$.

Imperfect redistribution in the context of other tax/transfer schemes may have other implications for public spending than those corresponding to the lump-sum tax. Consider the other special case of imperfect redistribution discussed above with $z^i = tw^i$, where t is a proportional income tax rate set by the locality. We have found:

Proposition 3 *Suppose the public good is financed by a proportional income tax at rate t, which means $y^l < y^h$. It follows that a mean preserving increase in the spread of income increases (decreases) the optimal provision of the public good if*

$$[u^l_{yy}(\cdot)y^l - u^h_{yy}(\cdot)y^h] + [u^l_y(\cdot) - u^h_y(\cdot)]$$

is positive (negative).

Proof: see the Appendix.

The first term in the formula of Proposition 3 is negative, if the degree of relative risk aversion is nonincreasing in private consumption. This is analogous to the condition imposed in Proposition 2 although based on relative, instead of absolute, risk aversion. The second term is positive and provides an unambiguous incentive to increase the provision. The reason is that a high income tax rate makes the distribution of private consumption across agents less sensitive to the distribution of gross income than what would be the case with a low income tax rate. The following result is a direct consequence of Proposition 3:

Corollary: *If the public good is financed by a proportional income tax, and if the degree of relative risk aversion is constant or increasing in private consumption, it follows that a mean preserving increase in the spread of income increases the optimal provision of the public good.*

To see this, let us define $\delta^i = -[u^i_{yy}y^i]/u^i_y > 0$ for $i = l, h$. It follows from Proposition 3 that

$$sign\ \frac{\partial g}{\partial w^h}_{\overline{I}=const} = \ sign\ [(1-\delta^l)u^l_y - (1-\delta^h)u^h_y]$$

which is unambiguously positive if $\delta^h \geq \delta^l$.

3 Migration

The main conclusion from the previous section is that, by relaxing the assumption about perfect redistribution in the benchmark model, the optimal provision of local public goods will be sensitive to the distribution of income in the locality. This section extends the benchmark model by allowing for migration across localities, which is important in the sense that migration affects the local tax base.

To simplify the notations, the or pre-migration population is normalized to equal one for both agent types, i.e. $n^l = n^h = 1$. Low income earners are immobile, and their preferences follow from the benchmark model in the previous section. High

income earners are assumed to be (imperfectly) mobile across localities.[5] If a high income earner in locality i decides to stay, he/she will receive the utility

$$u_i^h = u(y_i^h) + v(g_i) + \theta_i$$

where the subindex is used as an indicator of locality. The parameter $\theta_i \in [\underline{\theta}, \overline{\theta}]$ is interpreted as a 'taste for the home locality' among the high income earners, and is uniformly distributed with distribution function $F(\theta_i)$. If, on the other hand, the high income earner decides to migrate to another locality (here called j), he/she will receive

$$\overline{u}_j^h = u(y_j^h) + v(g_j)$$

Define by $\widetilde{\theta}_i$ the value of θ_i at which a high income earner is indifferent between staying in locality i and moving to locality j, i.e.

$$u(y_i^h) + v(g_i) + \widetilde{\theta}_i = \overline{u}_j^h,$$

It follows that the high income earner stays in locality i if $\theta_i \geq \widetilde{\theta}_i$, and the 'number of stayers' in locality i is $1 - F(\widetilde{\theta}_i)$.

There is also migration into locality i. High income earners in locality j receive the utility

$$u_j^h = u(y_j^h) + v(g_j) + \theta_j$$

if staying in locality j, where $\theta_j \in [\underline{\theta}, \overline{\theta}]$ is a uniformly distributed 'taste for home parameter'. The high income earner will migrate to locality i if $\theta_j < \widetilde{\theta}_j$, where

$$u(y_j^h) + v(g_j) + \widetilde{\theta}_j = \overline{u}_i^h$$

determines $\widetilde{\theta}_j$ (the value of θ_j at which the individual is indifferent between staying in j and moving to i), and $\overline{u}_i^h = u(y_i^h) + v(g_i)$ is the utility received after having migrated to locality i.

[5]The assumption that high income earners are mobile while low income earners are not, is made to reflect the relative mobility of educational groups, which appears to be the common empirical observation. See Eliasson et al. (1998) for an empirical illustration.

Let us now turn to the optimization problem facing the benthamite planner in locality i. The local welfare function can be written as

$$W_i = u_i^l + \int_{\widetilde{\theta}_i}^{\overline{\theta}} [\overline{u}_i^h + \theta_i] dF(\theta_i) + F(\widetilde{\theta}_j)\overline{u}_i^h$$

where $\overline{u}_i^h$ is defined as suggested above. The welfare function can be rewritten in a more convenient form if we collect the terms involving $\overline{u}_i^h$, i.e.

$$W_i = u_i^l + \widehat{n}^h \overline{u}_i^h + \int_{\widetilde{\theta}_i}^{\overline{\theta}} \theta_i dF(\theta_i) \tag{6}$$

where $\widehat{n}_i^h = 1 - F(\widetilde{\theta}_i) + F(\widetilde{\theta}_j)$ is interpretable as the 'relative number of high income earners' in locality i. To be able to focus the analysis on the role of migration, we shall provide the benthamite planner with the same policy instruments as in the benchmark model, which makes perfect redistribution a feasible option from a budget technical point of view. The budget constraint is written as

$$-z_i^l - \widehat{n}_i^h z_i^h - p_i g_i = 0 \tag{7}$$

The optimization problem of the benthamite planner is to maximize the local welfare function given by equation (6) subject to equation (7). Since the analysis only concerns the behavior of locality i, we can suppress the locality indicator and write the benthamite planner's problem as follows:

$$\underset{\lambda, z^l, z^h, g}{Max} \; u^l + \widehat{n}^h \overline{u}^h + \int_{\widetilde{\theta}}^{\overline{\theta}} \theta dF(\theta) - \lambda[z^l + \widehat{n}^h z^h + pg] \tag{8}$$

3.1 A Myopic Local Government

It is instructive to begin by analyzing a myopic local government, which does not recognize that its policy will affect the migratory behavior. As we are about to show, even if this assumption will imply perfect redistribution of private consumption, the distribution of private pre-tax income is, nevertheless, important for the provision of public goods.

A myopic local government means that the planner takes $\widehat{n}^h$ to be exogenous in the optimization, in which case the necessary conditions for problem (8) are equivalent to those of the benchmark model in Section 2, i.e.

$$\lambda;\ -z^l - \widehat{n}^h z^h - pg = 0 \tag{9}$$

$$z^l;\ u_y^l - \lambda = 0 \tag{10}$$

$$z^h;\ \widehat{n}^h[\overline{u}_y^h - \lambda] = 0 \tag{11}$$

$$g;\ (1+\widehat{n}^h)v_g - \lambda p = 0 \tag{12}$$

where $u_y^l = \partial u^l/\partial y^l$, $\overline{u}_y^h = \partial \overline{u}^h/\partial y^h$ and $v_g = \partial v/\partial g$. It follows from equations (10) and (11) that the optimal solution is characterized by perfect redistribution; $y^l = y^h$.

There is one important difference though between this model and the benchmark model set out in Section 2: even if the local government is myopic, the redistributive policy will affect the local tax base and, therefore, the ability to finance the local public goods. Many of the qualitative properties of the benchmark model will, nevertheless, continue to apply here as well. The comparative statics are

$$\partial g/\partial w^i > 0,\ \partial z^i/\partial w^k > 0,\ \partial z^l/\partial w^l < 0, \text{ and } \partial z^h/\partial w^h \gtrless 0 \text{ for } i,k = l,h \text{ and } i \neq k,$$

meaning that the responsiveness of the tax base to migration is not strong enough to fully offset the implications of perfect redistribution. In particular, a decrease of w^l reduces z^h despite the loss of tax base following from lower benefits to the high income earners. However, contrary to what was found in the benchmark model, we cannot show that an increase in w^h necessarily decreases z^h.

How will the optimal allocation of the public good respond to a mean preserving change in the spread of income? The mean income in the locality is given by $\overline{I} = (w^l + \widehat{n}^h w^h)/(1+\widehat{n}^h)$. Note first that

$$\frac{dw^l}{dw^h}_{\overline{I}=const} = \widehat{n}_y^h(\overline{I} - w^h) - \widehat{n}^h$$

where $\widehat{n}_y^h > 0$ is the effect on the net in-migration of high income earners following from an increase in private consumption by high income earners. From this formula, it can be seen that the relationship between w^h and w^l given a constant $\overline{I}$ does not only depend on the relative number of high income earners as in the benchmark model; it also depends on the change in (net) migration following additional spread of income. Since $\widehat{n}_y^h > 0$, the first term on the right hand side is negative, which means that the negative relationship between w^h and w^l, with the mean income held constant, is stronger here than in the benchmark model. In other words, to preserve the mean income of the locality, w^l must decrease more for a given increase in w^h than in the benchmark model because of the migration effect. We are now in the position to explain the main result of this subsection:

Proposition 4 *With a myopic local government, the solution to problem (8) implies that a mean preserving increase in the spread of income decreases the optimal provision of the public good.*

Proof: see the Appendix.

To see the intuition behind the proposition, it is useful to analyze the formula for the response in g;

$$\frac{\partial g}{\partial w^h}_{\overline{I}=const} = \widehat{n}_y^h(\overline{I} - w^h)\frac{\partial g}{\partial w^h} \tag{13}$$

which follows because $\partial g/\partial w^h = \widehat{n}^h \partial g/\partial w^l$ in the myopic equilibrium (see the Appendix). The intuition is that, since migration makes perfect redistribution more costly than in the benchmark model, there are less resources left over for public good allocation. Equation (13) also explains why the public good remains unaffected in the benchmark model, where $\widehat{n}_y^h = 0$.

3.2 The Complete Model

Let us now relax the assumption of a myopic local government. The planner is, instead, assumed to recognize that the policies undertaken by the locality affect migration and use this information when solving the optimization problem. To

simplify the notations as much as possible, note first that $\widetilde{\theta}_i = -\widetilde{\theta}_j$. The assumption that θ_i and θ_j are uniformly distributed on $[\underline{\theta}, \overline{\theta}]$ will then imply

$$\widehat{n}_y^h = \frac{\partial \widehat{n}^h}{\partial y^h} = -2C\frac{\partial \widetilde{\theta}}{\partial y^h} > 0$$

$$\frac{\partial[\int_{\widetilde{\theta}}^{\overline{\theta}} \theta dF(\theta)]}{\partial y^h} = \frac{1}{2}\widehat{n}_y^h \widetilde{\theta}$$

where $C = dF(\widetilde{\theta})/d\theta = \left[\overline{\theta} - \underline{\theta}\right]^{-1}$ is a constant, and $\partial\widetilde{\theta}/\partial y^h = -u_y^h < 0$. It follows that, when $\widehat{n}^h$ is treated as endogenous during optimization, the necessary conditions for problem (8) can be written

$$\lambda;\ -z^l - \widehat{n}^h z^h - pg = 0 \tag{14}$$

$$z^l;\ u_y^l - \lambda = 0 \tag{15}$$

$$z^h;\ \widehat{n}^h[u_y^h - \lambda] + \widehat{n}_y^h[\frac{u^h + \overline{u}_j^h}{2} - \lambda z^h] = 0 \tag{16}$$

$$g;\ (1 + \widehat{n}^h)v_g - \lambda p + \widehat{n}_g^h[\frac{u^h + \overline{u}_j^h}{2} - \lambda z^h] = 0 \tag{17}$$

where $\widehat{n}_g^h = -2C[\partial\widetilde{\theta}/\partial g] > 0$. Equations (14) and (15) are identical to their counterparts in the previous subsection, whereas the first order conditions for z^h and g now contain the 'extra' term $(u^h + \overline{u}_j^h)/2 - \lambda z^h$, which is positive since $z^h < 0$. In equation (16), we can interpret $\widehat{n}_y^h(u^h + \overline{u}_j^h)/2$ as the utility flow from in-migration when z^h increases marginally, while $\widehat{n}_y^h \lambda z^h$ measures the utility value of the flow of tax revenues. The third term on the right hand side of equation (17) can be given a similar interpretation in terms of g. By comparing the complete model set out here with the myopic local government of the previous subsection, we find:

Lemma: *The solution to problem (8), when $\widehat{n}^h$ is treated as endogenous during optimization, implies; (i) imperfect redistribution, $y^l < y^h$, and (ii) the optimal*

provision of the local public good may either exceed, be equal to or fall short of that corresponding to the myopic solution.

The first part of the lemma follows directly from equations (15) and (16). To see the second part, note that the responsiveness of $\widehat{n}^h$ to changes in z^h and g determine the differences between the solution to the complete model and the solution corresponding to the myopic local government. If increases in z^h and g give rise to substantial net in-migration, then the complete model is likely to imply higher z^h and higher g than the myopic model. If, on the other hand, $\widehat{n}^h$ is only weakly responsive to changes in z^h and g, it follows that the solution to the complete model may imply less provision of local public goods than the solution to the myopic version of the model. However, the reader should note that it is not possible to have both z^h and g lower here than in the myopic equilibrium.

Turning to the comparative statics properties with respect to w^h and w^l, the only unambiguous result is $\partial z^l/\partial w^l < 0$, which has the same interpretation as in the benchmark model. If, in addition, the difference in private consumption (or net income) between the two groups is 'sufficiently small'[6], we will find $\partial g/\partial w^l > 0$ and $\partial z^h/\partial w^l > 0$. The intuition is that, if the equilibrium is sufficiently close to perfect redistribution, then the optimal responses to a change of w^l are close to those of the myopic model. However, the comparative statics with respect to w^h remain ambiguous (even if the economy operates close to the myopic equilibrium). The reason is that changes in w^h will not only influence migration indirectly via the public budget constraint (in a way similar to w^l), it will also have a direct effect on the net migration to the locality.

As expected from the comparative statics analysis, a mean preserving increase in the spread of income can change the provision of the local public good in either direction. Formally;

$$\frac{\partial g}{\partial w^h}_{\overline{I}=const} = \frac{\partial g}{w^h} + [\widehat{n}^h_y(\overline{I} - w^h) - \widehat{n}^h]\frac{\partial g}{\partial w^l} \tag{18}$$

There are, at least, two important differences between the equilibrium in the com-

[6]The formal requirement is $u^h_y > \frac{2}{3}u^l_y$.

plete model and the myopic equilibrium discussed in the previous subsection: the complete model neither requires $\partial g/\partial w^l > 0$, unless the difference between y^h and y^l is 'sufficiently small', nor does it require $\partial g/\partial w^h = \widehat{n}^h \partial g/\partial w^l$. With these two observations at our disposal, it is easy to derive the following result;

Proposition 5 *If the private consumption levels of the two groups are sufficiently close in the sense that* $u_y^h \geq \frac{2}{3} u_y^l$, *a mean preserving increase in the spread of income can increase the provision of the public good only if* $\partial g/\partial w^h > \widehat{n}^h \partial g/\partial w^l$.

The first requirement in the proposition means that y^h and y^l are 'sufficiently close' in the sense of assuring that $\partial g/\partial w^l > 0$. As a consequence, we find that $\widehat{n}_y^h(\overline{I} - w^h)\partial g/\partial w^l < 0$ (like in the myopic equilibrium). It will then follow that a mean preserving increase in the spread of income decreases the provision of the public good, unless the allocation of the public good is very sensitive to changes in w^h. If it is not, the economy operates close enough to the myopic equilibrium to assure that Proposition 4 still applies.

To get some further intuition, note that the distribution of 'tastes for home' play an important role in determining the effects of a mean preserving increase in the spread of income. If the taste distribution is heterogenous (in the sense that $\overline{\theta} - \underline{\theta}$ is large), the responsiveness of population to policy is relatively small. Therefore, for a given initial distribution of income, a more heterogenous taste distribution makes redistribution less costly, implying that the decision maker will prefer more redistribution. As a consequence, the more heterogenous the taste distribution, the more likely it is that the private consumption levels of the two groups meet the requirement given in the Proposition above. On the other hand, if tastes are homogenous (i.e. $\overline{\theta} - \underline{\theta}$ is small), redistribution is relatively costly. In this case, the decision maker may not find it worthwhile to redistribute income to meet the requirement of the Proposition. Therefore, the derivative $\partial g/\partial w^l$ may be negative, while $\partial g/\partial w^h$ may still be positive, implying that a mean preserving increase in the spread of income is more likely to increase the public good provision than when the taste distribution is heterogenous.

4 Summary and Conclusions

This paper extends the analysis of local public spending by focusing attention on the importance of the distribution of income among local residents for the optimal provision of local public goods. The analysis is based on a two-type model, where the local residents can be either high income earners or low income earners, and the local government is assumed to act in accordance with a benthamite planner giving equal weight to the utilities of all local residents. The main purpose is to study how a mean preserving increase in the spread of income among the local residents affects the provision of the local public good. We would like to summarize the main results as follows:

- In a 'benchmark' version of the model, which does not allow for migration across localities and where the local decision maker has the means to accomplish perfect redistribution of private consumption, are we able to derive what resembles Warr's (1983) 'neutrality result'. This means that a change in the distribution of income, holding the average income constant, does not affect the optimal provision of local public goods. However, by restricting the means of redistribution or allowing for migratory behavior, we find that the provision of public goods is sensitive to the distribution of income in the locality.

- If, in the absence of migration, the local decision maker is unable to achieve perfect redistribution of private consumption, the results are sensitive to the degree of risk aversion with respect to private consumption. For example, we show that when taxes are lump-sum, the optimal provision of the public good decreases as a response to a mean preserving increase in the spread of income if the degree of absolute risk aversion is a nonincreasing function of private consumption.

- By allowing high income earners to migrate between localities, the 'neutrality result' will no longer hold even if perfect redistribution is a feasible option from the viewpoint of the decision maker. This is most clearly seen when the decision maker is myopic in the sense of not considering the migratory responses following from changes in the provision of the public good or the tax/transfer scheme. In this case, we show that a mean preserving increase in the spread of income unambigously

decreases the optimal provision of the local public good. If the decision maker takes migratory responses into consideration when determinaing the optimal policy, we can no longer sign the effect on public good provision following from a mean preserving spread of income. The reason is that the results become dependent upon how sensitive net migration is to local public policies as well as the difference in gross income between high and low income earners.

To conclude, the main empirical implication of this paper is that the spread of income generally matters for the provision of local public goods. It remains for future research to study whether the spread of income is an empirically important determinant of the expenditures on local public goods.

Appendix

Proof of Proposition 2:

When $z = z^l = z^h$, the first order conditions are given by

$$-nz - pg = 0$$

$$n^l \left(u_y^l - \lambda\right) + n^h \left(u_y^h - \lambda\right) = 0$$

$$nv_g - \lambda p = 0$$

where $n = n^h + n^l$. Differentiating the first-order conditions with respect to w^l and w^h, comparative statics can be found by solving the system

$$\begin{bmatrix} 0 & -n & -p \\ -n & n^l u_{yy}^l + n^h u_{yy}^h & 0 \\ -p & 0 & nv_{gg} \end{bmatrix} \begin{bmatrix} \partial\lambda/\partial w^i \\ \partial z/\partial w^i \\ \partial g/\partial w^i \end{bmatrix} = \begin{bmatrix} 0 \\ -n^i u_{yy}^i \\ 0 \end{bmatrix}, i = l, h.$$

Note that $\partial g/\partial w^i = -pnn^i u_{yy}^i / \left|\overline{H}\right|$, where $\left|\overline{H}\right| > 0$ is the determinant of the bordered Hessian matrix given above. Substituting the derivatives into

$$\frac{\partial g}{\partial w^h}_{\overline{I}=const} = \frac{\partial g}{\partial w^h} + \frac{\partial g}{\partial w^l} \cdot \frac{\partial w^l}{\partial w^h}_{\overline{I}=const}, \tag{A1}$$

and noting that $(\partial w^l/\partial w^h)_{\overline{I}=const} = -n^h/n^l$, we obtain $(\partial g/\partial w^h)_{\overline{I}=const} = (u_{yy}^l - u_{yy}^h)pnn^h/\left|\overline{H}\right|$. It follows that

$$sign\left[(\partial g/\partial w^h)_{\overline{I}=const}\right] = sign\left[u_{yy}^l - u_{yy}^h\right].$$

To relate this result to the degree of absolute risk aversion, rewrite the derivative as

$$(\partial g/\partial w^h)_{\overline{I}=const} = u_y^l a\left[(u_y^h/u_y^l)r^h - r^l\right],$$

where $r^i = -u_{yy}^i/u_y^i, i = l, h$ is the degree of absolute risk aversion, and $a = pnn^h/\left|\overline{H}\right|$ is positive. Since, by imperfect redistribution, $u_y^h/u_y^l < 1$, it follows that $r^h \leqslant r^l$ is sufficient for the public good provision to decrease.■

Proof of Proposition 3:

Under a linear income tax, $t > 0$, the budget constraint of the locality is given by

$$t(n^l w^l + n^h w^h) - pg = 0.$$

Forming the Lagrangian for this problem, the first-order conditions with respect to λ, t, and g are given by

$$t(n^l w^l + n^h w^h) - pg = 0$$
$$-n^l w^l (u_y^l - \lambda) - n^h w^h (u_y^h - \lambda) = 0$$
$$nv_g - \lambda p = 0,$$

where $n = n^h + n^l$. Differentiating the first-order conditions, gives the system

$$\begin{bmatrix} 0 & I & -p \\ I & \Omega_{tt} & 0 \\ -p & 0 & nv_{gg} \end{bmatrix} \begin{bmatrix} \partial\lambda/\partial w^i \\ \partial t/\partial w^i \\ \partial g/\partial w^i \end{bmatrix} = \begin{bmatrix} -tn^i \\ \omega_t^i n^i \\ 0 \end{bmatrix}, i = l, h,$$

where $I = n^l w^l + n^h w^h$, $\Omega_{tt} = n^l \left(w^l\right)^2 u_{yy}^l + n^h \left(w^h\right)^2 u_{yy}^h$, and $\omega_t^i = u_y^i - \lambda + u_{yy}^i y^i$. Solving for $\partial g/\partial w^i$ gives $\partial g/\partial w^i = -pn^i \left(I\omega_t^i + t\Omega_{tt}\right) / \left|\overline{H}\right|$. Finally, applying equation (A1), it is found that $(\partial g/\partial w^h)_{\overline{I}=const} = pIn^h \left[\left(u_y^l - u_y^h\right) + \left(u_{yy}^l y^l - u_{yy}^h y^h\right)\right]$.■

Proof of proposition 4:

Differentiating conditions (9), (10), (11), and (12), results in the system

$$\begin{bmatrix} 0 & -1 & -\Omega_{\lambda z^h} & -\Omega_{\lambda g} \\ -1 & u_{yy}^l & 0 & 0 \\ -\widehat{n}^h & 0 & \widehat{n}^h u_{yy}^h & 0 \\ -p & 0 & \widehat{n}_y^h v_g & \Omega_{gg} \end{bmatrix} \begin{bmatrix} \partial\lambda/\partial w^l & \partial\lambda/\partial w^h \\ \partial z^l/\partial w^l & \partial z^l/\partial w^h \\ \partial z^h/\partial w^l & \partial z^h/\partial w^h \\ \partial g/\partial w^l & \partial g/\partial w^h \end{bmatrix} = \begin{bmatrix} 0 & \widehat{n}_y^h z^h \\ -u_{yy}^l & 0 \\ 0 & -\widehat{n}^h u_{yy}^h \\ 0 & -\widehat{n}_y^h v_g \end{bmatrix},$$

where we naturally assume that $\Omega_{\lambda z^h} = \widehat{n}^h + \widehat{n}_y^h z^h$, the marginal cost of transfers to the high income earners, and $\Omega_{\lambda g} = p + \widehat{n}_g^h z^h$, the marginal cost of the public good, are positive. Finally, $\Omega_{gg} = (1 + \widehat{n}^h) v_{gg} + \widehat{n}_g^h v_g < 0$, where the sign requirement follows from fulfillment of the second order conditions. The public good responses to w^l and $w^h$are $\partial g/\partial w^l = \widehat{n}^h u_{yy}^l A / \left|\overline{H}\right| > 0$, and $\partial g/\partial w^h = \left(\widehat{n}^h\right)^2 u_{yy}^l A / \left|\overline{H}\right| > 0$, where $A = \widehat{n}_y^h v_g - p u_{yy}^h > 0$, and $\left|\overline{H}\right| < 0$. Note that $\partial g/\partial w^h = \widehat{n}^h \partial g/\partial w^l$. Applying equation (A1) and using $(dw^l/dw^h)_{\overline{I}=const} = \widehat{n}_y^h(\overline{I} - w^h) - \widehat{n}^h$, results in $(\partial g/\partial w^h)_{\overline{I}=const} = \widehat{n}^h \partial g/\partial w^l + \left(\widehat{n}_y^h(\overline{I} - w^h) - \widehat{n}^h\right) \partial g/\partial w^l = \widehat{n}_y^h(\overline{I} - w^h) \partial g/\partial w^l$.■

References

Barr, J.L. and O.A. Davis, (1966), An Elementary Political and Economic Theory of the Expenditures of Local Governments, *Southern Economic Journal* **33**, 149-65.

Bergstrom, T.C. and R.P. Goodman, (1973), Private Demands for Public Goods, *American Economic Review* **63**, 280-96.

Boadway, R., M. Marchand, and M. Vigneault, (1998), The conbsequences of Overlapping Tax Bases for Redistribution and Public Spending in a Federation, *Journal of Public Economics* **68**, 453-78.

Borchering, T.E. and R.T. Deacon, (1972), The Demand for Services of Non-Federal Government, *American Economic Review* **62**, 891-901.

Deacon, R.T. (1978) A Demand Model for the Local Public Sector. *Review of Economics and Statistics* **60**, 184-192.

Eliasson, K., U. Lindgren, and O. Westerlund, (1998), Geographical Labour Mobility: Migration or Commuting?, *mimeo*, Department of Economics, Umeå University.

Warr, P., (1983), The Private Provision of the Public Good is Independent of the Distribution of Income, *Economics Letters* **13**, 207-11.

Wildasin, D.E., (1986), *Urban Public Finance*, Harwood Academic Publishers, Chur, Switzerland.

Wildasin, D.E., (1991), Income Redistribution in a Common Labour Market, *American Economic Review* **81**, 757-74.